Privatization in an International Mixed Oligopoly: the Role of Product Differentiation under Price Competition

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Abstract. By developing a linear model in a two-country framework of international price competition, we show how the degree of product differentiation and the cross-country distribution of private firms affect the strategic privatization choices made by governments concerned with their own country’s welfare. More particularly, the work points out that sufficiently low product differentiation may lead public ownership to be optimally chosen to restrict competition in the country with the larger number of firms, and privatization to be global welfare enhancing in this case.

JEL Classification: F23, L13, L32
Keywords: Mixed oligopoly; price competition; strategic privatization; international markets.

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1 Introduction

This work develops a strategic game of privatization involving two governments concerned with their own country’s social welfare. In this two-country model, each government decides whether to privatize or not its own public (welfare-maximizing) firm which competes in a common international market with the other country’s public firm and a variable number of private (profit-maximizing) firms from both countries. Using a mixed oligopoly approach, we first derive the outcome of market competition in the presence of two public firms, one from each country. Such an outcome is then compared with that of a mixed market under privatization of one of the public firms (i.e., unilateral privatization) and that of an entirely privatized market, which allows us to determine the equilibrium choices of the strategic game.

A trend towards privatization of state-owned enterprises has been registered since the early 1980s in both the developed and the developing economies as a key feature of the deregulation or the transition processes underway in those countries. The growing public deficit and the need for improving external competitiveness and overall efficiency have been often listed as reasons for this trend. Privatization waves started in Europe following the post-war nationalizations and limited the role of the State as a central actor in many sectors such as network industries (i.e., telecommunications, transports, energy and utilities), banking and insurance, postal services, education and health. These sectors have also been increasingly exposed to international competition, in response to international liberalization and demand growth. The contribution of mixed oligopoly theory to the privatization debate is broadly acknowledged in the economic literature. Within this theory, indeed, a number of studies have discussed in the last three decades the strategic and efficiency reasons supporting the idea that the presence of public firms on markets acts as a regulation mechanism which improves resource allocation and enhances overall efficiency with respect to a privatized context. Welfare-maximization by a public firm, by inducing an output expansion that translates into a larger industry output, has been invoked as the main reason for social desirability of a mixed market. As shown by De Fraja and Delbono (1989) under the assumption of convex costs, the presence of a high producing public firm benefits social welfare, provided that the number of private firms is sufficiently low, case in which it favors consumer surplus more than it negatively affects cost allocation and private firms’ output and profits. De Fraja (1991) also demonstrates how the higher allocative efficiency induced by higher production can overcome the lower productive efficiency induced by an assumed managerial slackness in a mixed market relative to a privatized one.

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1 Public ownership is generally associated with welfare maximization objectives. This assumption, however, has been relaxed in many contexts by letting public firms maximize a weighted average of welfare and their own profits, which allows for partial privatization. The search for the optimal extent of privatization is a core issue of this literature; see Matsumura (1998) as a major reference in the field.

2 Convex costs are assumed to avoid the trivial case of quantity competition under constant marginal costs, thus guaranteeing a positive output for private firms.
provided that inefficiency of the public firm is low enough. By contrast, Cremer et al. (1989) show how the benefits from the presence of more than one public firm in the market are limited by a budget constraint which limits total output expansion. Public ownership is also shown to result in a higher degree of productive efficiency by providing most effective cost-reducing incentives through higher R&D investments (Poyago-Theotoky, 1998; Ishibashi and Matsumura, 2006; Cato, 2011; Gil-Moltò et al., 2011) or an appropriate incentive contract for the public manager (De Fraja, 1993).

The effects of the presence of public firms on international competition have been also considered in a number of works raising issues of interest for both industrial organization and international trade theory. The extent to which the competitive pressure exerted by private foreign firms alters competition on a mixed domestic market is central to the analyses carried out in a single-country perspective. While the assumption of quantity competition is spread in this strand of literature, price competition has been less extensively investigated, with the exception of Matsumura et al. (2009), Ohnishi (2010) and Chirco and Scrimitore (2010). A two-country approach has been used to investigate the extent to which interactions between governments affect unilateral or coordinated privatization (Dadpay and Heywood, 2006), strategic privatization (Bárcena-Ruiz and Garzón, 2005a), the optimal degree of privatization (Han and Ogawa, 2008), and strategic trade policy (Pal and White, 2003). Within this literature, both quantity competition and the existence of decreasing returns to scale, or of an efficiency gap between the public and the private firms under constant marginal costs, are common assumptions.

In this paper, a two-country model is developed under the assumption of price competition. This assumption, besides capturing a more plausible mode of competing on international markets, also allows us to run the model under

3 The higher costs exogenously attributed to public ownership relative to private ownership in De Fraja (1991) and the endogenous lower efficiency of the public high-producing firm under convex costs in De Fraja and Delbono (1989) motivate privatization in both contexts. Indeed, the objective of profit maximization pursued by all firms may enhance social welfare by improving ex-ante productive efficiency of the public firm in the former, and by equalizing firm production, thus also reducing total costs, in the latter. However, the belief that public ownership is cost-inefficient compared to private ownership has been often challenged and does not receive unanimous consensus, neither in the theoretical nor in the empirical literature (on this point see also Willner and Parker (2007), Paragraph 2.1, and the works referred to therein).

4 In such contexts, the presence of foreign private competitors on a domestic market has been shown to affect the optimal privatization policy (Chang, 2005; Chao and Yu, 2006; Matsumura and Tomaru, 2012; Nabin et al., 2014), market opening policy and cross-borders acquisitions (Fjell and Pal, 1996), strategic trade policy (Pal and White, 1998), as well as the cost-reducing incentives by the public firm (Tomaru, 2007).

5 Chirco and Scrimitore (2010) examine the outcome of price competition under product differentiation in an international one-country oligopoly. They find that the presence of a public firm on the domestic market, regardless of the degree of product differentiation, always succeeds in enforcing internal market discipline, since it induces all private firms to keep prices lower and react to international competition with further beneficial price reductions.

6 See also Bárcena-Ruiz and Garzón (2005b) and Xu et al. (2016) as valuable contributions in the field.
constant returns to scale and explore the impact of product differentiation on the strategic privatization choice. Indeed, the peculiar effects of price competition in a mixed oligopoly framework, originally highlighted by Ghosh and Mitra (2010), enlarge significantly the set of market configurations in which the public firms’ budget constraint is consistent with constant average and marginal costs, thus allowing us to focus on the welfare properties of firms’ strategic interactions and rule out any exogenous or endogenous technological asymmetry between public and private firms. In such a scenario, the choice to strategically privatize or not its state-controlled firm is made at the first stage of a game by each government which maximizes its own country welfare under free trade; at the last stage of the game, all firms compete offering imperfectly substitutable products and sharing the same linear technology. Therefore, the market structure arising as a Nash equilibrium of the game may entail a mixed market with a public firm from each country, or a mixed market with a public firm from one country only (i.e., unilateral privatization) or, finally, a fully privatized market. By solving the two stage game, we investigate the way in which both product differentiation and the cross-country distribution of firms affect the optimal privatization choice in each country. Moreover, we aim at exploring the extent to which such strategies are welfare-enhancing or welfare-detrimental for the market as a whole: by assuming that decisions regarding privatization could be delegated to a supra-national authority, we verify whether its choice would diverge from the non-cooperative decisions taken by governments.

Starting from a benchmark model with two public firms, one from each country, the paper highlights the forces shaping firms’ incentives in a mixed international market under a symmetric or an asymmetric distribution of firms across countries. In particular, it shows that sufficiently high product substitutability, by favoring market competition and positive spillovers from one country to the other, leads the public firm from the country with the larger number of firms, more concerned with its own firms’ profits, to enhance its own country’s welfare by setting a relatively high price. The aim of keeping firms’ profits high is also shown to lead to a price reversal between that public firm and its private rivals, provided that relevant asymmetries across countries exist. The strategic choice by that government of retaining public ownership to protect its firms’ profits is observed under such circumstances. Conversely, in those cases in which the price reversal is not feasible, due to scarce product substitutability and limited asymmetry in the cross-country distribution of firms, privatization turns out to be the optimal strategic choice of the most populated country. Keeping constant the assumption of asymmetric distribution of firms across countries, our analysis also shows that letting the controlled firm be public is the optimal choice of government.

The assumption on constant marginal costs can be restrictive in the analysis of mixed markets since it ignores economies of scale characterizing network industries and other sectors exposed to international competition where the presence of public firms is more likely. However, we believe that such an assumption can be more realistic than the assumption of convex costs more often introduced in the mixed oligopoly frameworks.

See Bárcena-Ruiz and Garzón (2005b), who take the same approach to investigate economic integration and privatization.
both governments, provided that product differentiation is high enough, while such a strategy is optimally chosen regardless of the degree of product differentiation under cross-country symmetry. In both circumstances, public ownership turns out to be optimal from a global welfare perspective, since it aims at enhancing country-specific welfare by inducing greater competition, which is also beneficial for the market as a whole. However, in the above cases in which one government’s strategy is motivated by a firm protectionist attitude, it is shown to hurt global welfare and diverge from that made by a supra-national authority. When product substitutability is high enough, the latter adopts privatization in at least one country as a global welfare enhancing strategy.

The paper is organized as follows. In Section 2 we develop our benchmark two-country model and we analyze the outcome of market competition in the presence of a public firm in each country. In Section 3 we discuss both the solution of the strategic privatization game and the optimal privatization policy which would be implemented by a supra-national authority. Finally, Section 4 gathers some conclusions.

2 The model

We consider a two-country economy, in which $S$ firms compete with respect to prices in a single oligopolistic market for a differentiated product. Each firm produces one variety: $m + 1$ varieties are produced by firms of country $H$ (domestic firms) and $n + 1$ varieties by firms of country $F$ (foreign firms), so that $S = m + n + 2$.

The consumers of the two countries are identical in tastes and size, the latter being normalized to 1 in each country. The representative consumer exhibits the following semilinear quadratic preferences:

$$U(q) = \sum_{s=1}^{S} q_s - \frac{1}{2} \left(1 - \gamma \right) \left( \sum_{s=1}^{S} q_s^2 \right) + \gamma \left( \sum_{s=1}^{S} q_s \right)^2 + q_0$$

where $q_0$ is a composite good produced in a perfectly competitive market, which absorbs all income effects of price changes, and $\gamma$ is the degree of product substitutability, ranging from 0 (absence of substitutability) to 1 (homogeneous products).9. These preferences imply that all the $S$ varieties of the differentiated product (independently of their being produced by domestic or foreign firms) enter symmetrically the utility function of domestic and foreign consumers. Maximization of $U(q)$ yields the linear direct demand for the generic variety $s$ from each country:

$$q_{s,dc}^C = \frac{1 - \gamma - (1 + \gamma (m + n)) p_s + \gamma P_s}{(1 - \gamma)(1 + \gamma (m + n + 1))}, \quad C = H, F$$

9 Our analysis is also robust to a model specification with the Shubik and Levitan (1980) demand function, which embodies product substitutability under the hypothesis that the market size is independent of the number of varieties and the degree of product substitutability.
so that the single market demand for variety $s$ is:

$$Q_s^d = q_s^{dH} + q_s^{dF} = 2\frac{1 - \gamma - (1 + \gamma (m + n)) p_s + \gamma P_s}{(1 - \gamma)(1 + \gamma (m + n + 1))}$$

(2)

where $P_s = \sum_{v \neq s} p_v$ is the sum of the prices of all varieties other than $s$.

We assume that among firms of country $H$, $m$ are private (profit maximizing), while the remaining firm is public (domestic welfare maximizing); similarly, among firms of country $F$, $n$ are private and one is public (foreign welfare maximizing). The production technology of each firm exhibits constant returns to scale and is identical across firms, so that neither the origin country nor the public or private nature of the firm affect its cost function. The constant average and marginal cost of production is $c<1$. In the sequel, the set of domestic private firms will be denoted by $\{H\}$ and its generic element by $h$, the set of foreign private firms by $\{F\}$ and its generic element by $f$, while the domestic public firm will be indexed by $i$ and the foreign public firm by $j$.

The public firm in country $H$ solves the following maximization problem:

$$\max_{p_i} W_H = CS_H + \pi_i + \Pi_H$$

(A)

where $\pi_i$ are the profits of the public firm $i$, $\Pi_H = \sum_{h \in \{H\}} \pi_h = \sum_{h \in \{H\}} (p_h - c) Q_h^d$ are the overall profits of the domestic private firms and $CS_H$ is the surplus of consumers of country $H$:

$$CS_H = \frac{(1 - \gamma) \left( \sum_{s=1}^S (q_s^{dH})^2 \right) + \gamma \left( \sum_{s=1}^S q_s^{dH} \right)^2}{2}$$

which can be expressed in terms of prices by using the domestic demand functions given by (1). The solution of problem (A) yields the best reply of the public firm of the home country as a function of the foreign public firm price $p_j$ and the (domestic and foreign) private firms’ prices:

$$p_i = \frac{1 - \gamma + 2c (1 + \gamma n) + \gamma \left( p_j + 3 \sum_{h \in \{H\}} p_h + \sum_{f \in \{F\}} p_f \right)}{3(1 + \gamma (m + n))}$$

(3)

Similarly, the public firm in the foreign country faces the following problem:

$$\max_{p_j} W_F = CS_F + \pi_j + \Pi_F$$

(B)

where $\pi_j$ are the profits of the public firm $j$, $\Pi_F = \sum_{f \in \{F\}} (p_f - c) q_f^d$ are the overall profits of foreign private firms and $CS_F$ is the surplus of consumers of country $F$:

$$CS_F = \frac{(1 - \gamma) \left( \sum_{s=1}^S (q_s^{dF})^2 \right) + \gamma \left( \sum_{s=1}^S q_s^{dF} \right)^2}{2}$$
The solution of problem (B) gives the reaction function for firm $j$:

$$
p_j = \frac{1 - \gamma + 2e(1 + \gamma m) + \gamma \left( p_i + \sum_{h \in H} p_h + 3 \sum_{f \in F} p_f \right)}{3(1 + \gamma (m + n))}
$$  \hspace{1cm} (4)

Let us now consider the optimal behavior of each private domestic firm. Maximizing $\pi_h$ with respect to $p_h$ we obtain the optimal reply function of firm $h$:

$$
p_h = \frac{1 - \gamma + e(1 + \gamma (m + n)) + \gamma \left( p_i + p_j + \sum_{k \in (H-h)} p_k + \sum_{f \in F} p_f \right)}{2(1 + \gamma (m + n))}
$$  \hspace{1cm} (5)

where $\sum_{k \in (H-h)} p_k$ denotes the sum of the prices of the private domestic firms other than $h$. In the same way, maximization of $\pi_f$ with respect to $p_f$ gives the optimal reply function of firm $f$:

$$
p_f = \frac{1 - \gamma + e(1 + \gamma (m + n)) + \gamma \left( p_i + p_j + \sum_{h \in (H)} p_h + \sum_{g \in (F-f)} p_g \right)}{2(1 + \gamma (m + n))}
$$  \hspace{1cm} (6)

where $\sum_{g \in (F-f)} p_g$ denotes the sum of the prices of the private foreign firms other than $f$.

Summing (5) over the $m$ domestic firms and (6) over the $n$ foreign firms and using (4) and (5), we obtain the following solution for the equilibrium prices of the public firms:

$$
p^*_i = \frac{(1-\gamma)(2\gamma(2m+n)+\gamma+2)+\gamma m(3\gamma m+2\gamma+5)+3\gamma^2 n(2m+n)+2\gamma(\gamma+2)+7\gamma n}{(\gamma+2)(1-\gamma)+\gamma(n+m)(\gamma(3(n+m)-2)+9)}
$$  \hspace{1cm} (7)

$$
p^*_j = \frac{(1-\gamma)(2\gamma(2n+m)+\gamma+2)+\gamma n(3\gamma n+2\gamma+5)+3\gamma^2 m(2n+m)+2\gamma(\gamma+2)+7\gamma n}{(\gamma+2)(1-\gamma)+\gamma(n+m)(\gamma(3(n+m)-2)+9)}
$$  \hspace{1cm} (8)

By substituting $p^*_i, p^*_j$, and the aggregate equilibrium prices of private firms into (4) and (5), we obtain the individual prices of the private domestic and foreign firms:

$$
p^*_h = p^*_j = \frac{3\gamma(\gamma+1)(3\gamma^2 m(2n+m)+2\gamma(\gamma+2)+7\gamma n)}{(\gamma+2)(1-\gamma)+\gamma(n+m)(\gamma(3(n+m)-2)+9)}
$$  \hspace{1cm} (9)

Inspection of the equilibrium prices allows to establish the following Proposition.

**Proposition 1.** The equilibrium prices of the domestic and foreign private firms always coincide. The prices set by the two public firms are such that $p^*_i = p^*_j$ if $m = n$, while $p^*_i \not\geq p^*_j$ if $m \not\geq n$. If $n > m$ then there is a value $\gamma_f = \frac{4}{n-m} \leq 1$ such that $p^*_f > p^*_j = p^*_h$ when $\gamma > \gamma_f$. Similarly, if $m > n$ then there is a value $\gamma_h = \frac{4}{m-n} \leq 1$ such that $p^*_i > p^*_h = p^*_j$ when $\gamma > \gamma_h$. \hspace{1cm} [Proof] It follows from (7), (8) and (9).
same market conditions. But if they are unevenly distributed across countries, the objective functions of the two public firms differ, with the profit component of welfare having a higher relative weight for the public firm operating in the country with the largest number of private firms. Therefore, the optimal reaction of this public firm to any given profile of the prices of the rivals is to set a higher price than the one which maximizes welfare for the other public firm. Indeed, while the marginal benefit in terms of higher consumer surplus of a price reduction is the same for both public firms — produced quantities affecting the consumer surplus of both countries symmetrically and independently of the origin country — the marginal cost in terms of lower domestic profits is higher for the public firm of the country with the higher number of private firms. The balance is therefore obtained at a higher price.

These considerations also help to understand the second statement of Proposition 1, that in the presence of an asymmetry in the cross-country distribution of firms, the price of the public firm can be higher than that of the private firms. Suppose that most of the private firms belong to the foreign country. If the asymmetry is sufficiently large, the public firm of the domestic country perceives a strong incentive to set its price very close to marginal cost, for any given profile of the prices set by the rivals; this implies that all the other firms (foreign and domestic) face a downward shift of their demand functions. Under these tougher demand conditions, for the public foreign firm the marginal benefit on the consumer surplus of a price reduction is very low, and the balance with its marginal cost in terms of domestic profits may occur at a price higher than the individual profit-maximizing price. When its marginal impact on the consumer surplus through price changes becomes very low, a welfare-maximizing behavior at the margin resembles a collusive behavior, which in our framework results into a protectionist-like attitude.

This result extends to price competition the idea already put forth by Dadpay and Heywood (2006) in a quantity-setting framework with homogeneous product. In their model, however, the degree of asymmetry required for one country’s public firm to produce less than the private firms depends on that country’s share in market demand. In our model, the reversal in the level of prices occurs beyond a threshold level of the degree of product differentiation, which in turn depends on the cross-country asymmetry in the distribution of firms: the price reversal never occurs if the difference in the number of firms is equal to 1, while it occurs for \( \gamma > 1/|n - m| \) if this difference is greater than 1. This interplay between the degree of asymmetry and the degree of product differentiation is worth stressing. A higher degree of asymmetry implies a greater difference in the objective function of the public firms and in their desired aggressiveness; high values of \( \gamma \) imply that the markets of the various firms become highly connected, creating strong spillovers of the price decisions of each firm on the demand faced by the others. When the asymmetry is reduced to a difference of one firm, the objective function of the public firms are quite similar and no scope arises for a price reversal for all values of \( \gamma \). But when the difference is greater than 1, for \( \gamma > 1/|n - m| \) the undesired effects in the more populated country of the aggressiveness of the public firm of the
less populated country become so strong to induce the public firm of the latter to take a protectionist-like role which shows itself through a price reversal at equilibrium.

3 Privatization policy

The previous discussion has enlightened the key role of the asymmetry in the cross-country distribution of firms (and the related asymmetry in the public firms’ objective functions) on the equilibrium prices. We have shown that the incentive of each of the public firms to adopt a more or less aggressive behavior is defined by the interplay between this asymmetry and the degree of product substitutability. The interesting question arises whether such an interplay can affect the choice of privatizing or not the public firm in each country. We tackle this issue in a twofold perspective. First, we assume that the privatization choice is unilaterally made by governments interested in their own country’s welfare. In this non-cooperative case, the interaction between governments takes the form of a strategic privatization game, the Nash equilibrium of which is discussed in the next subsection. Through the analysis of the effects of governments’ strategic decisions on each country’s and global welfare, we then turn to the issue of the socially optimal market structure, i.e. the structure which would be optimally chosen by a supra-national authority according to a ‘cooperative’ view.

3.1 The strategic privatization game

We develop a game in which Country $H$, with $m+1$ firms, and Country $F$, with $n+1$ firms, non-cooperatively decide whether to implement a privatization policy or not, i.e. whether to privatize their controlled firms before competing with respect to prices. The pay-off matrix for given $m$ and $n$ is shown in Figure 1, where $S$ denotes the strategy to let the controlled firm be state-owned, and $P$ denotes the privatization strategy.

![Figure 1](attachment:image.png)

We recall that (i) $W_{H}^{SS}(\gamma)$ and $W_{F}^{SS}(\gamma)$ (with $SS$ capturing that both firms are state-owned) are obtained by substituting the equilibrium prices in (7), (8) and (9) in the social welfare functions given by (A) and (B); (ii) $W_{H}^{PP}(\gamma)$

9
and $W_{FP}^P (\gamma)$ are the welfare outcomes when competition involves only privatized firms in each country, and the number of private firms in each country is increased by one to take into account the newly privatized firms; (iii) the expressions for welfare in the asymmetric cases, for example $W_{HP}^P (\gamma)$ and $W_{FP}^P (\gamma)$, have been computed by solving a two-country model where the overall demand coming from the two countries is satisfied only by private firms in the privatizing country - in our example the $n + 1$ firms of the privatizing country $F$ - and by the public firm and the private firms ($m$ in our example) in the other. Finally, when $m = n$, clearly $W_{HP}^{SS} (\gamma) = W_{HP}^{SS} (\gamma)$ and $W_{FP}^{PP} (\gamma) = W_{FP}^{PP} (\gamma)$ in the symmetric cases, while $W_{HP}^{SP} (\gamma) = W_{FP}^{PS} (\gamma)$ and $W_{FP}^{PS} (\gamma) = W_{FP}^{PS} (\gamma)$ under unilateral privatization.

A numerical evaluation of the welfare functions of Table 1 for different given values of $m$ and $n$ yields the results summarized in the following proposition.

**Proposition 2.** (a) If the distribution of firms across the two countries is symmetric, i.e. $m = n$, the unique Subgame Perfect Nash Equilibrium (SPNE) in dominant strategies of the privatization game is SS. (b) If the distribution of firms is asymmetric, e.g. $m < n$, there exists a threshold value of $\gamma$, $\gamma (m, n) < \gamma^f$, such that SS is the SPNE for $\gamma \in [0, \gamma)$ and for $\gamma \in (\gamma^f, 1]$, while $SP$ is the SPNE for $\gamma \in (\gamma^f, 1]$. For $\gamma = \gamma$ and $\gamma = \gamma^f$ both SS and SP are equilibria.

**Proof.** As to part (a) of Proposition 2, if $m = n$ numerical computation shows that $W_{HP}^{SS} (\gamma) > W_{FP}^{PS} (\gamma)$ and $W_{FP}^{SP} (\gamma) > W_{FP}^{PP} (\gamma)$ for all values of $\gamma$, so that non-privatizing is a dominant strategy for each country. If, on the contrary, the distribution of firms is asymmetric and, for example, $n > m$, computation shows that for all $\gamma$, $W_{HP}^{SS} (\gamma) > W_{HP}^{PS} (\gamma)$ and $W_{FP}^{SP} (\gamma) > W_{FP}^{PP} (\gamma)$, so that non-privatizing is still a dominant strategy for the less populated country $H$. Given that $H$ optimally chooses to preserve public ownership, the SPNE is identified through a comparison between $W_{HP}^{SS} (\gamma)$ and $W_{FP}^{PS} (\gamma)$. Given $m$ and $n$, the latter shows that (i) there is a threshold value of $\gamma$, $\gamma (m, n) < \gamma^f$, at which the $W_{FP}^{PS} (\gamma)$ function intersects the $W_{HP}^{SS} (\gamma)$ from below, and (ii) the $W_{FP}^{PS} (\gamma)$ function intersects the $W_{HP}^{SS} (\gamma)$ from above at $\gamma^f$. This defines an interval $(\gamma^f, 1]$ at which privatizing is the optimal choice of the most populated country $F$, given that $H$ does not privatize. Therefore the equilibrium market structure is SS for $\gamma \in [0, \gamma)$ and for $\gamma \in (\gamma^f, 1]$, while it is $SP$ for $\gamma \in (\gamma^f, 1]$, while both SS and SP are equilibria for $\gamma = \gamma$ and $\gamma = \gamma^f$. ■

In order to point out the forces leading to the results in Proposition 2, we recall that when private firms are symmetrically distributed between the two countries ($m = n$), the objectives of the two governments are perfectly aligned. In a world of private firms only, for all $\gamma$ each of them finds it optimal to move to a mixed structure in order to pursue its welfare objectives through higher consumer surplus; moreover, since the behavior of public firms in the

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10 The market outcomes of cases (ii) and (iii) are derived in Appendix A.
two countries reflects the same balance between consumer surplus and profits, for all \( \gamma \) each government benefits from the presence of a public firm in the other country and finds it optimal to reinforce the competitive pressure of the latter by adopting itself a mixed structure.

Conversely, product differentiation crucially affects one country’s privatization choice in the presence of an asymmetric distribution of private firms. If, for example, \( n > m \), the optimal balance between consumers’ surplus and firms’ profits in the two countries is no more the same - the government of \( H \) favouring a more competitive environment than that preferred by \( F \). As \( \gamma \) increases, the reciprocal spillovers of the price choices in each country strengthen, amplifying the perceived effects of the decisions taken in the country.

When \( \gamma \) is sufficiently low, namely \( \gamma \in \left[ 0, \frac{1}{2} \right) \), the toughness of competition is low and spillovers are weak. The market structure and the related price decisions in each country have a weak effect on the welfare of the other country. Therefore, the welfare objectives of both governments are best achieved by adopting a mixed structure, independently of the decision of the other. Notice that this interval decreases in size – \( \gamma \) becoming smaller – as the difference in the number of firms increases, widening the unbalance in the government objectives.

As \( \gamma \) increases, market competition becomes tougher and the cross-country spillover of market and price decisions intensifies. For the country \( H \)’s more aggressive government, this stronger interaction with a less aggressive government reinforces \( S \) as dominant strategy. As far as \( F \) is concerned, for sufficiently high \( \gamma \), namely for \( \gamma > \frac{1}{2} \), the aggressive attitude of a public firm in country \( H \) creates a competitive pressure in the market which is ‘too’ inconsistent with country \( F \)’s own optimal balance of consumers’ surplus and profits. The incentive perceived by the government of \( F \) is to counteract this excess of competition, and restore its preferred balance between the two components of welfare. Clearly, this cannot be achieved through a public firm in \( F \) adding competitive pressure on the private firms. In such circumstances, there is a range of values of \( \gamma \) at which the highest welfare in country \( F \) is achieved by softening competition through a privatization of the public firm. This range covers all \( \gamma > \frac{1}{2} \), when the asymmetry between countries amounts to a difference of one firm, i.e. when \( \gamma^f = 1 \) and the price reversal highlighted in Proposition 1 never occurs. When the cross-country difference in the number of private firms is higher, privatization is the optimal choice of country \( F \) in the interval \( \left( \frac{1}{2}, 1 \right) \); but for all \( \gamma \) at which the price reversal occurs – i.e. when \( \gamma \in \left( \frac{1}{2}, 1 \right) \) – choosing a mixed market in which the public firm plays a protectionist role turns out to be the best defensive strategy of country \( F \).

As an example, Table 1 shows the threshold values of \( \gamma \) for different values of \( n \), given \( m = 5 \). Notice that as the difference in the population of firms increases, the size of the unilateral privatization interval decreases, while the interval in which country \( F \) chooses a mixed market in order to soften competition widens.
Table 1
The threshold values of the unilateral privatization interval (m=5)

<table>
<thead>
<tr>
<th>n</th>
<th>γ'</th>
<th>γP</th>
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<tbody>
<tr>
<td>6</td>
<td>0.63</td>
<td>1</td>
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3.2 The global welfare perspective

We consider now the decision made by a supra-national authority on whether the public firms in country H and country F should be privatized or rather kept as public by the governments.

In order to assess the impact of this cooperative approach to privatization, we compare aggregate social welfare, defined as the sum of social welfare in the two countries $W(\gamma) = W_H(\gamma) + W_F(\gamma)$, in the different market structures. This leads us to introduce the following Propositions.

**Proposition 3** (a) If private firms are symmetrically distributed, i.e. $m=n$, the non-cooperative equilibrium SS is globally efficient (maximizes aggregate social welfare) if $\gamma \in [0, \hat{\gamma}^P)$, where $\hat{\gamma}^P = \hat{\gamma}^P(m)$ is close to 1. For $\gamma \in (\hat{\gamma}^P, 1]$ the optimal choice of a supra-national authority is PP. If $\gamma = \hat{\gamma}^P$, PP and SS generate the same global maximum welfare.

**Proof.** See Appendix B. ■

**Proposition 4** If the distribution of private firms is asymmetric with a difference of one firm, e.g. $n = m + 1$, the following holds: (i) for $\gamma \in [0, \hat{\gamma}^P(m, n)]$ the non-cooperative equilibrium SS coincides with the globally efficient solution; (ii) there exists a value of $\gamma$, $\tilde{\gamma}^P(m, n) > \gamma$ and close to 1, such that for $\gamma \in (\tilde{\gamma}^P, 1]$ the non-cooperative equilibrium SP is not globally efficient, and the optimal choice of a supra-national authority is SS (iii) for $\gamma \in (\hat{\gamma}^P, 1]$ the non-cooperative equilibrium SP is not globally efficient, and the optimal choice of a supra-national authority is PP. If $\gamma = \hat{\gamma}^P$, PP and SS generate the same maximum welfare.

**Proof.** See Appendix B. ■

**Proposition 5** If the distribution of private firms is asymmetric with a difference of more than one firm, e.g. $n > m + 1$, the following holds: (i) for
The objective of a supra-national authority is to maximize global welfare and this is obviously achieved in the most competitive environment, i.e. when prices are closer to marginal costs and the overall quantity produced is higher. This is the key force driving its choices of the optimal market structure.

Proposition 3 deals with the symmetric distribution case, in which the non-cooperative solution is an $SS$ configuration for all $\gamma$. The latter is indeed the most competitive, thus the globally efficient, market structure unless $\gamma$ is not too close to 1. Clearly, there exists a threshold value $\gamma^C$ very close to 1 and decreasing in the number of firms, beyond which the products being almost homogenous makes the standard Bertrand competition between private firms more efficient than a market structure with public firms. The competitive pressure on prices of profit maximizing firms becomes stronger than that exerted by the action of public firms whose objective function includes the sum of firms profits.

Propositions 4 and 5 deal with the case of asymmetric distribution of firms. The global welfare properties of the non-cooperative solution and the globally efficient solution are assessed with respect to the relevant intervals of the differentiation parameter $\gamma$ highlighted in Proposition 2. When products are highly differentiated, i.e. $\gamma \in [0, \gamma^l]$, the Nash equilibrium of the strategic game coincides with the choice made by a supra-national authority. Within this interval, the effects of the misalignment of the countries’ objective functions are weak due to weak spillovers; both countries choose unilaterally a mixed structure as a way of enhancing welfare through increased market competitiveness and, in so doing, behave consistently with the global welfare objective of a supra-national authority. However, for $\gamma > \gamma^l$, the Nash equilibrium of the strategic privatization game does not correspond to the market structure chosen by a supra-national authority. As shown in the previous section, in the interval $\gamma > \gamma^l$ the $SP$ equilibrium for $n = m+1$, and both the $SP$ and the $SS$ Nash equilibrium for $n > m+1$, are associated to country F’s incentive to soften competition and limit the profit-detrimental effects of the increasingly tough aggressiveness of the public firm in country H. In the above interval, the strategies adopted by country F (or by both countries for $\gamma$ close to 1) at the Nash equilibrium are not globally efficient. For $\gamma \in (\gamma^l, \gamma^f)$ – or $\gamma \in (\gamma^l, \gamma^P)$ when $n = m+1$ – a supra-national authority would choose $SS$ (and not $SP$), which lets global welfare be enhanced through a more competitive public firm in country F. Conversely, when $n > m+1$, the

\[
\gamma \in [0, \gamma (m, n)] \text{ the non-cooperative equilibrium } SS \text{ coincides with the globally efficient solution; (ii) For } \gamma \in (\gamma (m, n), \gamma^f) \text{ the non-cooperative equilibrium } SP \text{ is not globally efficient, and the optimal choice of a supra-national authority is } SS - SS \text{ and } SP \text{ generating the same maximum welfare for } \gamma = \gamma^f; (iii) there exists a value of } \gamma, \gamma^P (m, n) > \gamma^l \text{ and close to 1, such that for } \gamma \in (\gamma^l, \gamma^P) \text{ the non-cooperative equilibrium } SS \text{ is not globally efficient, and the optimal choice of a supra-national authority is } SP \text{ as long as } \gamma \in (\gamma^l, \gamma^f), \text{ and } PP \text{ when } \gamma \in (\gamma^P, 1] - SP \text{ and } PP \text{ generating the same maximum welfare for } \gamma = \gamma^P. \]

**Proof** See Appendix B. ■
supra-national authority would choose: a) $SP$ (and not $SS$) at $\gamma \in (\gamma^I, \gamma^P)$, which reveals that privatization in country $F$ turns out to be global welfare enhancing in that interval of $\gamma$ in which the price reversal occurs and public ownership is kept for protectionist purposes; b) $PP$ (and not $SS$) at $\gamma \in (\gamma^P, 1]$ – or $\gamma \in (\gamma^P, 1]$ when $n = m + 1$ – i.e., in those intervals where values of $\gamma$ very close to 1 allow to exploit the welfare-enhancing properties of Bertrand competition among private firms.

The lesson which can be drawn from these results is that in a global perspective the scope for welfare enhancing privatizations is much wider than that perceived by self-interested governments. As in closed economies, under price competition a fully private market is more socially desirable when products are almost homogeneous. But in open markets another effect is at work. If the behaviour of the public firms is driven by country-specific objectives, there is a large set of values of $\gamma$ for which the country with the larger share of private firms may rely on its public firm to relax competition; in all these cases maximization of global welfare requires this public firm to be privatized, with an increase of the consumers’ surplus which more than counterbalances the overall decrease in private profits.

4 Concluding remarks

The issue of strategic privatization of state-owned firms on globalized and integrated markets needs further theoretical investigation. We tackle this issue in a two-country framework of oligopolistic competition where technologically identical firms offer differentiated products and compete with respect to prices. On the one hand, our analysis confirms that, as long as private firms are symmetrically distributed across countries, the choice of a mixed market structure in each country is a Nash equilibrium for any degree of product differentiation. On the other hand, it also shows that in the presence of asymmetries in the cross-country distribution of firms, and therefore of misalignment of objectives between the two governments, there are intervals of the differentiation parameter within which unilateral privatization by the country with the largest number of firms is the equilibrium outcome. Moreover, we have shown that when the asymmetry is large enough and products are sufficiently substitutes, the motivation behind the government’s choice of preserving a public firm in the market may be very different in the two countries: it is the desire to enhance competition for the less firm-populated country and the attempt to soften profit-detrimental competition for the other one. When both the choice of privatizing and that of keeping public ownership are aimed at protecting firms’ profit margin, they are clearly inefficient in an aggregate perspective. Indeed, the assessment of the global welfare properties of the equilibria arising in the strategic privatization game points out their inefficiency when they imply a profit defensive strategy by the most populated country, which prevents competition from achieving the highest welfare in the whole market. Moreover, it highlights that, when prod-
ucts are almost homogeneous, the very competitive outcome of price competition among private firms ensures global welfare optimality of privatization in both countries. The policy implications to be drawn from these findings are that in international markets where governments compete through their public firms against private firms, supra-national coordination may be required in order to attain welfare gains.

Appendix A

Consider the following variation of the two-country model described in Section 2: country $F$ privatizes its public firm, so that its $n+1$ firms are all private, and firm $i$ in country $H$ is the only public firm in the single market. By solving the welfare maximization problem of firm $i$ and the profit maximization problem of the private firms in $F$ and $H$, the following equilibrium prices are obtained:

$$p^*_i = \frac{3\gamma^2(m^2+r^2)+(6)\gamma c+(4+5c)(1-\gamma)\gamma m+(c(2+5\gamma)+2(1-\gamma)\gamma r+(1-\gamma)(2-\gamma)(1+2c)}{3\gamma^2(m^2+r^2)+(6)\gamma c+(4+5c)(1-\gamma)\gamma m+(9-7\gamma)\gamma r+3(1-\gamma)(2-\gamma)}$$  \hspace{1cm} (A1)

$$p^*_h = p^*_f = \frac{3\gamma^2(m^2+r^2)+(6)\gamma c+(3+6c)(1-\gamma)\gamma m+(c(0-4\gamma)+3(1-\gamma)\gamma r+(1-\gamma)(3(1+c)-(2+\gamma))}{3\gamma^2(m^2+r^2)+(6)\gamma c+(3+6c)(1-\gamma)\gamma m+(9-7\gamma)\gamma r+3(1-\gamma)(2-\gamma)}$$  \hspace{1cm} (A2)

where $r = n + 1$.

The expressions for $W^P_H(\gamma)$ and $W^P_F(\gamma)$ can then be obtained by using (A1) and (A2) into the demand functions (1) and (2), and then substituting the latter into the definitions of $W_H$ and $W_F$.

If country $H$ privatizes, so that its $m+1$ firms are all private and firm $j$ in country $F$ is the only public firm in the single market, the equilibrium prices are:

$$p^*_j = \frac{3\gamma^2(n^2+k^2)+(6)\gamma c+(4+5c)(1-\gamma)\gamma n+(c(7-5\gamma)+2(1-\gamma)\gamma k+(1-\gamma)(2-\gamma)(1+2c)}{3\gamma^2(n^2+k^2)+(6)\gamma c+(4+5c)(1-\gamma)\gamma n+(9-7\gamma)\gamma k+3(1-\gamma)(2-\gamma)}$$  \hspace{1cm} (A3)

$$p^*_h = p^*_f = \frac{3\gamma^2(n^2+k^2)+(6)\gamma c+(3+6c)(1-\gamma)\gamma n+(c(0-4\gamma)+3(1-\gamma)\gamma k+(1-\gamma)(3(1+c)-(2+\gamma))}{3\gamma^2(n^2+k^2)+(6)\gamma c+(3+6c)(1-\gamma)\gamma n+(9-7\gamma)\gamma k+3(1-\gamma)(2-\gamma)}$$  \hspace{1cm} (A4)

where $k = m + 1$. Notice that when $r = k$, (A3) and (A4) respectively coincide with (A1) and (A2). By using (A3) and (A4) and following the same procedure described above, we obtain the welfare functions $W^P_H(\gamma)$ and $W^P_F(\gamma)$.

Finally, if both countries privatize so that the $m+1$ firms in $H$ and the $n+1$ firms in $F$ are all private, the equilibrium prices are:

$$p^*_h = p^*_f = \frac{1 - \gamma + c(1-2\gamma) + c\gamma(h+k)}{2 + \gamma(h+k-3)}$$

where again $k = m + 1$ is the number of private firms in country $H$, and $r = n + 1$ is the number of private firms in country $F$. Given (A5), $W^P_H(\gamma)$ and $W^P_F(\gamma)$ can then be easily derived.
Appendix B

Proof of Proposition 3. We recall that under a symmetric distribution of firms, $SS$ is the optimal strategic choice of the two countries for all values of $\gamma$. However, by computing the global welfare functions for different values of $m = n$, we obtain that there are three threshold values of $\gamma$, $\gamma(m)$, $\gamma^P(m)$, and $\gamma^P(m)$ -- with $\gamma^P(m) < \gamma^P(m) < \gamma^P(m)$ -- which define the following global welfare rankings of the different market structures:

- $W^{SS}(\gamma) > W^{SP}(\gamma) > W^{PP}(\gamma)$ for $\gamma \in [0, \gamma^P)$ with $W^{SP}(\gamma) = W^{PP}(\gamma)$ when $\gamma = \gamma^P$.
- $W^{SS}(\gamma) > W^{PP}(\gamma) > W^{SP}(\gamma)$ for $\gamma \in (\gamma^P, \gamma^P)$ with $W^{SS}(\gamma) = W^{PP}(\gamma)$ when $\gamma = \gamma^P$.
- $W^{PP}(\gamma) > W^{SS}(\gamma) > W^{SP}(\gamma)$ for $\gamma \in (\gamma^P, \gamma^P)$ with $W^{SS}(\gamma) = W^{SP}(\gamma)$ when $\gamma = \gamma^P$.
- $W^{PP}(\gamma) > W^{SP}(\gamma) > W^{SS}(\gamma)$ for $\gamma \in (\gamma^P, 1]$.

According to the above inequalities, $SS$ is globally efficient market structure in the interval $[0, \gamma^P)$, while $PP$ is efficient in the interval $(\gamma^P, 1]$. Table B1 synthesizes the values of the above thresholds for different $m(= n)$.11

Table B1

<table>
<thead>
<tr>
<th>$m$</th>
<th>$\hat{\gamma}$</th>
<th>$\gamma^P$</th>
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Proof of Proposition 4. Consider the case of asymmetric distribution of firms, with a difference of 1 firm, e.g., $n = m + 1$. By computing the global welfare functions for different values of $m$ and $n = m + 1$, we obtain that (i)
there is no value of $\gamma$ for which the $PS$ configuration welfare-dominates the others; (ii) there are two threshold values of $\gamma$, $\tilde{\gamma} (m, n)$, and $\tilde{\gamma}^P (m, n)$ – with $\tilde{\gamma} < \tilde{\gamma}^P$ – which define the following global welfare rankings of the remaining market structures:

- $W^{SS} (\gamma) > W^{SP} (\gamma) > W^{PP} (\gamma)$ for $\gamma \in [0, \tilde{\gamma})$ with $W^{SP} (\gamma) = W^{PP} (\gamma)$ when $\gamma = \tilde{\gamma}$
- $W^{SS} (\gamma) > W^{PP} (\gamma) > W^{SP} (\gamma)$ for $\gamma \in \left(\tilde{\gamma}, \tilde{\gamma}^P\right)$ with $W^{SS} (\gamma) = W^{PP} (\gamma)$ when $\gamma = \tilde{\gamma}^P$
- $W^{PP} (\gamma) > W^{SS} (\gamma) > W^{SP} (\gamma)$ for $\gamma \in \left(\tilde{\gamma}^P, 1\right)$

According to the above inequalities, $SS$ is the globally efficient market structure in the interval $[0, \tilde{\gamma}^P)$, while $PP$ is efficient in the interval $\left(\tilde{\gamma}^P, 1\right]$. Table B2 provides examples of the above threshold values of $\gamma$.

**Table B2**

<table>
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<th>$n = m + 1$</th>
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</tr>
<tr>
<td>4</td>
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**Proof of Proposition 5.** Consider the case of asymmetric distribution of firms, with a difference in the number of firms greater than 1, e.g., $n > m + 1$. By computing the global welfare functions for different values of $m$ and $n$, we obtain that (i) there is no value of $\gamma$ for which the $PS$ configuration welfare-dominates the others; (ii) there are three threshold values of $\gamma$, $\gamma^f$, $\gamma (m, n)$, and $\gamma^P (m, n)$ – with $\gamma^f < \gamma < \gamma^P$ – which define the following global welfare rankings of the remaining market structures:

- $W^{SS} (\gamma) > W^{SP} (\gamma) > W^{PP} (\gamma)$ for $\gamma \in [0, \gamma^f)$ with $W^{SS} (\gamma) = W^{SP} (\gamma)$ and when $\gamma = \gamma^f$
- $W^{SP} (\gamma) > W^{SS} (\gamma) > W^{PP} (\gamma)$ for $\gamma \in (\gamma^f, \gamma)$ with $W^{SS} (\gamma) = W^{PP} (\gamma)$ when $\gamma = \gamma$
- $W^{SP} (\gamma) > W^{PP} (\gamma) > W^{SS} (\gamma)$ for $\gamma \in (\gamma, \gamma^P)$ with $W^{SP} (\gamma) = W^{PP} (\gamma)$ when $\gamma = \gamma^P$
\[ W^{PP}(\gamma) > W^{SP}(\gamma) > W^{SS}(\gamma) \text{ for } \gamma \in (\gamma^P, 1] \]

According to the above inequalities, \( SS \) is the globally efficient market structure in the interval \([0, \gamma^f]\), while \( SP \) is efficient in the interval \((\gamma^f, \gamma^P]\), and \( PP \) in the interval \((\gamma^P, 1]\). Table B3 provides some examples of the threshold values of \( \gamma \) in this case.

**Table B3**  
The welfare ranking. The threshold values of \( \gamma \)  
in the asymmetric case (\( m = 5 \))

<table>
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<th>( n )</th>
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<th>( \tau )</th>
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