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EERI
Economics and Econometrics Research Institute
Avenue Louise
1050 Brussels
Belgium

Tel: +32 2271 9482
Fax: +32 2271 9480
www.eeri.eu

Testing of Parameter's Instability in a Balanced Panel: An Application to Real Effective Exchange Rate for SAARC Countries

By

Varun Agiwal, Jitendra Kumar* and Sumit Kumar Sharma

Department of Statistics, Central University of Rajasthan, Bandersindri, Ajmer, India

Email: varunagiwal.stats@gmail.com; vjitendrav@gmail.com; sumitkmaharshi@gmail.com

Abstract

Present paper considers structural break in panel AR(1) model which allows instability in mean, variance and autoregressive coefficient. This model is extension of univariate model proposed by Meligkotsiduo *et al.* (2004) and review of existing panel data time series model considering break studied by Levin *et al.* (2002), Pesaran (2004), Bai (2010), Liu *et al.* (2011), Wachter and Tzavalis (2012). Paper dealt the identification of structural break by comparing the posterior probability of all possible models like break on all three parameters, only two parameters, one parameter and there is no break. A simulation study is carried out to validate the derived theorems. An Empirical analysis on Real Exchange Rate of India and its neighboring countries (SAARC countries including China) are also carried out. The present study is correctly identifying the common break on 1991 which happened due to second gulf war and international debt crisis.

Keywords: Panel autoregressive model, Structural break, Prior and Posterior probability

JEL Classification: C11, C12, C23

***Corresponding to:** Department of Statistics, Central University of Rajasthan, Bandersindri, Ajmer, 305817, India. Tel.: +91-8875755284
E-mail address: vjitendrav@gmail.com

1. Introduction

An economy development of a nation mainly depends upon various factors such as income, interest rate, GDP, exchange rate, unemployment rate, production, inflation etc. Sometimes financial, political, environmental conditions may down the economy growth which can seem in terms of economic crisis, change of a policy, technology change etc. These changes may be analyzed with the help of structural break model in time series as well as economics. The existing research and literature on structural change at various levels had been analyzed in several applications in time series and econometric data such as gross domestic product (Wang and Zivot 2000), real interest rate, real exchange rate and consumer price index (Meligkotsidou 2011) and import-export series (Kumar et al. 2012). Although, several studies had holding single as well as multiple breaks in level/trend/ autoregressive coefficient or/and error variance for making significant inference in univariate and panel data time series models (see Bai and Perron 1998, Wang and Zivot 2000, Shao and Zhang 2010, Kim 2011). In addition, this concept had been equally popular in both classical and Bayesian approach under certain mild assumptions and prior information. However, it is well developed framework for testing and estimating the patterns of structural break model with the help of model selection procedure. For this, structural change can be classified in such a way that one adopt hypothesis testing problem and other takes model selection approach. To known the stationarity of series is very important part of time series and unit root hypothesis is well known concept for this propose. Generally, unit root hypothesis with or without structural break was developed by several researchers with real economical situation, see Zivot and Andrews (1992), Perron (1997), Newbold *et al.* (2001), Kumar *et al.* (2012) and other hypothesis was identified the location and number of break points present in the series. This hypothesis was well developed by Vogelsang (1997), Bai and Perron

(1998). The literature on model selection procedure first introduced in change point by Kim and Maddala (1991), Wang and Zivot (2000) under Bayesian approach which uses the BIC criterion for selecting the best model among the models with different numbers of breaks.

In comparison to vast literature on estimation, testing and identification of structural change for univariate series, there has been a growing literature for panel and multivariate time series in recent years among various authors. Li *et al.* (2015) introduced a penalized principal component estimation procedure for estimation and detection of break points in interactive fixed effect panel data model and applied to an environmental Kuznets curve related for controlling to energy consumption. Detection and position of multiple breaks in multivariate time series had been analyzed by Preuss *et al.* (2015) which introduced new nonparametric procedure referred as MuBreD procedure. MuBreD is based on a comparison of estimated spectral distribution on different segments of the observed times series. Recently, Shin and Hwang (2017) developed panel mean change CUSUM test for detection of break in panel process and illustrated by Asian country stock price indices. Sengupta (2017) analyzed the break testing using likelihood ratio in spatial panel model and proposed break-date estimator to determine the break-location.

This article presents a single structural break point in all coefficients of a panel data setup that includes the analysis for real exchange effective rate of India and its neighboring countries. It is well known under Bayesian approach that posterior odds ratio or Bayes factor easily handle the problem for determining the presence of break point. Keeping in mind to consider those components which mainly change the structure of the series, a general fully structural break panel AR(1) time series model is studied. Posterior probabilities are obtained for testing the structural change hypothesis using the prior assumption. Then, a simulation as well as an empirical study is carried out to explore the presence of break point in REER data set of SAARC summit including China.

2. Structural break model and its sub-models

In this section, describe a fully structural break model as a function of panel AR(1) time series model. Let { y_{it} } be a stochastic process with observed values on cross section unit $i=1, 2, \ldots, n$ and time $t=1, 2, \ldots, T$ under the assumption that time dimension is same for each cross section. This process { y_{it} } is known as balanced panel data time series model and if a break at time point T_B toward change the intercept coefficient from μ_{i1} to μ_{i2}. Then, the model is specified as

$$y_{it} = \begin{cases} \mu_{i1} + u_{it} & \text{for } t = 1,2,3,\ldots,T_B \\ \mu_{i2} + u_{it} & \text{for } t = T_B + 1,\ldots,T \end{cases} \tag{1}$$

where stochastic error term u_{it} follows panel AR(1) model with structure change on autoregressive coefficient and error variance at similar break point *i.e.* T_B and ε_{it} are independently and identical distributed random variable with mean zero and unknown variance.

$$u_{it} = \begin{cases} \rho_1 u_{it-1} + \sigma_1 \varepsilon_{it} & \text{for } t = 1,2,3,\ldots,T_B \\ \rho_2 u_{it-1} + \sigma_2 \varepsilon_{it} & \text{for } t = T_B + 1,\ldots,T \end{cases} \tag{2}$$

Operating equation (2) in equation (1) becomes

$$y_{it} = \begin{cases} \rho_1 y_{it-1} + (1-\rho_1)\mu_{i1} + \sigma_1 \varepsilon_{it} & \text{for } t = 1,2,3,\ldots,T_B \\ \rho_2 y_{it-1} + (1-\rho_2)\mu_{i2} + \sigma_2 \varepsilon_{it} & \text{for } t = T_B + 1,\ldots,T \end{cases} \tag{3}$$

This model is a panel data structural break model having change at T_B in all model parameters *i.e.* intercept, variance and autoregressive coefficient. The initial value y_{i0} for the stochastic process in (3) is assumed to be known. To recognize the impact of structure change in the model, test the break point hypothesis under different circumstances which may be occurring due to instability in model parameters. Proposed panel data structural break model (3) has several well known sub-models which were studied by various researchers, details are listed in appendix table-A1. These sub-models (hypotheses) were analysis in reference to cross-section dependence test, unit root hypothesis, estimating the break points and detecting structural break via model selection criterion under classical approach. However, these models may further analyze the presence of break point from a Bayesian point of view with the help of posterior odds ratio. In present paper, main focus on break point testing in all coefficients under Bayesian perspective which are jointly changing the structure of the series using posterior odds ratio. The proposed hypothesis is also comparing with existing models like break in one and two parameters only.

3. Prior Distribution

Primarily assumption for implementing a Bayesian approach is that consider an unknown quantity of model parameter(s). In general, a functional form is used for parameters which contains some information known as prior information. It gives brief idea about the parameter behavior and definitely provided a better explanation. For our reference (Schotman and van Dijk 1991, Meligkotsidou 2011), following prior distribution is considered for obtaining posterior probability j=1, 2 and i=1, 2,…, n.

$$P(\rho_j) \propto \frac{1}{(1-l_j)}; \quad l_j < \rho_j < 1; l_j > -1 \tag{4}$$

$$P\left(\sigma_j^2\right) = \frac{d_j^{c_j}}{\Gamma c_j}\left(\sigma_j^2\right)^{-c_j-1} \exp\left[-\frac{d_j}{\sigma_j^2}\right]; \quad c_j, d_j > 0 \tag{5}$$

$$P(\mu_{ij}) = \frac{1}{(2\pi)^{\frac{n}{2}} \tau_j^n \sigma_j^n} \exp\left[-\frac{1}{2\tau_j^2 \sigma_j^2} (\mu_{ij} - \vartheta_{ij})^2 \right] \tag{6}$$

4. Posterior Probability

The present study is targeting to explore the time series model with structural break on all coefficients. Model may be written in various forms where break is present at single as well as multiple coefficient(s). Present section derives posterior probability/marginal likelihood with the assist of likelihood function and prior distribution. Here, we are obtaining the posterior probability for proposed model as well as sub-models. Let us define following notations to express the posterior probability:

$$A(\rho_1) = \tau_1^2 T_B (1-\rho_1)^2 + 1$$

$$B(\rho_2) = \tau_2^2 (T - T_B)(1-\rho_2)^2 + 1$$

$$C(\rho_1) = \tau_1^2 (1-\rho_1) \sum_{t=1}^{T_B} (y_{it} - \rho_1 y_{i,t-1}) + \vartheta_{i1}$$

$$D(\rho_2) = \tau_2^2 (1-\rho_2) \sum_{t=T_B+1}^{T} (y_{it} - \rho_2 y_{i,t-1}) + \vartheta_{i2}$$

$$E(\rho_1) = d_1 + \frac{1}{2\tau_1^2} \sum_{i=1}^{n} \left\{ \tau_1^2 \sum_{t=1}^{T_B} (y_{it} - \rho_1 y_{i,t-1})^2 - \frac{[C(\rho_1)]^2}{A(\rho_1)} + \vartheta_{i1}^2 \right\}$$

$$F(\rho_2) = d_2 + \frac{1}{2\tau_2^2} \sum_{i=1}^{n} \left\{ \tau_2^2 \sum_{t=T_B+1}^{T} (y_{it} - \rho_2 y_{i,t-1})^2 - \frac{[D(\rho_2)]^2}{B(\rho_2)} + \vartheta_{i2}^2 \right\}$$

$$G(\rho_1) = \tau^2 T_B (1-\rho_1)^2 + 1$$

$$H(\rho_2) = \tau^2 (T - T_B)(1-\rho_2)^2 + 1$$

$$I(\rho_1) = \tau^2 (1-\rho_1) \sum_{t=1}^{T_B} (y_{it} - \rho_1 y_{i,t-1}) + \vartheta_{i1}$$

$$J(\rho_2) = \tau^2 (1-\rho_2) \sum_{t=T_B+1}^{T} (y_{it} - \rho_2 y_{i,t-1}) + \vartheta_{i2}$$

$$K(\rho_1, \rho_2) = d + \frac{1}{2\tau^2} \sum_{i=1}^{n} \left\{ \tau^2 \sum_{t=1}^{T_B} (y_{it} - \rho_1 y_{i,t-1})^2 + \tau^2 \sum_{t=T_B+1}^{T} (y_{it} - \rho_2 y_{i,t-1})^2 - \frac{[I(\rho_1)]^2}{G(\rho_1)} - \frac{[J(\rho_2)]^2}{H(\rho_2)} + \vartheta_{i1}^2 + \vartheta_{i2}^2 \right\}$$

$$L(\rho)=\tau_1^2 T_B(1-\rho)^2+1$$
$$M(\rho)=\tau_2^2(T-T_B)(1-\rho)^2+1$$
$$N(\rho)=\tau_1^2(1-\rho)\sum_{t=1}^{T_B}\left(y_{it}-\rho y_{i,t-1}\right)+\vartheta_{i1}$$
$$O(\rho)=\tau_2^2(1-\rho)\sum_{t=T_B+1}^{T}\left(y_{it}-\rho y_{i,t-1}\right)+\vartheta_{i2}$$
$$P(\rho)=d_1+\frac{1}{2\tau_1^2}\sum_{i=1}^{n}\left\{\tau_1^2\sum_{t=1}^{T_B}\left(y_{it}-\rho y_{i,t-1}\right)^2+\vartheta_{i1}^2-\frac{[N(\rho)]^2}{L(\rho)}\right\}$$
$$Q(\rho)=d_2+\frac{1}{2\tau_2^2}\sum_{i=1}^{n}\left\{\tau_2^2\sum_{t=T_B+1}^{T}\left(y_{it}-\rho y_{i,t-1}\right)^2+\vartheta_{i2}^2-\frac{[O(\rho)]^2}{M(\rho)}\right\}$$
$$R(\rho_1,\rho_2)=\tau^2 T_B(1-\rho_1)^2+\tau^2(T-T_B)(1-\rho_2)^2+1$$
$$S(\rho_1,\rho_2)=\tau^2(1-\rho_1)\sum_{t=1}^{T_B}\left(y_{it}-\rho_1 y_{i,t-1}\right)+\tau^2(1-\rho_2)\sum_{t=T_B+1}^{T}\left(y_{it}-\rho_2 y_{i,t-1}\right)+\vartheta_i$$
$$T(\rho_1,\rho_2)=d+\frac{1}{2\tau^2}\sum_{i=1}^{n}\left\{\tau^2\sum_{t=1}^{T_B}\left(y_{it}-\rho_1 y_{i,t-1}\right)^2+\tau^2\sum_{t=T_B+1}^{T}\left(y_{it}-\rho_2 y_{i,t-1}\right)^2+\vartheta_i^{\,2}-\frac{[S(\rho_1,\rho_2)]^2}{R(\rho_1,\rho_2)}\right\}$$
$$U(\rho)=\tau^2 T_B(1-\rho)^2+1$$
$$V(\rho)=\tau^2(T-T_B)(1-\rho)^2+1$$
$$W(\rho)=\tau^2(1-\rho)\sum_{t=1}^{T_B}\left(y_{it}-\rho y_{i,t-1}\right)+\vartheta_{i1}$$
$$X(\rho)=\tau^2(1-\rho)\sum_{t=T_B+1}^{T}\left(y_{it}-\rho y_{i,t-1}\right)+\vartheta_{i2}$$
$$Y(\rho)=d+\frac{1}{2\tau^2}\sum_{i=1}^{n}\left\{\vartheta_{i1}^2+\vartheta_{i2}^2+\tau^2\sum_{t=1}^{T}\left(y_{it}-\rho y_{i,t-1}\right)^2-\frac{[W(\rho)]^2}{U(\rho)}-\frac{[X(\rho)]^2}{V(\rho)}\right\}$$
$$\phi(\rho)=\tau^2 T(1-\rho)^2+1$$
$$\psi(\rho)=\tau^2(1-\rho)\sum_{t=1}^{T}\left(y_{it}-\rho y_{i,t-1}\right)+\vartheta_i$$
$$Z(\rho)=d+\frac{1}{2\tau^2}\sum_{i=1}^{n}\left\{\vartheta_i^2+\tau^2\sum_{t=1}^{T}\left(y_{it}-\rho y_{i,t-1}\right)^2-\frac{[\psi(\rho)]^2}{\phi(\rho)}\right\}$$
$$\gamma(\rho,\mu_i)=d_1+\frac{1}{2}\sum_{i=1}^{n}\left\{\sum_{t=1}^{T_B}\left(y_{it}-\rho y_{i,t-1}-(1-\rho)\mu_i\right)^2\right\}$$
$$\lambda(\rho,\mu_i)=d_2+\frac{1}{2}\sum_{i=1}^{n}\left\{\sum_{t=T_B+1}^{T}\left(y_{it}-\rho y_{i,t-1}-(1-\rho)\mu_i\right)^2\right\}$$

$$\eta(\rho_1, \mu_i) = d_1 + \frac{1}{2}\sum_{i=1}^{n}\left\{\sum_{t=1}^{T_B}\left(y_{it} - \rho_1 y_{i,t-1} - (1-\rho_1)\mu_i\right)^2\right\}$$

$$\omega(\rho_2, \mu_i) = d_2 + \frac{1}{2}\sum_{i=1}^{n}\left\{\sum_{t=T_B+1}^{T}\left(y_{it} - \rho_2 y_{i,t-1} - (1-\rho_2)\mu_i\right)^2\right\}$$

Using the above notation, posterior probability under the proposed model satisfying the hypothesis $H^1 : \rho_1 \neq \rho_2, \mu_{i1} \neq \mu_{i2}, \sigma_1^2 \neq \sigma_2^2$ which considers break in mean, variance as well as autoregressive coefficient is obtained as

$$P\left(y\middle|H^1\right) = \int_{l_1}^{1}\int_{l_2}^{1} \frac{d_1^{c_1} d_2^{c_2}}{(2\pi)^{\frac{nT}{2}}(1-l_1)(1-l_2)\Gamma c_1 \Gamma c_2 [A(\rho_1)]^{\frac{n}{2}}[B(\rho_2)]^{\frac{n}{2}}} \frac{\Gamma\left(\frac{nT_B}{2} + c_1\right)}{[E(\rho_1)]^{\frac{nT_B}{2}+c_1}} \frac{\Gamma\left(\frac{n(T-T_B)}{2} + c_2\right)}{[F(\rho_2)]^{\frac{n(T-T_B)}{2}+c_2}} d\rho_1 d\rho_2 \tag{7}$$

However, many researchers getting many applications with the models which do not consider break point in all coefficients because all coefficients may not be much affected. Due to this reason, proposed model have been analyzed in a particular form by several authors likes model contains no break, break in one coefficient, break in two coefficients. Wachter and Tzavalis (2012) studied the panel AR(1) model considering break in mean and autoregressive coefficients and developed break detecting testing procedure with the help of 2-step generalized method of moments. However, this model is not tested under Bayesian approach yet so we have derived the posterior probability for $H^2 : \rho_1 \neq \rho_2, \mu_{i1} \neq \mu_{i2}, \sigma_1^2 = \sigma_2^2 = \sigma^2$ and get the expression

$$P\left(y\middle|H^2\right) = \int_{l_1}^{1}\int_{l_2}^{1} \frac{d^{c}}{(2\pi)^{\frac{nT}{2}}(1-l_1)(1-l_2)\Gamma c [G(\rho_1)]^{\frac{n}{2}}[H(\rho_2)]^{\frac{n}{2}}} \frac{\Gamma\left(\frac{nT}{2} + c\right)}{[K(\rho_1, \rho_2)]^{\frac{nT}{2}+c}} d\rho_1 d\rho_2 \tag{8}$$

Liu *et al.* (2011) proposed a model selection approach for testing structural break in panel varying coefficients model and apply to OECD health expenditure data to see the performance. Present work explored this by considering a hypothesis $H^5 : \rho_1 \neq \rho_2, \mu_{i1} = \mu_{i2}, \sigma_1^2 = \sigma_2^2$ and obtained the posterior probability

$$P(y|H^5) = \int_{l_1}^{1}\int_{l_2}^{1} \frac{d^c}{(2\pi)^{\frac{nT}{2}}(1-l_1)(1-l_2)\Gamma c[R(\rho_1,\rho_2)]^{\frac{n}{2}}} \frac{\Gamma\left(\frac{nT}{2}+c\right)}{[T(\rho_1,\rho_2)]^{\frac{nT}{2}+c}} d\rho_1 d\rho_2 \tag{9}$$

Bai (2010) considered break in mean or/and variance in panel AR(1) model for estimating the parameters using least square or quasi-maximum likelihood method. He also obtained limiting distribution of estimated break point and expanded in multiple break points. The model gives extension for hypothesis testing problem. So, we purposed to test the presence of break point in mean and variance, equivalent hypothesis $H^4 : \rho_1 = \rho_2, \mu_{i1} \neq \mu_{i2}, \sigma_1^2 \neq \sigma_2^2$ and get the posterior probability

$$P(y|H^4) = \int_{l}^{1} \frac{d_1^{c_1} d_2^{c_2}}{(2\pi)^{\frac{nT}{2}}(1-l)\Gamma c_1 \Gamma c_2 [L(\rho)]^{\frac{n}{2}}[M(\rho)]^{\frac{n}{2}}} \frac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{[P(\rho)]^{\frac{nT_B}{2}+c_1}} \frac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{[Q(\rho)]^{\frac{n(T-T_B)}{2}+c_2}} d\rho \tag{10}$$

Consider break point in mean $H^6 : \rho_1 = \rho_2, \mu_{i1} \neq \mu_{i2}, \sigma_1^2 = \sigma_2^2$, the posterior probability is defined as

$$P(y|H^6) = \int_{l}^{1} \frac{d^c}{(2\pi)^{\frac{nT}{2}}(1-l)\Gamma c[U(\rho)]^{\frac{n}{2}}[V(\rho)]^{\frac{n}{2}}} \frac{\Gamma\left(\frac{nT}{2}+c\right)}{[Y(\rho)]^{\frac{nT}{2}+c}} d\rho \tag{11}$$

and under H[7] when break is present in variance only, the posterior probability is,

$$P\left(y\middle|H^{7}\right)=\int_{l}^{1}\int_{R^{n}}\frac{d_1^{c_1}d_2^{c_2}}{(2\pi)^{\frac{nT}{2}+\frac{n}{2}}(1-l)\Gamma c_1\Gamma c_2\tau^{n}\sigma^{n}}\frac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{\left[\gamma(\rho,\mu_i)\right]^{\frac{nT_B}{2}+c_1}}\frac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{\left[\lambda(\rho,\mu_i)\right]^{\frac{n(T-T_B)}{2}+c_2}}\exp\left[-\frac{1}{2\tau^2\sigma^2}\sum_{i=1}^{n}(\mu_i-\vartheta_i)^2\right]d\mu d\rho. \tag{12}$$

Moreover, Pesaran (2004) proposed error cross section dependence (CD) test for panel data model and verified the robustness of CD test to single or multiple breaks in the slope coefficient and/or error variances under classical approach. For our purpose, we test the break point hypothesis $H^3:\rho_1\neq\rho_2,\mu_{i1}=\mu_{i2}=\mu_i,\sigma_1^2\neq\sigma_2^2$ for this model using posterior probability, as given below

$$P\left(y\middle|H^{3}\right)=\int_{l_1}^{1}\int_{l_2}^{1}\int_{R^{n}}\frac{d_1^{c_1}d_2^{c_2}}{(2\pi)^{\frac{nT+n}{2}}(1-l_1)(1-l_2)\Gamma c_1\Gamma c_2\tau^{n}\sigma^{n}}\frac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{\left[\eta(\rho_1,\mu_i)\right]^{\frac{nT_B}{2}+c_1}}\frac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{\left[\omega(\rho_2,\mu_i)\right]^{\frac{n(T-T_B)}{2}+c_2}}\exp\left[-\frac{1}{2\tau^2\sigma^2}\sum_{i=1}^{n}(\mu_i-\vartheta_i)^2\right]d\mu d\rho_1 d\rho_2. \tag{13}$$

For unit root hypothesis, Levin *et al.* (2002) considered three simple panel AR(1) model contain without intercept, with intercept and with intercept and trend components to test that series is stationary or have unit root. A t-statistic was computed and finds the asymptotic properties of the limiting distribution. In our model, no break is consider in coefficients reduce form of model is simple panel AR(1) model with intercept and posterior probability for corresponding hypothesis $H^8:\rho_1=\rho_2,\mu_{i1}=\mu_{i2},\sigma_1^2=\sigma_2^2$ is

$$P\left(y\middle|H^{8}\right)=\int_{l}^{1}\frac{d^{c}}{(2\pi)^{\frac{nT}{2}}(1-l)\Gamma c\left[\phi(\rho)\right]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT}{2}+c\right)}{\left[Z(\rho)\right]^{\frac{nT}{2}+c}}d\rho \tag{14}$$

The integral in equation (7) to (14) are difficult to evaluate especially when the prior follows uniform distribution. Therefore, exact or approximate analytical results are obtained by numerical techniques.

5. Posterior Odds Ratio

Under Bayesian perspective, posterior odds ratio is used in decision making for hypothesis testing problem and then selecting a suitable model using posterior probability. The posterior odds ratio is the combination of prior odds ratio with the Bayes factor of the null and alternative hypothesis that means it is the ratio of the posterior probability for the observed series under the given hypothesis. If each prior probability is deemed equally likely associated with each hypothesis, POR reduces to Bayes factor. The method which is used for hypothesis testing is Jeffrey's hypothesis testing criterion using following steps:

Object: To test $H_0 : \theta_0 \in \Theta_0$ vs $H_1 : \theta_1 \in \Theta_1$ where Θ_0 and Θ_1 is a set of parameters

Step-1: Consider a prior distribution $\pi(\theta)$ for every parameter as a random variable.

Step-2: Write the likelihood function for a given series under a particular hypothesis.

Step-3: Compute posterior probability/distribution under null (β_0) and alternative hypothesis (β_1).

Step-4: Reject H_0 if the posterior odds ratio (β_{01}) is less than one, otherwise accept.

The following posterior odds ratio (POR) is obtained for our proposed model using the posterior probability derived in equation (7) to (14) when alternative hypothesis considering break in all coefficients. Hypothesis testing for the model which consider break in all coefficients with other particular form are given below by POR$_1$ to POR$_7$. However all other particular forms may also be tested by POR$_8$ to POR$_{28}$ which are given in Appendix Table-A2.

POR$_1$: For testing the null hypothesis that series have break in mean and autoregressive coefficient $H^2 : \rho_1 \neq \rho_2, \mu_{i1} \neq \mu_{i2}, \sigma_1^2 = \sigma_2^2,$ against the alternative hypothesis that series have break in mean, variance and autoregressive coefficient $H^1 : \rho_1 \neq \rho_2, \mu_{i1} \neq \mu_{i2}, \sigma_1^2 \neq \sigma_2^2$

$$\beta_{01}^1 = \frac{p_0}{1-p_0} \frac{\int_{l_1}^{1}\int_{l_2}^{1} \frac{d^c}{\Gamma c [G(\rho_1)]^{\frac{n}{2}} [H(\rho_2)]^{\frac{n}{2}}} \frac{\Gamma\left(\frac{nT}{2}+c\right)}{[K(\rho_1,\rho_2)]^{\frac{nT}{2}+c}} d\rho_1 d\rho_2}{\int_{l_1}^{1}\int_{l_2}^{1} \frac{d_1^{c_1} d_2^{c_2}}{\Gamma c_1 \Gamma c_2 [A(\rho_1)]^{\frac{n}{2}} [B(\rho_2)]^{\frac{n}{2}}} \frac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{[E(\rho_1)]^{\frac{nT_B}{2}+c_1}} \frac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{[F(\rho_2)]^{\frac{n(T-T_B)}{2}+c_2}} d\rho_1 d\rho_2}$$

POR$_2$: For testing the null hypothesis that series have break in variance and autoregressive coefficient $H^3 : \rho_1 \neq \rho_2, \mu_{i1} = \mu_{i2}, \sigma_1^2 \neq \sigma_2^2,$ against the alternative hypothesis that series have break in mean, variance and autoregressive coefficient $H^1 : \rho_1 \neq \rho_2, \mu_{i1} \neq \mu_{i2}, \sigma_1^2 \neq \sigma_2^2$

$$\beta_{01}^2 = \frac{p_0}{1-p_0} \frac{\int_{l_1}^{1}\int_{l_2}^{1}\int_{R^n} \frac{1}{(2\pi)^{\frac{n}{2}} \tau^n \sigma^n [\eta(\rho_1,\mu_i)]^{\frac{nT_B}{2}+c_1} [\omega(\rho_2,\mu_i)]^{\frac{n(T-T_B)}{2}+c_2}} \exp\left[-\frac{1}{2\tau^2\sigma^2}\sum_{i=1}^{n}(\mu_i-\vartheta_i)^2\right] d\mu d\rho_1 d\rho_2}{\int_{l_1}^{1}\int_{l_2}^{1} \frac{1}{[A(\rho_1)]^{\frac{n}{2}} [B(\rho_2)]^{\frac{n}{2}} [E(\rho_1)]^{\frac{nT_B}{2}+c_1} [F(\rho_2)]^{\frac{n(T-T_B)}{2}+c_2}} d\rho_1 d\rho_2}$$

POR$_3$: For testing the null hypothesis that series have break in mean and variance $H^4 : \rho_1 = \rho_2, \mu_{i1} \neq \mu_{i2}, \sigma_1^2 \neq \sigma_2^2,$ against the alternative hypothesis that series have break in mean, variance and autoregressive coefficient $H^1 : \rho_1 \neq \rho_2, \mu_{i1} \neq \mu_{i2}, \sigma_1^2 \neq \sigma_2^2$

$$\beta_{01}^{3}=\frac{p_0}{1-p_0}\frac{\int_{l}^{1}\frac{1}{(1-l)[L(\rho)]^{\frac{n}{2}}[M(\rho)]^{\frac{n}{2}}[P(\rho)]^{\frac{nT_B}{2}+c_1}[Q(\rho)]^{\frac{n(T-T_B)}{2}+c_2}}d\rho}{\int_{l_1}^{1}\int_{l_2}^{1}\frac{1}{(1-l_1)(1-l_2)[A(\rho_1)]^{\frac{n}{2}}[B(\rho_2)]^{\frac{n}{2}}[E(\rho_1)]^{\frac{nT_B}{2}+c_1}[F(\rho_2)]^{\frac{n(T-T_B)}{2}+c_2}}d\rho_1 d\rho_2}$$

POR₄: For testing the null hypothesis that series have break in autoregressive coefficient $H^5:\rho_1\neq\rho_2,\mu_{i1}=\mu_{i2},\sigma_1^2=\sigma_2^2,$ against the alternative hypothesis that series have break in mean, variance as well as autoregressive coefficient $H^1:\rho_1\neq\rho_2,\mu_{i1}\neq\mu_{i2},\sigma_1^2\neq\sigma_2^2$

$$\beta_{01}^{4}=\frac{p_0}{1-p_0}\frac{\int_{l_1}^{1}\int_{l_2}^{1}\frac{d^c}{\Gamma c[R(\rho_1,\rho_2)]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT}{2}+c\right)}{[T(\rho_1,\rho_2)]^{\frac{nT}{2}+c}}d\rho_1 d\rho_2}{\int_{l_1}^{1}\int_{l_2}^{1}\frac{d_1^{c_1}d_2^{c_2}}{\Gamma c_1\Gamma c_2[A(\rho_1)]^{\frac{n}{2}}[B(\rho_2)]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{[E(\rho_1)]^{\frac{nT_B}{2}+c_1}}\frac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{[F(\rho_2)]^{\frac{n(T-T_B)}{2}+c_2}}d\rho_1 d\rho_2}$$

POR₅: For testing the null hypothesis $H^6:\rho_1=\rho_2,\mu_{i1}\neq\mu_{i2},\sigma_1^2=\sigma_2^2$ that series have break in mean against the alternative hypothesis that series have break in mean, variance as well as autoregressive coefficient $H^1:\rho_1\neq\rho_2,\mu_{i1}\neq\mu_{i2},\sigma_1^2\neq\sigma_2^2$

$$\beta_{01}^{5}=\frac{p_0}{1-p_0}\frac{\int_{l}^{1}\frac{d^c}{(1-l)\Gamma c[U(\rho)]^{\frac{n}{2}}[V(\rho)]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT}{2}+c\right)}{[Y(\rho)]^{\frac{nT}{2}+c}}d\rho}{\int_{l_1}^{1}\int_{l_2}^{1}\frac{d_1^{c_1}d_2^{c_2}}{(1-l_1)(1-l_2)\Gamma c_1\Gamma c_2[A(\rho_1)]^{\frac{n}{2}}[B(\rho_2)]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{[E(\rho_1)]^{\frac{nT_B}{2}+c_1}}\frac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{[F(\rho_2)]^{\frac{n(T-T_B)}{2}+c_2}}d\rho_1 d\rho_2}$$

POR$_6$: For testing the null hypothesis $H^7:\rho_1=\rho_2,\mu_{i1}=\mu_{i2},\sigma_1^2\neq\sigma_2^2$ that series have break in variance against the alternative hypothesis that series have break in mean, variance as well as autoregressive coefficient $H^1:\rho_1\neq\rho_2,\mu_{i1}\neq\mu_{i2},\sigma_1^2\neq\sigma_2^2$

$$\beta_{01}^6=\frac{p_0}{1-p_0}\frac{\int_l^1\int_{R^n}\frac{1}{(2\pi)^{\frac{n}{2}}(1-l)\tau^n\sigma^n[\gamma(\rho,\mu_i)]^{\frac{nT_B}{2}+c_1}[\lambda(\rho,\mu_i)]^{\frac{n(T-T_B)}{2}+c_2}}\exp\left[-\frac{1}{2\tau^2\sigma^2}\sum_{i=1}^{n}(\mu_i-\vartheta_i)^2\right]d\mu d\rho}{\int_{l_1}^1\int_{l_2}^1\frac{1}{(1-l_1)(1-l_2)[A(\rho_1)]^{\frac{n}{2}}[B(\rho_2)]^{\frac{n}{2}}[E(\rho_1)]^{\frac{nT_B}{2}+c_1}[F(\rho_2)]^{\frac{n(T-T_B)}{2}+c_2}}d\rho_1 d\rho_2}$$

POR$_7$: For testing the null hypothesis $H^8:\rho_1=\rho_2,\mu_{i1}=\mu_{i2},\sigma_1^2=\sigma_2^2$ that series have no break against the alternative hypothesis that series have break in mean, variance as well as autoregressive coefficient $H^1:\rho_1\neq\rho_2,\mu_{i1}\neq\mu_{i2},\sigma_1^2\neq\sigma_2^2$

$$\beta_{01}^7=\frac{p_0}{1-p_0}\frac{\int_l^1\frac{d^c}{(1-l)\Gamma c[\phi(\rho)]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT}{2}+c\right)}{[Z(\rho)]^{\frac{nT}{2}+c}}d\rho}{\int_{l_1}^1\int_{l_2}^1\frac{d_1^{c_1}d_2^{c_2}}{(1-l_1)(1-l_2)\Gamma c_1\Gamma c_2[A(\rho_1)]^{\frac{n}{2}}[B(\rho_2)]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{[E(\rho_1)]^{\frac{nT_B}{2}+c_1}}\frac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{[F(\rho_2)]^{\frac{n(T-T_B)}{2}+c_2}}d\rho_1 d\rho_2}$$

6. Simulation Study

In this section, a simulation study has been presented to illustrate the application of various theoretical outcome developed in previous section on the basis of generated series from equation (3) with different sizes of series T = {40, 60, 80} and for different break point T_B = {T/4, T/2, 3T/4}. For a particular set of parameters and the simulated series, compute posterior odds ratio (POR) of our models and replicated the process 10,000 times. Without loss of generality, we have taken $\rho_1 = 0.90$ and $\rho_2 = 0.95$ with hyper parameter value is $l_1 = 0.8\rho_1$ and $l_2 = 0.8\rho_2$. For generation series, initial value of intercept $\{\mu_{.1}, \mu_{.2}\}$ = {(0.01, 0.012, 0.014), (0.02, 0.022, 0.024)} and error variance (σ_1^2, σ_2^2) = (0.009, 0.01) corresponding hyper parameter value are $\upsilon_{i1} = y_{i0}$, $\upsilon_{i2} = \bar{y}_i$, $\tau_1^2 = \tau_2^2 = 4$ and $c_1 = d_1 = c_2 = d_2 = 0.01$. The expressions of posterior probability are coming in a complicated form which do not analysis numerical or without any integration technique. In present simulation, use a Monte Carlo integration method to approximating the value of integral. However, this method of approximation will guarantee a particular rate of convergence only when the integral has continuous derivatives to a certain order.

Using integration method, find the value of posterior probability for different combination of hypothesis. For better interpretation, one may separate POR in three different groups as per the presence of break point in the parameters. These groups are (i) break in all parameters (ii) break in any two parameters (iii) break in any one parameter only. These groups are further divided according to their alternative hypothesis. All statement from (i) to (iii) are tested, taking various null hypothesis which are listed in appendix Table-A1. As there are several values and our purpose is to test our proposed model with other models and conclude the inference therein. If the value of POR is less than one, null hypothesis is to be rejected otherwise accepted. In details all cases are discussed as below:

6.1: Break in all parameters

Considering posterior odds ratio from β_{01}^{1} to β_{01}^{7} where the alternative hypothesis is formulated by proposed model which allows break in mean, variance and autoregressive coefficient. The null hypothesis against similar alternative hypothesis has been tested one by one and obtained the POR value in Table-1.

Table-1: Posterior odds ratio when alternative hypothesis is break in all parameters								
T	TB	β_{01}^{1}	β_{01}^{2}	β_{01}^{3}	β_{01}^{4}	β_{01}^{5}	β_{01}^{6}	β_{01}^{7}
40	10	2.59E-02	1.50E-01	5.08E-01	7.31E-03	2.39E-02	6.41E-02	7.14E-03
	20	8.52E-03	4.44E-01	5.96E-01	1.58E-03	4.37E-03	3.81E-01	4.88E-04
	30	7.64E-03	9.31E-03	2.87E-01	3.99E-03	2.04E-03	1.14E-01	2.00E-04
60	15	1.21E-02	2.33E-02	2.38E-01	3.94E-03	1.38E-02	3.70E-04	1.89E-03
	30	3.61E-02	5.88E-03	1.17E-01	2.67E-04	2.50E-03	1.45E-02	1.02E-04
	45	5.57E-04	1.28E-04	1.87E-01	2.62E-04	1.13E-04	1.54E-06	3.85E-05
80	20	5.40E-05	2.09E-03	4.56E-01	1.86E-05	2.79E-05	6.71E-05	9.65E-06
	40	5.18E-05	4.03E-03	2.39E-01	1.99E-05	1.21E-05	3.89E-04	5.11E-06
	60	1.18E-05	2.64E-02	1.03E-01	3.50E-08	1.66E-07	4.32E-06	4.08E-10

From Table-1, the results for simulated data are very small with varying time series and also with different break point. One may conclude that as a size of the series increases, value of POR reduces and if break point considers about to lower and upper quartile of the series, POR is less as compare to middle break point. This turns out when series is divided in an equal part. The table also shows that our proposed model is always accepted to support the break series and make the model reliable and stationary. This may happened because break is also considered in random coefficient which is having more impact as compare to other parameters.

6.2: Break in any two parameters

Consider the situation when break is present only on two parameters. Sometimes one may be getting a time series which is affecting to some parameters not all parameters. To handle this type of scenario, we choose a particular suitable model which is considering break in any two parameters. The proposed model contained three parameters and the combination of parameters is (A) break in mean and autoregressive coefficient (B) break in variance and autoregressive coefficient (C) break in mean and variance. Then, considering this as an alternative hypothesis and remaining all possible hypotheses are considering as a null and making inference. The results for all combination of hypothesis are summarized in Table-2.

Table-2: Posterior odds ratio when alternative hypothesis is break in mean and autoregressive coefficient							
T	TB	β_{01}^{8}	β_{01}^{9}	β_{01}^{10}	β_{01}^{11}	β_{01}^{12}	β_{01}^{13}
40	10	5.80E+00	1.96E+01	2.82E-01	9.21E-01	2.47E+00	2.75E-01
	20	5.21E+01	6.99E+01	1.85E-01	5.13E-01	4.47E+01	5.73E-02
	30	1.22E+00	3.76E+01	5.22E-01	2.67E-01	1.49E+01	2.62E-02
60	15	1.92E+00	1.96E+01	3.24E-01	1.37E-01	3.05E-02	1.55E-01
	30	1.63E-01	3.23E+00	7.40E-03	6.91E-02	4.01E-01	2.82E-03
	45	2.30E-01	3.35E+02	4.70E-01	2.03E-01	2.77E-03	6.91E-02
80	20	3.87E+01	8.44E+03	3.44E-01	5.16E-01	1.24E+00	1.79E-01
	40	7.78E+01	4.60E+03	3.84E-01	2.33E-01	7.50E+00	9.86E-02
	60	2.23E+03	8.74E+03	2.96E-03	1.40E-02	3.65E-01	3.45E-05

It is evident from Table-2 that ignoring break in error variance of parent series, value of POR gradually increases and cross the value 1 when size of series increases *i.e.* in this situation accept the null hypothesis. These hypothesis consider break in error variance, see $\beta_{01}^{8}, \beta_{01}^{9}, \beta_{01}^{12}$. In other terms, randomness of the model is depending upon error variance and this change the structure of the series more. So, discarding break in variance tends to series mean constant and this increase the POR value. Form Table-2, the remaining POR is less than one concluded that alternative hypothesis is true.

Table-3: Posterior odds ratio when alternative hypothesis is break in autoregressive coefficient and error variance						
T	TB	β_{01}^{14}	β_{01}^{15}	β_{01}^{16}	β_{01}^{17}	β_{01}^{18}
40	10	3.38E+00	4.86E-02	1.59E-01	4.26E-01	4.75E-02
	20	1.34E+00	3.55E-03	9.83E-03	8.58E-01	1.10E-03
	30	3.08E+01	4.28E-01	2.19E-01	1.22E+01	2.15E-02
60	15	1.02E+01	1.69E-01	5.93E-01	1.59E-02	8.10E-02
	30	1.98E+01	4.55E-02	4.25E-01	2.47E+00	1.73E-02
	45	1.46E+03	2.04E+00	8.81E-01	1.20E-02	3.00E-01

80	20	2.18E+02	8.89E-03	1.33E-02	3.21E-02	4.62E-03
	40	5.92E+01	4.93E-03	3.00E-03	9.64E-02	1.27E-03
	60	3.91E+00	1.32E-06	6.29E-06	1.63E-04	1.54E-08

If we do not consider break in mean in our specified model, hypothesis is coming only for break in autoregressive coefficient and error variance and testing for remaining null hypothesis be carried out in Table-3. As we know that error term affect the series much more than other parameters like trend component, intercept etc. and then fluctuation in the series after the break point is more due to break in error term which may or may not depend upon series. This directly impacts the model's stationarity. Hence, POR is always greater than 1 which can be seen in β_{01}^{14}. For other null hypothesis, accept the alternative model with varying size of the series.

Table-4: Posterior odds ratio when alternative hypothesis is break in mean and error variance

T	TB	β_{01}^{19}	β_{01}^{20}	β_{01}^{21}	β_{01}^{22}
40	10	1.44E-02	4.70E-02	1.26E-01	1.41E-02
	20	2.65E-03	7.33E-03	6.40E-01	8.20E-04
	30	1.39E-02	7.11E-03	3.96E-01	6.99E-04
60	15	1.66E-02	5.82E-02	1.56E-03	7.95E-03
	30	2.29E-03	2.14E-02	1.24E-01	8.73E-04
	45	1.40E-03	6.05E-04	8.27E-06	2.06E-04
80	20	4.07E-05	6.11E-05	1.47E-04	2.12E-05
	40	8.34E-05	5.07E-05	1.63E-03	2.14E-05
	60	3.38E-07	1.61E-06	4.18E-05	3.95E-09

Table-4 is considering break in mean and error variance i.e. ignore the break in autoregressive coefficient. Here it is revealed that proposed hypothesis significantly affecting the correct model. Hence all null hypotheses are rejected because POR is less than one.

6.3: Break in any one parameter only

In real scenario, change in any parameter value may affected the whole series specially, for example if ρ is equal to one means series is unit root otherwise stationary. Similarly, this type of cases is also taken in our account to make appropriate conclusion about the remaining model under all situations. Table-5 summarized the results for analysis of break point in one parameter.

Table-5: Posterior odds ratio when alternative hypothesis is break in							
		Autoregressive Coefficient			**Mean**		**Error**
T	TB	β_{01}^{23}	β_{01}^{24}	β_{01}^{25}	β_{01}^{26}	β_{01}^{27}	β_{01}^{28}
40	10	3.27E+00	8.77E+00	9.77E-01	2.69E+00	2.99E-01	1.11E-01
	20	2.77E+00	2.42E+02	3.09E-01	8.73E+01	1.12E-01	1.28E-03
	30	5.12E-01	2.85E+01	5.03E-02	5.58E+01	9.83E-02	1.76E-03
60	15	3.51E+00	9.41E-02	4.80E-01	2.68E-02	1.37E-01	9.58E-02
	30	9.34E+00	5.42E+01	3.81E-01	5.81E+00	4.08E-02	7.03E-03
	45	4.31E-01	5.89E-03	1.47E-01	1.37E-02	3.40E-01	9.25E-01
80	20	1.50E+00	3.62E+00	5.20E-01	2.41E+00	3.46E-01	1.44E-01
	40	6.08E-01	1.95E+01	2.57E-01	3.21E+01	4.22E-01	1.31E-02
	60	4.75E+00	1.23E+02	1.17E-02	2.60E+01	2.46E-03	9.45E-05

From Table-5, if hypothesis considers break in autoregressive coefficient which is not making better understanding either it is accepted or rejected depend. Here we have considered only the situation of stationary series. Then, this null hypothesis may consider that there is break in mean or variance which sometimes affects the series because POR is greater than one. This shows that break present in autoregressive coefficient is also affecting the series.

7. Application

South Asian Association for Regional Cooperation (SAARC) is regional organization of South Asia countries, namely Afghanistan, Bangladesh, Bhutan, India, Nepal, Maldives, Pakistan and Sri Lanka. It brings together for strengthen the economical, technological, social and cultural development among the associated countries and mentor for establishing the relation with developed nations to get support and assistance. There are various levels when member countries work together by undertaking joint/connected programs (Shaheen (2013)). Hence, a relationship can be well established for economic policies of mutual interest. In this section, consider annual historical data of real effective exchange rate (REER) of SAARC country for the period 1979 to 2016. There are eight countries out of which REER series of Afghanistan is available since 2002 so this is not included in the analysis. The purpose of present application is to examine the presence of structural break in the parameters. Firstly, a natural way is used for determining the number and location of break point in individual series which is well developed by Zeileis *et al.* (2002). For all countries, the most preferred break point and its position are summarized in Table-6.

Table-6: Number of identified break points for REER data set

Country	Number of Breaks	T_B
Bangladesh	0	NA
Bhutan	1	1991
India	1	1991
Maldives	1	1992
Nepal	1	1991
Pakistan	1	1991
Sri Lanka	1	2004
China	1	1991

Present study emphasis only common break point in balanced panel which is occurred at the year 1991 except Bangladesh, Maldives and Sri Lanka. This was the year when economic conditions changed due to second Gulf war, oil crisis, political instability and international trading. For analysis purpose, consider these countries which are having similar break point as a panel and dropped others from the analysis. The most appropriate model for this data set is selected among the proposed and per-existing model by using Akaike information criterion (AIC) and Bayes information criterion (BIC) considering break at T_B =1991. For information criterion value, likelihood function is evaluated at the posterior mean. The model which has the lowest AIC and BIC value is the best suitable model for REER data and shown in Table-7.

Table-7:Model selection for REER data set			
Model	LogL	AIC	BIC
PAR (ρ_1, ρ_2, μ_{i1}, μ_{i2}, σ_1,	463.6662	947.3325	977.2416
PAR(ρ, μ_{i1}, μ_{i2}, σ_1, σ_2)	465.3792	948.7585	978.9803
PAR(ρ_1, ρ_2, μ_{i1}, μ_{i2}, σ)	472.1972	962.3945	990.8776
PAR(ρ_1, ρ_2, μ_i, σ_1, σ_2)	471.5557	957.1115	979.2650
PAR(ρ, μ_i, σ_1, σ_2)	478.7935	969.5871	988.5758
PAR(ρ, μ_{i1}, μ_{i2}, σ)	474.0607	964.1214	989.4397
PAR(ρ_1, ρ_2, μ_i, σ)	486.0198	984.0396	1003.0280
PAR(ρ, μ_i, σ)	490.4496	990.8991	1006.7230

A testing of hypothesis is performed after indentifying the position of break. To determine the value of posterior odds ratio for test the instability of parameter(s). Table-8 shows the values of POR toward considering the ratio of various posterior probabilities with condition that probability of favoring of null and alternative is equal *i.e.* prior probability is equal to 1.

Table-8: POR value at break point 1991								
	H^1	H^2	H^3	H^4	H^5	H^6	H^7	H^8
H^1	1.00E+00							
H^2	3.52E-16	1.00E+00						
H^3	6.24E-05	1.77E+11	1.00E+00					
H^4	5.51E-02	1.57E+14	8.82E+02	1.00E+00				
H^5	3.56E-17	1.01E-01	5.71E-13	6.47E-16	1.00E+00			
H^6	2.34E-17	6.65E-02	3.75E-13	4.25E-16	6.57E-01	1.00E+00		
H^7	2.04E-06	5.80E+09	3.27E-02	3.70E-05	5.73E+10	8.72E+10	1.00E+00	
H^8	3.36E-18	9.54E-03	5.37E-14	6.09E-17	9.41E-02	1.43E-01	1.64E-12	1.00E+00

Table-8 display appropriate conclusion about our proposed model and shows that all null hypothesis in REER is reject when alternative hypothesis consider break in mean, error variance and autoregressive coefficient *i.e.* for POR_1-POR_7, the posterior odds ratio is less than one. This demonstrates that model is containing break in all parameters which raise an appropriate interpretation and make the series stationary as comparison with other models. For other hypothesis similar results are obtained because ratio of posterior probability is less than one. But series instability is more due to presence of break in mean as well as variance or break in variance only because of posterior probability in favor of this hypothesis which is higher as compare to others model. This impact also observed by partially effected variables. Since whole series may be stationary for a particular model. However, in a structural break model at a fixed break point the series is converting a non-stationary. Therefore, change in the autoregressive coefficient has been also tested the unit root hypothesis before/ after the break point and then achieved the outcomes. The outcomes give productive results for this data set and show that real effect exchange rate is changing due to more variation in their import and export of commodities, level shifting by oil crisis and political governance of the countries.

8. Conclusion

In the present study, we deal with break point problem in PAR (1) time series model under Bayesian framework. The impact of break point in parameter(s) has been recorded using posterior odds ratio and observed that break is present of REER series for all coefficients which is our proposed model. This change on coefficients may be handled by proper implementing economic policy to the nation. It is also noticed that proper handling of break is important before modeling the series. The similar results are observed for simulated data also. Overall, we can concluded that if break in variance is present in any model, model is more unstable and fluctuations of the series is more because of more randomness is taken by error term and this change the structure of the series up and down.

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Data Source: Real effective exchange rates for 178 countries: a new database.

http://bruegel.org/publications/datasets/real-effective-exchange-rates-for-178-countries-a-new-database/.

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Appendix

Table-A1: Formulation of Hypothesis		
Hypothesis	Model	Reference
$H^1 : \rho_1 \neq \rho_2, \mu_{i1} \neq \mu_{i2}, \sigma_1^2 \neq \sigma_2^2$	$y_{it} = \begin{cases} \rho_1 y_{i,t-1} + (1-\rho_1)\mu_{i1} + \sigma_1 \varepsilon_{it} \\ \rho_2 y_{i,t-1} + (1-\rho_2)\mu_{i2} + \sigma_2 \varepsilon_{it} \end{cases}$	Proposed
$H^2 : \rho_1 \neq \rho_2, \mu_{i1} \neq \mu_{i2}, \sigma_1^2 = \sigma_2^2 = \sigma^2$	$y_{it} = \begin{cases} \rho_1 y_{i,t-1} + (1-\rho_1)\mu_{i1} + \sigma \varepsilon_{it} \\ \rho_2 y_{i,t-1} + (1-\rho_2)\mu_{i2} + \sigma \varepsilon_{it} \end{cases}$	Wachter and Tzavalis (2012)
$H^3 : \rho_1 \neq \rho_2, \mu_{i1} = \mu_{i2} = \mu_i, \sigma_1^2 \neq \sigma_2^2$	$y_{it} = \begin{cases} \rho_1 y_{i,t-1} + (1-\rho_1)\mu_i + \sigma_1 \varepsilon_{it} \\ \rho_2 y_{i,t-1} + (1-\rho_2)\mu_i + \sigma_2 \varepsilon_{it} \end{cases}$	Pesaran (2004)
$H^4 : \rho_1 = \rho_2, \mu_{i1} \neq \mu_{i2}, \sigma_1^2 \neq \sigma_2^2$	$y_{it} = \begin{cases} \rho y_{i,t-1} + (1-\rho)\mu_{i1} + \sigma_1 \varepsilon_{it} \\ \rho y_{i,t-1} + (1-\rho)\mu_{i2} + \sigma_2 \varepsilon_{it} \end{cases}$	Bai (2010)
$H^5 : \rho_1 \neq \rho_2, \mu_{i1} = \mu_{i2}, \sigma_1^2 = \sigma_2^2$	$y_{it} = \begin{cases} \rho_1 y_{i,t-1} + (1-\rho_1)\mu_i + \sigma \varepsilon_{it} \\ \rho_2 y_{i,t-1} + (1-\rho_2)\mu_i + \sigma \varepsilon_{it} \end{cases}$	Liu *et al.* (2011)
$H^6 : \rho_1 = \rho_2, \mu_{i1} \neq \mu_{i2}, \sigma_1^2 = \sigma_2^2$	$y_{it} = \begin{cases} \rho y_{i,t-1} + (1-\rho)\mu_{i1} + \sigma \varepsilon_{it} \\ \rho y_{i,t-1} + (1-\rho)\mu_{i2} + \sigma \varepsilon_{it} \end{cases}$	Bai (2010)
$H^7 : \rho_1 = \rho_2, \mu_{i1} = \mu_{i2}, \sigma_1^2 \neq \sigma_2^2$	$y_{it} = \begin{cases} \rho y_{i,t-1} + (1-\rho)\mu_i + \sigma_1 \varepsilon_{it} \\ \rho y_{i,t-1} + (1-\rho)\mu_i + \sigma_2 \varepsilon_{it} \end{cases}$	Bai (2010)
$H^8 : \rho_1 = \rho_2, \mu_{i1} = \mu_{i2}, \sigma_1^2 = \sigma_2^2$	$y_{it} = \rho y_{i,t-1} + (1-\rho)\mu_i + \sigma \varepsilon_{it}$	Levin *et al.* (2002)

Table-A2: Formulation of Posterior Odds Ratio	
Hypothesis	Posterior Odds Ratio (POR)
Null(H_3): Break in variance and autoregressive coefficient Alternative(H_2): break in mean and autoregressive coefficient	$\beta_{01}^{8}=\frac{p_0}{1-p_0}\frac{\int_{l_1}^{1}\int_{l_2}^{1}\int_{R^n}\frac{d_1^{c_1}d_2^{c_2}}{(2\pi)^{\frac{n}{2}}\Gamma c_1\Gamma c_2\tau^n\sigma^n}\frac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{[\eta(\rho_1,\mu_i)]^{\frac{nT_B}{2}+c_1}}\frac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{[\omega(\rho_2,\mu_i)]^{\frac{n(T-T_B)}{2}+c_2}}\exp\left[-\frac{1}{2\tau^2\sigma^2}\sum_{i=1}^{n}(\mu_i-\vartheta_i)^2\right]d\mu d\rho_1 d\rho_2}{\int_{l_1}^{1}\int_{l_2}^{1}\frac{d^c}{\Gamma c[G(\rho_1)]^{\frac{n}{2}}[H(\rho_2)]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT}{2}+c\right)}{[K(\rho_1,\rho_2)]^{\frac{nT}{2}+c}}d\rho_1 d\rho_2}$
Null(H_4): break in mean and variance Alternative(H_2): break in mean and autoregressive coefficient	$\beta_{01}^{9}=\frac{p_0}{1-p_0}\frac{\int_{l}^{1}\frac{d_1^{c_1}d_2^{c_2}}{(1-l)\Gamma c_1\Gamma c_2[L(\rho)]^{\frac{n}{2}}[M(\rho)]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{[P(\rho)]^{\frac{nT_B}{2}+c_1}}\frac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{[Q(\rho)]^{\frac{n(T-T_B)}{2}+c_2}}d\rho}{\int_{l_1}^{1}\int_{l_2}^{1}\frac{d^c}{(1-l_1)(1-l_2)\Gamma c[G(\rho_1)]^{\frac{n}{2}}[H(\rho_2)]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT}{2}+c\right)}{[K(\rho_1,\rho_2)]^{\frac{nT}{2}+c}}d\rho_1 d\rho_2}$
Null(H_5): break in autoregressive coefficient Alternative(H_2): break in mean and autoregressive coefficient	$\beta_{01}^{10}=\frac{p_0}{1-p_0}\frac{\int_{l_1}^{1}\int_{l_2}^{1}\frac{1}{[R(\rho_1,\rho_2)]^{\frac{n}{2}}[T(\rho_1,\rho_2)]^{\frac{nT}{2}+c}}d\rho_1 d\rho_2}{\int_{l_1}^{1}\int_{l_2}^{1}\frac{1}{[G(\rho_1)]^{\frac{n}{2}}[H(\rho_2)]^{\frac{n}{2}}[K(\rho_1,\rho_2)]^{\frac{nT}{2}+c}}d\rho_1 d\rho_2}$
Null(H_6): break in mean Alternative(H_2): break in mean and autoregressive coefficient	$\beta_{01}^{11}=\frac{p_0}{1-p_0}\frac{\int_{l}^{1}\frac{1}{(1-l)[U(\rho)]^{\frac{n}{2}}[V(\rho)]^{\frac{n}{2}}[Y(\rho)]^{\frac{nT}{2}+c}}d\rho}{\int_{l_1}^{1}\int_{l_2}^{1}\frac{1}{(1-l_1)(1-l_2)[G(\rho_1)]^{\frac{n}{2}}[H(\rho_2)]^{\frac{n}{2}}[K(\rho_1,\rho_2)]^{\frac{nT}{2}+c}}d\rho_1 d\rho_2}$
Null(H_7): break in variance Alternative(H_2): break in mean and autoregressive coefficient	$\beta_{01}^{12}=\frac{p_0}{1-p_0}\frac{\int_{l}^{1}\int_{R^n}\frac{d_1^{c_1}d_2^{c_2}}{(2\pi)^{\frac{n}{2}}(1-l)\Gamma c_1\Gamma c_2\tau^n\sigma^n}\frac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{[\gamma(\rho,\mu_i)]^{\frac{nT_B}{2}+c_1}}\frac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{[\lambda(\rho,\mu_i)]^{\frac{n(T-T_B)}{2}+c_2}}\exp\left[-\frac{1}{2\tau^2\sigma^2}\sum_{i=1}^{n}(\mu_i-\vartheta_i)^2\right]d\mu d\rho}{\int_{l_1}^{1}\int_{l_2}^{1}\frac{d^c}{(1-l_1)(1-l_2)\Gamma c[G(\rho_1)]^{\frac{n}{2}}[H(\rho_2)]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT}{2}+c\right)}{[K(\rho_1,\rho_2)]^{\frac{nT}{2}+c}}d\rho_1 d\rho_2}$
Null(H_8): no break Alternative(H_2): break in mean and autoregressive coefficient	$\beta_{01}^{13}=\frac{p_0}{1-p_0}\frac{\int_{l}^{1}\frac{1}{(1-l)\Gamma c[\phi(\rho)]^{\frac{n}{2}}[Z(\rho)]^{\frac{nT}{2}+c}}d\rho}{\int_{l_1}^{1}\int_{l_2}^{1}\frac{1}{(1-l_1)(1-l_2)\Gamma c[G(\rho_1)]^{\frac{n}{2}}[H(\rho_2)]^{\frac{n}{2}}[K(\rho_1,\rho_2)]^{\frac{nT}{2}+c}}d\rho_1 d\rho_2}$

Null(H_4): break in mean and variance Alternative(H_3): Break in variance and autoregressive coefficient	$\beta_{01}^{14}=\frac{p_0}{1-p_0}\frac{\int_l^1\frac{1}{(1-l)[L(\rho)]^{\frac{n}{2}}[M(\rho)]^{\frac{n}{2}}[P(\rho)]^{\frac{nT_B}{2}+c_1}[Q(\rho)]^{\frac{n(T-T_B)}{2}+c_2}}d\rho}{\int_{l_1}^1\int_{l_2}^1\int_{R^n}\frac{1}{(2\pi)^{\frac{n}{2}}(1-l_1)(1-l_2)\tau^n\sigma^n[\eta(\rho_1,\mu_i)]^{\frac{nT_B}{2}+c_1}[\omega(\rho_2,\mu_i)]^{\frac{n(T-T_B)}{2}+c_2}}\exp\left[-\frac{1}{2\tau^2\sigma^2}\sum_{i=1}^{n}(\mu_i-\vartheta_i)^2\right]d\mu d\rho_1 d\rho_2}$
Null(H_5): break in autoregressive coefficient Alternative(H_3): Break in variance and autoregressive coefficient	$\beta_{01}^{15}=\frac{p_0}{1-p_0}\frac{\int_{l_1}^1\int_{l_2}^1\frac{d^c}{\Gamma c[R(\rho_1,\rho_2)]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT}{2}+c\right)}{[T(\rho_1,\rho_2)]^{\frac{nT}{2}+c}}d\rho_1 d\rho_2}{\int_{l_1}^1\int_{l_2}^1\int_{R^n}\frac{d_1^{c_1}d_2^{c_2}}{(2\pi)^{\frac{n}{2}}\Gamma c_1\Gamma c_2\tau^n\sigma^n}\frac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{[\eta(\rho_1,\mu_i)]^{\frac{nT_B}{2}+c_1}}\frac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{[\omega(\rho_2,\mu_i)]^{\frac{n(T-T_B)}{2}+c_2}}\exp\left[-\frac{1}{2\tau^2\sigma^2}\sum_{i=1}^{n}(\mu_i-\vartheta_i)^2\right]d\mu d\rho_1 d\rho_2}$
Null(H_6): break in mean Alternative(H_3): Break in variance and autoregressive coefficient	$\beta_{01}^{16}=\frac{\frac{p_0}{1-p_0}\int_l^1\frac{d^c}{(1-l)\Gamma c[U(\rho)]^{\frac{n}{2}}[V(\rho)]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT}{2}+c\right)}{[Y(\rho)]^{\frac{nT}{2}+c}}d\rho}{\int_{l_1}^1\int_{l_2}^1\int_{R^n}\frac{d_1^{c_1}d_2^{c_2}}{(2\pi)^{\frac{n}{2}}(1-l_1)(1-l_2)\Gamma c_1\Gamma c_2\tau^n\sigma^n}\frac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{[\eta(\rho_1,\mu_i)]^{\frac{nT_B}{2}+c_1}}\frac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{[\omega(\rho_2,\mu_i)]^{\frac{n(T-T_B)}{2}+c_2}}\exp\left[-\frac{1}{2\tau^2\sigma^2}\sum_{i=1}^{n}(\mu_i-\vartheta_i)^2\right]d\mu d\rho_1 d\rho_2}$
Null(H_7): break in variance Alternative(H_3): Break in variance and autoregressive coefficient	$\beta_{01}^{17}=\frac{p_0}{1-p_0}\frac{\int_l^1\int_{R^n}^{\infty}\frac{1}{(1-l)[\gamma(\rho,\mu_i)]^{\frac{nT_B}{2}+c_1}[\lambda(\rho,\mu_i)]^{\frac{n(T-T_B)}{2}+c_2}}\exp\left[-\frac{1}{2\tau^2\sigma^2}\sum_{i=1}^{n}(\mu_i-\vartheta_i)^2\right]d\mu d\rho}{\int_{l_1}^1\int_{l_2}^1\int_{R^n}^{\infty}\frac{1}{(1-l_1)(1-l_2)[\eta(\rho_1,\mu_i)]^{\frac{nT_B}{2}+c_1}[\omega(\rho_2,\mu_i)]^{\frac{n(T-T_B)}{2}+c_2}}\exp\left[-\frac{1}{2\tau^2\sigma^2}\sum_{i=1}^{n}(\mu_i-\vartheta_i)^2\right]d\mu d\rho_1 d\rho_2}$
Null(H_8): No break Alternative(H_3): Break in variance and autoregressive coefficient	$\beta_{01}^{18}=\frac{p_0}{1-p_0}\frac{\int_l^1\frac{d^c}{(1-l)\Gamma c[\phi(\rho)]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT}{2}+c\right)}{[Z(\rho)]^{\frac{nT}{2}+c}}d\rho}{\int_{l_1}^1\int_{l_2}^1\int_{R^n}^{\infty}\frac{d_1^{c_1}d_2^{c_2}}{(2\pi)^{\frac{n}{2}}(1-l_1)(1-l_2)\Gamma c_1\Gamma c_2\tau^n\sigma^n}\frac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{[\eta(\rho_1,\mu_i)]^{\frac{nT_B}{2}+c_1}}\frac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{[\omega(\rho_2,\mu_i)]^{\frac{n(T-T_B)}{2}+c_2}}\exp\left[-\frac{1}{2\tau^2\sigma^2}\sum_{i=1}^{n}(\mu_i-\vartheta_i)^2\right]d\mu d\rho_1 d\rho_2}$
Null(H_5): break in autoregressive coefficient Alternative(H_4): break in mean and variance	$\beta_{01}^{19}=\frac{p_0}{1-p_0}\frac{\int_{l_1}^1\int_{l_2}^1\frac{d^c}{(1-l_1)(1-l_2)\Gamma c[R(\rho_1,\rho_2)]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT}{2}+c\right)}{[T(\rho_1,\rho_2)]^{\frac{nT}{2}+c}}d\rho_1 d\rho_2}{\int_l^1\frac{d_1^{c_1}d_2^{c_2}}{(1-l)\Gamma c_1\Gamma c_2[L(\rho)]^{\frac{n}{2}}[M(\rho)]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{[P(\rho)]^{\frac{nT_B}{2}+c_1}}\frac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{[Q(\rho)]^{\frac{n(T-T_B)}{2}+c_2}}d\rho}$

Null(H_6): break in mean Alternative(H_4): break in mean and variance	$\beta_{01}^{20}=\dfrac{p_0}{1-p_0}\dfrac{\int_l^1 \dfrac{d^c}{\Gamma c[U(\rho)]^{\frac{n}{2}}[V(\rho)]^{\frac{n}{2}}}\dfrac{\Gamma\left(\frac{nT}{2}+c\right)}{[Y(\rho)]^{\frac{nT}{2}+c}}d\rho}{\int_l^1 \dfrac{d_1^{c_1}d_2^{c_2}}{\Gamma c_1\Gamma c_2[L(\rho)]^{\frac{n}{2}}[M(\rho)]^{\frac{n}{2}}}\dfrac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{[P(\rho)]^{\frac{nT_B}{2}+c_1}}\dfrac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{[Q(\rho)]^{\frac{n(T-T_B)}{2}+c_2}}d\rho}$
Null(H_7): break in variance Alternative(H_4): break in mean and variance	$\beta_{01}^{21}=\dfrac{p_0}{1-p_0}\dfrac{\int_l^1\int_{R^n}\dfrac{1}{(2\pi)^{\frac{n}{2}}\tau^n\sigma^n[\gamma(\rho,\mu_i)]^{\frac{nT_B}{2}+c_1}[\lambda(\rho,\mu_i)]^{\frac{n(T-T_B)}{2}+c_2}}\exp\left[-\frac{1}{2\tau^2\sigma^2}\sum_{i=1}^{n}(\mu_i-\vartheta_i)^2\right]d\mu d\rho}{\int_l^1\dfrac{1}{[L(\rho)]^{\frac{n}{2}}[M(\rho)]^{\frac{n}{2}}[P(\rho)]^{\frac{nT_B}{2}+c_1}[Q(\rho)]^{\frac{n(T-T_B)}{2}+c_2}}d\rho}$
Null(H_8): No break Alternative(H_4): break in mean and variance	$\beta_{01}^{22}=\dfrac{p_0}{1-p_0}\dfrac{\int_l^1 \dfrac{d^c}{\Gamma c[\phi(\rho)]^{\frac{n}{2}}}\dfrac{\Gamma\left(\frac{nT}{2}+c\right)}{[Z(\rho)]^{\frac{nT}{2}+c}}d\rho}{\int_l^1 \dfrac{d_1^{c_1}d_2^{c_2}}{\Gamma c_1\Gamma c_2[L(\rho)]^{\frac{n}{2}}[M(\rho)]^{\frac{n}{2}}}\dfrac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{[P(\rho)]^{\frac{nT_B}{2}+c_1}}\dfrac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{[Q(\rho)]^{\frac{n(T-T_B)}{2}+c_2}}d\rho}$
Null(H_6): break in mean Alternative(H_5): break in autoregressive coefficient	$\beta_{01}^{23}=\dfrac{p_0}{1-p_0}\dfrac{\int_l^1\dfrac{1}{(1-l)\Gamma c[U(\rho)]^{\frac{n}{2}}[V(\rho)]^{\frac{n}{2}}[Y(\rho)]^{\frac{nT}{2}+c}}d\rho}{\int_{l_1}^1\int_{l_2}^1\dfrac{1}{(1-l_1)(1-l_2)\Gamma c[R(\rho_1,\rho_2)]^{\frac{n}{2}}[T(\rho_1,\rho_2)]^{\frac{nT}{2}+c}}d\rho_1 d\rho_2}$
Null(H_7): break in variance Alternative(H_5): break in autoregressive coefficient	$\beta_{01}^{24}=\dfrac{p_0}{1-p_0}\dfrac{\int_l^1\int_{R^n}\dfrac{d_1^{c_1}d_2^{c_2}}{(2\pi)^{\frac{n}{2}}(1-l)\Gamma c_1\Gamma c_2\tau^n\sigma^n}\dfrac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{[\gamma(\rho,\mu_i)]^{\frac{nT_B}{2}+c_1}}\dfrac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{[\lambda(\rho,\mu_i)]^{\frac{n(T-T_B)}{2}+c_2}}\exp\left[-\frac{1}{2\tau^2\sigma^2}\sum_{i=1}^{n}(\mu_i-\vartheta_i)^2\right]d\mu d\rho}{\int_{l_1}^1\int_{l_2}^1\dfrac{d^c}{(1-l_1)(1-l_2)\Gamma c[R(\rho_1,\rho_2)]^{\frac{n}{2}}}\dfrac{\Gamma\left(\frac{nT}{2}+c\right)}{[T(\rho_1,\rho_2)]^{\frac{nT}{2}+c}}d\rho_1 d\rho_2}$
Null(H_8): No break Alternative(H_5): break in autoregressive coefficient	$\beta_{01}^{25}=\dfrac{p_0}{1-p_0}\dfrac{\int_l^1\dfrac{1}{(1-l)\Gamma c[\phi(\rho)]^{\frac{n}{2}}[Z(\rho)]^{\frac{nT}{2}+c}}d\rho}{\int_{l_1}^1\int_{l_2}^1\dfrac{1}{(1-l_1)(1-l_2)\Gamma c[R(\rho_1,\rho_2)]^{\frac{n}{2}}[T(\rho_1,\rho_2)]^{\frac{nT}{2}+c}}d\rho_1 d\rho_2}$

Null(H_7): break in variance Alternative(H_6): break in mean	$$\beta_{01}^{26}=\frac{p_0}{1-p_0}\frac{\int_l^1\int_{R^n}\frac{d_1^{c_1}d_2^{c_2}}{(2\pi)^{\frac{n}{2}}\Gamma c_1\Gamma c_2\tau^n\sigma^n}\frac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{[\gamma(\rho,\mu_i)]^{\frac{nT_B}{2}+c_1}}\frac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{[\lambda(\rho,\mu_i)]^{\frac{n(T-T_B)}{2}+c_2}}\exp\left[-\frac{1}{2\tau^2\sigma^2}\sum_{i=1}^n(\mu_i-\vartheta_i)^2\right]d\mu d\rho}{\int_l^1\frac{d^c}{\Gamma c[U(\rho)]^{\frac{n}{2}}[V(\rho)]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT}{2}+c\right)}{[Y(\rho)]^{\frac{nT}{2}+c}}d\rho}$$
Null(H_8): no break Alternative(H_6): break in mean	$$\beta_{01}^{27}=\frac{p_0}{1-p_0}\frac{\int_l^1\frac{1}{[\phi(\rho)]^{\frac{n}{2}}[Z(\rho)]^{\frac{nT}{2}+c}}d\rho}{\int_l^1\frac{1}{[U(\rho)]^{\frac{n}{2}}[V(\rho)]^{\frac{n}{2}}[Y(\rho)]^{\frac{nT}{2}+c}}d\rho}$$
Null(H_8): no break Alternative(H_7): break in variance	$$\beta_{01}^{28}=\frac{p_0}{1-p_0}\frac{\int_l^1\frac{d^c}{\Gamma c[\phi(\rho)]^{\frac{n}{2}}}\frac{\Gamma\left(\frac{nT}{2}+c\right)}{[Z(\rho)]^{\frac{nT}{2}+c}}d\rho}{\int_l^1\int_{R^n}\frac{d_1^{c_1}d_2^{c_2}}{(2\pi)^{\frac{n}{2}}\Gamma c_1\Gamma c_2\tau^n\sigma^n}\frac{\Gamma\left(\frac{nT_B}{2}+c_1\right)}{[\gamma(\rho,\mu_i)]^{\frac{nT_B}{2}+c_1}}\frac{\Gamma\left(\frac{n(T-T_B)}{2}+c_2\right)}{[\lambda(\rho,\mu_i)]^{\frac{n(T-T_B)}{2}+c_2}}\exp\left[-\frac{1}{2\tau^2\sigma^2}\sum_{i=1}^n(\mu_i-\vartheta_i)^2\right]d\mu d\rho}$$