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Aleksandar Vasilev

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EERI
Economics and Econometrics Research Institute
Avenue Louise
1050 Brussels
Belgium

Tel: +32 2271 9482
Fax: +32 2271 9480
www.eeri.eu

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JEL classification: H22, J46, D51, D91, O41
Keywords: Progressive taxation; Informal Sector; Equilibrium (In)determinacy

*Aleksandar Vasilev is a CERGE-EI Affiliate Fellow and an Asst. Professor at the Department of Economics, American University in Bulgaria, 1 Georgi Izmirliev Sq., Blagoevgrad 2700, Bulgaria. Tel: 00 359 73 888 482. All errors and omissions are mine alone. E-mail for correspondence: avasilev@aubg.bg.

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1 Introduction and Motivation

Tax policies, and in particular personal income taxation policies, are known to affect households' incentives to invest in physical capital, and their decisions to provide labor services to businesses. The analysis of the effect of tax policies within the framework of exogenous growth models with a representative agent is relatively recent, e.g., King and Rebelo (1990). This paper adds to earlier research by focusing on the official-unofficial sector labor choice, and the unofficial sector technology is viewed as an alternative (labor-intensive) way to produce goods and services. Following Hansen (1985), who argues that variation in official hours worked is due to variations in employment, workers only need to decide whether to participate or not in the official sector. That is, labor in the official sector can be regarded as indivisible (as in Rogerson 1988), while in the the grey economy an individual can supply any number of hours, *i.e.*, labor in the unregistered sector is divisible. As in Vasilev (2015b), each individual face a multi-stage decision. It has to choose first whether or not to work full-time in the official labor market. Then, conditional on not working in the registered economy, whether to work in the grey economy, and if so, to choose many hours to supply there. The presence of the unofficial sector, and the participation decision margin create interesting interactions in the model, as shown in Vasilev (2015b).

As in Chen and Guo (2015) and Vasilev (2016), the focus in this paper is to examines the instability effect of progressive taxation in the case of Bulgaria pre-2008 and compare and contrast the results to the flat tax reform regime in place as of 2008. Importantly, our work differs from that earlier study. While our findings are qualitatively similar to that in Chen and Guo (2013, 2015), here there is no endogenous growth, and the mechanism is based on labor allocation between the official and unofficial sector. By investment in physical capital, the after-tax marginal productivity of labor is kept from decreasing, as compared to the return to labor in the official sector. Earnings from the grey economy are not taxed, though, which creates a sector-specific externality, which as pointed out in Farmer (1999), could create indeterminacy.

Our results come in stark contrast to Guo and Lansing (1988) who argue that a sufficiently progressive tax schedule can stabilize a real-business-cycle model, which possesses an indeterminate steady-state against fluctuations driven by "animal spirits." Indeed, in standard Keynesian setups, progressivity of the tax system is regarded as an automatic stabilizer. This is no longer the case in our model with unofficial sector. The reason is that since output estimates generally impute the size of the unofficial sector, but income taxes are levied on official production only, grey economy sector produces increasing returns to scale.[1]

The theoretical setup used in this paper to study the flat tax reform in Bulgaria follows the setup in Vasilev (2015b), which follows Conesa *et al.* (2001) and augments their framework with a sufficiently-detailed government sector to capture the distortionary effect of

[1] This is easily established using the specific functional forms for official and unofficial production provided later in the paper.

personal income taxation in Bulgaria. From early 1990s, up until Dec. 31, 2007, Bulgaria applied progressive income taxation on personal income,[2] with tax brackets for 2007 reported in Table 1 below.

Table 1: Progressive Income Taxation in Bulgaria in 2007

Monthly taxable income (in BGN)	Tax owed
0-200	Zero-bracket amount
200-250	20% on the amount earned above BGN 200
250-600	BGN 10 + 22% on the excess over BGN 250
> 600	BGN 87 + 24% on the excess over BGN 600

Source: author's calculations.

As of January 1, 2008, a proportional (flat) tax rate of 10% on personal income was introduced. At the same time, , who were previously paying no taxes, . To compensate workers at the bottom of the income distribution, who suddenly faced a positive tax rate, the monthly minimum wage was increased: it went up in several steps eventually reaching BGN 420 as of Jan. 2016. Overall, under proportional taxation system featuring a lower effective income tax rate than the corresponding rate under the progressive regime, a significant reallocation of labor from unregistered activities to the official sector was observed (Vasilev 2015b). This relocation was driven by the increase to after-tax return to labor in the registered economy, and thus making working in the grey sector less attractive. In addition, since labor and capital are complements in the production of registered output at the aggregate level, the increase in official employment increases the marginal productivity of capital. In turn, the higher return to physical capital provides a strong incentive for households to increase capital accumulation, thus enhancing the productive capacity of the economy. This generates a saddle-path dynamics by decreasing the magnitude of the IRS due to the shrinking of the unofficial output.

The rest of the paper is organized as follows: Section 2 presents the model setup and defines the equilibrium system. Section 3 describes the data used and the calibration procedure. Section 4 characterizes the model economy's long-run behavior under both the progressive and proportional income taxation regimes. Section 5 evaluates the model stability around the steady-state for both the progressive taxation and flat-tax regimes. Section 6 concludes.

2 Model Setup

2.1 Description of the model:

The model setup follows closely the framework utilized in Vasilev (2015b). There is a continuum of ex-ante identical agents ("households") distributed uniformly on the $[0, 1]$ interval.

[2] The description of the progressive tax system in Bulgaria in this section follows the structure used in Vasilev (2015a).

Each single-member household in the model economy is infinitely-lived, and maximizes

$$\sum_{t=0}^{\infty} \beta^t [\ln c_t + \alpha \ln(l_t)], \tag{2.1}$$

where c_t is consumption at time t, and l_t is the leisure enjoyed at time t. The parameter β is the discount factor, with $0 < \beta < 1$, and $\alpha > 0$ is the relative weight attached to the utility of leisure.[3]

Next, the household has a time endowment of unity in each period, which could be used for work in the official, the unofficial ("black market") sector, or enjoyed as leisure, so that

$$h_t^m + h_t^b + l_t = 1, \tag{2.2}$$

where $h_t^m \in \{0; \bar{h}\}$ is the indivisible time devoted to working in the official sector in period t, and $h_t^b \in [0, 1]$ is the (divisible) time spent in the unofficial sector in period t. Also, $h_{bt} = 0$ whenever $h_t^m = \bar{h}$, hence $0 \leq h_{mt} + h_{bt} \leq 1$. The hourly wage rate in the official ("market") and the unofficial ("black market") sectors is denoted by w_t^m and w_t^b, respectively.

Following the arguments in Rogerson (1988) and Hansen (1985), it can be easily shown that in equilibrium it must be the case that a proportion μ_t ($0 < \mu_t < 1, \forall t$) of the agents in the economy are working in the unofficial sector, while the rest, $1 - \mu_t$, will be supplying labor services in the official sector. Workers in the official sector will receive consumption c_t^m, while those working in the unofficial sector will consume c_t^b.[4] In equilibrium, consumption across sectors will be equalized: $c_t^m = c_t^b = c_t$. Note that μ_t can be interpreted also as the probability of being chosen to work in the unofficial sector in period t. This probability is determined endogenously in the model, as workers would seek for the optimal balance between the net return from working across the sectors (at the margin).

In addition to the labor income generated, each household saves by investing i_t in physical capital. As an owner of capital, the household receives gross interest income $r_t k_t$ from renting the capital to the firms; r_t is the before-tax return to private capital, and k_t denotes physical capital stock in the beginning of period t. Each household's physical capital evolves according to the following law of motion:

$$k_{t+1} = i_t + (1 - \delta) k_t, \tag{2.3}$$

where $0 < \delta < 1$ is the depreciation rate on capital.

Finally, the households own all firms in the economy, and receive an equal share of total profit (π_t) in the form of dividends. The households' aggregate budget constraint is

$$c_t + i_t \leq (1 - \tau_t)[r_t k_t + w_t^m (1 - \mu_t) \bar{h}] + \mu_t w_t^b h_t^b + \pi_t, \tag{2.4}$$

[3]Non-separability and elasticity of labor supply aspects are not relevant as additional propagation mechanisms of indeterminacy in this setup due to the assumed indivisibility of labor in the official sector.

[4]This in turn means that everyone working in the unofficial sector will choose the same amount of hours.

where, as in Guo and Lansing (1998), tax schedule is represented by the following function:

$$\tau_t = \eta \left(\frac{y_t^o}{y^o} \right)^{\phi}, \tag{2.5}$$

where τ_t denotes the tax rate on total (capital and labor) registered income, *i.e*, $y_t^o = r_t k_t^h + w_t^m (1-\mu_t)\bar{h}$, and y is the steady-state level of household's income. In addition, $0 < \eta < 1$ and $0 \leq \phi < 1$, where ϕ measures the progressivity of the tax system, and η is the average effective tax rate in steady state.
The reformulated aggregate problem of households is now to maximize

$$\sum_{t=0}^{\infty} \beta^t [\ln(c_t) + (1-\mu_t)\alpha \ln(1-\bar{h}) + \mu_t \alpha \ln(1-h_t^b)], \tag{2.6}$$

s.t.

$$c_t + k_{t+1} - (1-\delta)k_t = (1-\tau_t)[r_t k_t + w_t^m (1-\mu_t)\bar{h}] + \mu_t w_t^b h_t^b + \pi_t. \tag{2.7}$$

The households acts competitively by taking prices $\{w_t^m, w_t^b, r_t\}_{t=0}^{\infty}$, income tax schedule τ_t as given, and chooses allocations $\{c_t, i_t, k_t, \mu_t, h_t^b\}_{t=0}^{\infty}$ to maximize Eq.(2.6) s.t Eqs.(2.2)-(2.5) and (2.7), and the initial condition $\{k_0\}$ for physical capital stock.

The optimality conditions from the household's problem, together with the transversality condition (TVC) for physical capital are as follows:

$$\begin{aligned}
c_t : c_t^{-1} &= \lambda_t & (2.8)\\
k_{t+1} : \lambda_t &= \beta \lambda_{t+1} \left[(1-\delta) + \left(1 - (1+\phi)\tau_t \right) r_{t+1} \right] & (2.9)\\
\mu_t : \alpha \left[\ln(1-h_t^b) - \ln(1-\bar{h}) \right] &= \lambda_t \left[\left(1 - (1+\phi)\tau_t \right) w_t^m \bar{h} - w_t^b h_t^b \right] & (2.10)\\
h_{bt} : \alpha(1-h_t^b)^{-1} &= \lambda_t w_t^b & (2.11)\\
TVC : \lim_{t\to\infty} \beta^t c_t^{-1} k_{t+1} &= 0, & (2.12)
\end{aligned}$$

where λ_t is the Lagrangian multiplier on the household's budget constraint. In Eq. (2.8), the household consumes at a point where marginal utility from consumption equals the marginal cost imposed on the budget. Eq. (2.9) describes the optimal capital stock allocations chosen in any two contiguous periods. Participation rate in Eq. (2.10) is chosen so that the the net return from working an extra hour unofficially equals the net cost of doing so. From Eq. (2.11), hours in the grey economy will be chosen so that the disutility of unofficial work at the margin equals the return to labor in the grey economy. The last expression, Eq. (2.12), is the TVC, or the boundary condition imposed on capital.

2.2 Stand-in Firm: market sector

There is also a representative private firm in the model economy. It produces a homogeneous final product using a production function that requires physical capital k_t and labor $H_t^m =$

$(1-\mu_t)\bar{h}$. The production function is as follows

$$y_t^o = Ak_t^\theta (H_t^m)^{1-\theta}, \tag{2.13}$$

where y_t^o denotes official output produced in period t, A measures the level of total factor productivity, and $0 < \theta < 1$ denote the productivity of physical capital and $1-\theta$ captures the productivity of labor.

The representative firm acts competitively by taking prices $\{w_t^m, r_t\}_{t=0}^\infty$, and chooses $k_t, H_t^m, \forall t$ to maximize firm's static profit:

$$\pi_t = Ak_t^\theta (H_t^m)^{1-\theta} - r_t k_t - w_t^m H_t^m. \tag{2.14}$$

In equilibrium profit is zero in all periods. In addition, efficiency labor and capital receive their marginal products, *i.e.*

$$r_t = \theta \frac{y_t^o}{k_t}, \tag{2.15}$$

$$w_t^m = (1-\theta)\frac{y_t^o}{H_t^m}. \tag{2.16}$$

2.3 Stand-in Firm: unofficial sector

Each worker in the unofficial sector has access to an individual production function that uses only labor, given by Bh_t^γ.[5] As in Conesa *et al.* (2001), the labor intensive specification for the production process in the unregistered economy seems to be an adequate approximation to reality. Each firm in the unofficial sector will then hire labor h_{bt} in every period to maximize static profit

$$\max_{h_t^b} B(h_t^b)^\gamma - w_t^b h_t^b. \tag{2.17}$$

With free entry, there are zero profits, hence the implicit wage in the unofficial sector equals

$$w_t^b = B(h_t^b)^{\gamma-1}. \tag{2.18}$$

2.4 Government sector

The government collects tax revenue from registered labor and capital income to finance government expenditure, which are then spent on wasteful government consumption $\{g_t\}_{t=0}^\infty$. The government budget constraint is then

$$\tau_t \Big[r_t k_t + w_t^m (1-\mu_t)\bar{h} \Big] = g_t. \tag{2.19}$$

Government takes prices $\{w_t^m, r_t\}_{t=0}^\infty$ and allocations $\{k_t, \mu_t\}_{t=0}^\infty$ as given. The income tax schedule $\{\tau_t\}_{t=0}^\infty$ will be vary with income, while government consumption $\{g_t\}_{t=0}^\infty$ will be residually determined: it will adjust to ensure the government budget constraint is balanced in every time period.

[5] In equilibrium, there will be μ_t of those, so aggregate unofficial output equals $y_t^b = \mu_t B h_t^\gamma$.

2.5 Decentralized Competitive Equilibrium

2.5.1 Definition

Given the initial conditions for the state variable k_0, a Decentralized Competitive Equilibrium (DCE) is defined to be a sequence of prices $\{r_t, w_t^m, w_t^b\}_{t=0}^{\infty}$, allocations $\{c_t, i_t, k_t, \mu_t, h_t^b, g_t\}_{t=0}^{\infty}$, income tax schedule $\{\tau_t\}_{t=0}^{\infty}$ such that (i) households' expected utility is maximized; (ii) the stand-in firm in the official sector maximizes profit every period; (iii) wage rate in the unofficial sector is such that profits in the grey economy are zero every period; (iv) government budget is balanced in each time period; (iv) all markets clear.[6]

3 Data and model calibration

The model is calibrated to Bulgarian data at annual frequency. The period under investigation is 1993-2014 where 1993-2007 is when taxation was progressive, and 2008-14 is the flat tax regime. Data on the output, household consumption, private fixed investment shares in output, employment rate, the average wage rate, and the minimum wage rate was obtained from the National Statistical Institute (NSI). Table 2 on the next page summarizes the values of all model parameters.

Table 2: Model Parameters

Param.	Value	Definition	Source
β	0.959	Discount factor	Calibrated
θ	0.429	Capital income share	Data avg.
γ	0.571	Labor intensity underground production	Set
μ	0.533	Participation rate, unofficial sector	Data avg.
δ	0.050	Depreciation rate of physical capital	Set
α	0.611	Relative weight on leisure in utility function	Calibrated
η	{0.11; 0.14}	Average effective income tax rate (flat *vs.* progr.)	Data avg.
ϕ	{0; 0.43}	Progressivity parameter (flat *vs.* progr.)	Data avg.
A	1.072	Steady-state level of total factor productivity	Calibrated
B	0.910	Scale parameter underground production function	Calibrated

The values were obtained following a standard approach adopted in quantitative macroeconomics. Physical capital income share is set to its average value $\theta = 0.429$, and the labor income share is $1-\theta = 0.551$. Parameter $\gamma = 0.571$ of the grey economy production function is chosen equal to the labor intensity in the the official sector. Next, we use Vasilev's (2015b) estimate that $\delta = 0.05$, and that $K/Y = 3.491$. Next, we compute the average effective tax rate $\eta = 0.14$ and the (gross) degree of progressivity was computed to be $1+\phi = 1.43$ for the progressive regime, and $\eta = 0.11, \phi = 0$ for the flat tax. Next, from the steady-state Euler equation, we can calibrate the discount factor $\beta = 0.959$. The relative weight on leisure in

[6]The system of equations is provided in the Appendix

the household's utility function, parameter α, will be set to match the steady-state participation rate in the registered sector in Bulgaria over the period $1-\mu = 0.467$ (NSI 2015). We assume a typical household will work full-time, or on average $\bar{h} = 1/3$, in the official sector.

Technology in the underground sector is assumed to be such that workers working full time in the grey economy would earn the minimum wage. Thus parameter B will be set to match the ratio between the (average) market wage and the minimum wage (for total hours 1/3 worked in the unregistered sector), or $w^m/w^b = 2.51$. Normalizing steady-state official output to unity, we obtain $A = 1.072$ and $B = 0.910$.

4 Steady-State

Once model parameters were obtained, the steady-state ratios for the model calibrated to Bulgarian data were obtained. The results are reported in Table 3 on the next page for both tax regimes. In particular, keeping discount factor and depreciation rate constant, a lower

Table 3: Data Averages and Long-run solution

	Description	Data	Model (progr.)	Model (flat tax)
c/y	Consumption-to-output ratio	0.674	0.685	0.685
i/y	Fixed investment-to-output ratio	0.201	0.175	0.205
g/y	Gov't consumption-to-output ratio	0.176	0.140	0.110
k/y	Physical capital-to-output ratio	3.491	3.491	4.115
$w^m(1-\mu)\bar{h}/y$	Labor share in output	0.571	0.571	0.571
rk/y	Capital share in output	0.429	0.429	0.429
$\bar{h}$	Share of time spent working	0.333	0.333	0.333
μ	Employment rate in the grey economy	0.217	0.533	0.000

effective tax rate and no progressivity will raise the after-tax real interest rate. In turn, that would increase capital stock, and lower the employment rate in the unofficial sector, relocate that labor toward the official sector, and ultimately increase consumption.

5 Stability of Equilibrium Dynamics

The equilibrium system is now log-linearized around its unique steady-state, and after simplification, it can be represented by a system of two first-order difference equations in consumption and physical capital:

$$\begin{pmatrix} \hat{c}_{t+1} \\ \hat{k}_{t+1} \end{pmatrix} = \begin{pmatrix} A1 & A2 \\ A3 & A4 \end{pmatrix} \begin{pmatrix} \hat{c}_t \\ \hat{k}_t, \end{pmatrix} \tag{5.1}$$

where scalars $A1$, $A2$, $A3$, $A4$ are functions of model parameters. There are two distinct, real characteristic roots:

$$\lambda_1 = \frac{(A1 + A4) + \sqrt{(A1 - A4)^2 + 4A2.A3}}{2}$$
$$\lambda_2 = \frac{(A1 + A4) - \sqrt{(A1 - A4)^2 + 4A2.A3}}{2}$$

For Bulgaria under the progressive taxation regime (1993-2007), we obtain the following values:

$$A1 = 0.96, A2 = -0.01, A3 = -0.19, A4 = 0.23$$
$$\lambda_1 = 0.97, \lambda_2 = 0.24.$$

Given that the reduced-form representation of the equilibrium system features two characteristic roots that are less than unity, the model features global stability (indeterminacy or "sink dynamics"). Intuitively, this means that the Bulgarian economy under the progressive taxation regime can reach the steady state with either high or low consumption. As in Farmer (1999), the unofficial sector generates a sector-specific externality, as unregistered income is not taxed. In addition, unofficial output adds to total production in the computation of gross domestic product and thus the framework creates increasing returns to scale.

In contrast, for Bulgaria under the proportional (flat) tax regime (2008-2014) we obtain

$$A1 = 0.8, A2 = 0, A3 = 14.7, A4 = -2.17$$
$$\lambda_1 = 0.8, \lambda_2 = -2.17.$$

Now the model exhibits saddle-path stability, with one stable and one unstable root. Under proportional taxation, which features a lower effective tax rate, all labor relocates to the official sector, and there is no grey economy ($\mu = 0$). Therefore, aggregate output is the official production, which is produced using a constant-returns-to-scale technology.[7] We discuss the results for the (lack of) indeterminacy in detail in the following section.

6 Discussion

In this section we argue that the model discussed in this paper with informal sector is an isomorphic problem to a setup with increasing returns to scale and/or sector-specific externality. This is because total output in this framework is the sum of official production and the total unofficial output. Registered output is produced using a Cobb-Douglas function, which features constant returns to scale (CRS). In the official sector everyone works a full working week, so the only way to upscale hours is to increase the participation rate $1 - \mu$. On the other hand, production function in the grey economy features a decreasing returns

[7]When computing the terms, $\mu = 0$ creates problems, as we need to divide by zero, thus we set it to a very small positive value (0.001).

to scale (DRS). However, when we aggregate over individual unregistered production, the aggregate unofficial production function already features increasing returns to scale (IRS). Since hours in the unofficial sectors are set to $\bar{h}$, again the only way to increase total output is to increase the participation rate in the unofficial sector μ. Thus, aggregate production function in the grey economy becomes an AL-type one. However, since $\mu \in [0, 1]$, there will be no endogenous growth, and thus no balanced growth path. Overall, aggregate official and unofficial production function features IRS, as it is a sum of a function featuring CRS and another, which features IRS.

The existence of IRS in this setup are easy to justify, as a grey sector is always an option, and official GDP figures try to impute the size of unregistered activity in national accounts. Also, the unofficial sector is treated differently than the official sector, as taxes are based on registered production only. Thus the presence of an informal sector generates externalities in production. Also, Farmer (1999) has shown that the presence of IRS can produce indeterminate equilibria, as long as the increasing returns are large enough. In this case the magnitude of the IR is represented by the size of the unofficial sector relative to overall production. The size of the grey economy is driven by two parameters - μ and B, where the former is set to its average data value, and the latter is obtained from the restriction imposed on the ratio of the two wage rates.)

The other link to externalities is the two-sector modelling choice in our framework. In contrast to Benhabib and Farmer (1996), who model an economy with separate production processes for consumption and investment, in this paper consumption, investment, and government spending can be financed with proceeds from both official and unofficial production. The trade-off between producing unofficial output and official output in our model is driven by the labor allocation decision. Even though the two technologies produce the same goods, there is a different treatment in the model between the two sectors. The informal output is non-tradable, and not directly observable. The other aspect of externality generated by the presence of the informal sector in the model setup is that it is a non-competitive sector, as the wage rate in the grey economy differs from the marginal productivity of labor in that sector. This is because the sector is a monopolistic one: the firm faces a downward-sloping demand curve for labor in the unofficial sector. Therefore, in equilibrium the black market wage will feature a fixed mark-up $0 < 1/\gamma < 1$ over the marginal cost (or equivalently, the wage features a mark-up above the marginal productivity of labor). This pricing rule is obtained when we impose the zero-profit condition in the sector, which is in the spirit of free entry in models with monopolistic competition.

Finally, to a certain extent the externality in the model comes from an initial non-convexity - that each household is allowed to work either full-time in the official sector or nothing, while it can supply any number of hours in the grey sector (but cannot work in both sectors simultaneously). So in a sense we have a double non-convexity. We smooth the first binary choice by introducing a participation lottery a la Rogerson (1988) and Hansen (1985). The participation rate in the model is chosen optimally by households. Still, that is an incomplete participation even though markets are complete, as households are not allowed to participate in both sectors. So even without aggregate uncertainty, extrinsic uncertainty (uncertainty that does not affect model primitives) may matter for equilibrium. For example, given certain beliefs, different allocations can be produced, even though nothing fundamental has changed. These are also called "self-fulfilling prophesies," which are at the core of the Keynesian view of business cycles. As pointed out in Farmer (1999), one reason for such beliefs to occur in our model could be the *incomplete participation*, that despite the existence of complete set of markets, agents are not allowed to transact and trade labor services in all of them.

7 Conclusions

We show that in a exogenous growth model with informal economy calibrated to Bulgarian data under the progressive taxation regime (1993-2007), the economy exhibits equilibrium indeterminacy due to the the presence of an unofficial production. These results are in line with the findings in Benhabib and Farmer (1994, 1996) and Farmer (1999). Also, the findings in this paper are in contrast to Guo and Lansing (1988) who argue that progressive taxation works as an automatic stabilizer. Under the flat tax regime (2008-14), the economy calibrated to Bulgarian data displays saddle-path stability. The decrease in the average effective tax rate addresses the indeterminacy issue and eliminates the "sink" dynamics.

Appendix: Equilibrium System of Equations

$$
\begin{aligned}
c_t^{-1} &= \lambda_t \\
\tau_t &= \eta(y_t^o/y^o)^{\phi} \\
\lambda_t &= \beta\lambda_{t+1}\left[(1-\delta)+\left(1-(1+\phi)\tau_{t+1}\right)r_{t+1}\right] \\
\alpha[\ln(1-h_t^b)-\ln(1-\bar{h})] &= \lambda_t\left[\left(1-(1+\phi)\tau_t\right)w_t^m\bar{h}-w_t^b h_t^b\right] \\
\alpha(1-h_t^b)^{-1} &= \lambda_t w_t^b \\
H_t^m &= (1-\mu_t)\bar{h} \\
r_t &= \theta\frac{y_t^o}{k_t} \\
w_t &= (1-\theta)\frac{y_t^o}{H_t^m} \\
w_t^b &= B(h_t^b)^{\gamma-1} \\
g_t &= \tau[r_t k_t + w_t H_t^m] \\
y_t^b &= B(h_t^b)^{\gamma} \\
y_t^o &= A k_t^{\theta}(H_t^m)^{1-\theta} \\
y_t &= y_t^o + y_t^b \\
y_t &= c_t + k_{t+1} - (1-\delta)k_t + g_t
\end{aligned}
$$

References

Benhabib, J. and R.E. Farmer (1994). ”Indeterminacy and increasing returns.” *Journal of Economic Theory* 63: 19-46.
Benhabib, J. and R.E. Farmer (1996). ”Indeterminacy and sector-specific externalities.” *Journal of Monetary Economics* 37: 421-43.
Conesa, J.C., C.D. Moreno, and J. E. G. Sanchez (2001) ”Underground economy and aggregate fluctuations,” *Spanish Economic Review* 3, pp. 41-53.
Farmer, R.E. (1999) *Macroeconomics of self-fulfilling prophesies*, 2nd ed. The MIT Press: London: England.
Guo, J.-T. and K.J. Lansing. (1998) ”Indeterminacy and Stabilization Policy.” *Journal of Economic Theory* 82: 481-490.
Hansen, G. (1985) ”Indivisible Labor and the Business Cycle,” *Journal of Monetary Economics*, 16, pp. 309-327.
King, R.G. and S. Rebelo. (1990) ”Public Policy and Economic Growth: Developing Neoclassical Implications,” *Journal of Political Economy*, 98, S127-S150.
NSI (2015). National Statistical Institute Database. Available on-line at www.nsi.bg. Accessed on Dec. 14, 2015.
Rogerson, R. (1988) ”Indivisible Labor, Lotteries and Equilibrium,” *Journal of Monetary Economics*, 21, pp. 3-16.
Vasilev, A. (2015a) ”The welfare effect of flat income tax reform: the case of Bulgaria,” *Eastern European Economics*, 53 (3), 205-20.
Vasilev, A. (2015b) ”Welfare gains from the adoption of proportional taxation in a general-equilibrium model with a grey economy: the case of Bulgaria’s 2008 flat tax reform,” *Economic Change and Restructuring* 48 (2): 169-185.
Vasilev, A.Z. (2016) ”Progressive taxation and (in)stability in an edogenous growth model with human capital accumulation,” *Journal of Economics and Econometrics* 59(2): 1-15.