Progressive taxation and (in)stability in an endogenous growth model with human capital accumulation

Aleksandar Vasilev
Progressive taxation and (in)stability in an endogenous growth model with human capital accumulation: the case of Bulgaria

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Abstract

We show that in an endogenous growth model with human accumulation calibrated to Bulgarian data under the progressive taxation regime (1993-2007), the artificial economy exhibits equilibrium indeterminacy. These results are in line with the recent findings in Chen and Guo (2015) in the context of an AK endogenous growth model. Also, the findings are in contrast to Guo and Lansing (1988) who argue that progressive taxation works as an automatic stabilizer. Progressive taxation in our setup lead to equilibrium indeterminacy. This indeterminacy result could explain, at least partially, why the economic performance under the progressive taxation regime in Bulgaria was not impressive.

JEL classification: E32, E62, O41

Keywords: Progressive Income Taxation; Endogenous growth; Human capital; Equilibrium (In)determinacy

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1 Introduction and Motivation

Fiscal policies, such as taxation policies, are known to have supply-side effects. More specifically, taxes affect households’ incentives to invest in capital, and their labor supply decisions. In the standard one-sector deterministic general-equilibrium model, a tax on income decreases the after-tax return to physical capital. The high tax rate would then discourage the household from investing in capital stock, and thus in the absence of exogenous technological progress, the growth rate monotonically decreases to zero. Lucas (1988) extended the framework to allow for the existence of human capital as well, which interacts with physical capital due to the existence of complementarities between the two types of capital in the production function. The human capital accumulation will be then the channel through which the decrease in the marginal product of capital will be slowed down.\footnote{1}

However, a major limitation of previous studies using representative agent setups within a dynamic stochastic general equilibrium models focus on the average effective tax rate. The setup in this paper will thus follow Lucas’s (1988) spirit and take that broader view of capital, while at the same time introduce progressive taxation as in Vasilev (2015). The progressivity of the tax schedule will introduce an additional distortion on capital accumulation and labor supply, and would additionally lower the after-tax marginal returns to the factors of production. Overall, the simultaneous accumulation of both types of capital by the household would then guarantee sustained economic growth, or the economy would feature so-called balanced growth path (BGP). To allow for better tractability, the model will assume that labor supply is perfectly inelastic, and government that redistributes revenue back to taxpayers in the form of government transfers.\footnote{2}

Next, as in Chen and Guo (2015), the paper examines the instability effect of progressive taxation.

\footnote{1}{The analysis of the effect of fiscal policies in exogenous and endogenous growth models is relatively recent, e.g., King and Rebelo (1990), Lucas (1990), Stokey and Rebelo (1995), Ortigueira (1998), and the references therein.}

\footnote{2}{Note that our model is isomorphic to a setup with logarithmic utility of leisure and labor supply choice. In addition, the results do not change qualitatively (while substantially complicating the algebra) if we assume instead that the government spends on wasteful consumption or on utility-enhancing public goods, or when government investment augments private production.}
taxation in the case of Bulgaria and compare and contrast the results to the flat tax reform regime in place as of 2008.\(^3\) Importantly, our work differs from that earlier study. While our findings are qualitatively similar to that in Chen and Guo (2013, 2015), here the endogenous growth is driven by investment in human capital, which leads to the after-tax marginal productivity of labor to decrease much slower as compared to a setup without human capital accumulation. In addition, the paper will utilize a carefully calibrated general-equilibrium model to match Bulgaria’s post-communist behavior will demonstrate that progressive taxation created indeterminacy, which could explain the feeble growth during the period 1993-2007. Thus, the indeterminacy results that the model generates are in the particular context for the parameter values characterizing Bulgarian economy.

Our results come in stark contrast to Guo and Lansing (1988) who argue that a sufficiently progressive tax schedule can stabilize a real-business-cycle model, which possesses an indeterminate steady-state against fluctuations driven by "animal spirits." Indeed, in standard Keynesian setups, progressivity of the tax system is regarded as an automatic stabilizer. This is no longer the case in our model with human capital accumulation and thus an endogenously-growing economy.

The rest of the paper is organized as follows. Section 2 describes the model setup and defines the equilibrium balanced growth path. Section 3 describes the Bulgarian flat tax reform and how it compares to the previous progressive taxation regime. Section 4 presents the data, describes the calibration procedure, and reports the computed steady-states under the two tax regimes. Section 5 lays down the equilibrium stationary (detrended) system, and analyzes local stability under both taxation regimes. Section 6 concludes.

2 Model Setup

2.1 Description of the model:

The setup follows the one presented in Vasilev (2015): There is a representative household, as well as a representative firm. The household owns the physical capital and efficiency labor, which it supplies to the firm. The perfectly-competitive firm produces output using labor and capital. The government uses tax revenues from labor and capital income to finance government transfers.

2.2 Representative Household

There is an infinitely-lived representative household in the model economy, and no population growth. Total time available to the household is normalized to unity. The household maximizes the following utility function

\[ \sum_{t=0}^{\infty} \beta^t \ln(c_t), \]

where \( c_t \) is consumption at time \( t \), and the household does not value leisure. The parameter \( \beta \) is the discount factor, with \( 0 < \beta < 1 \). The instantaneous logarithmic utility function is increasing and concave in consumption, and satisfies the Inada conditions. Next, the household has an endowment of one unit of time in each period \( t \), which is supplied inelastically to labor services in all periods, i.e., \( h_t = 1, \forall t \).

The hourly wage rate is \( w_t \). However, the wage is paid per efficiency unit of labor, i.e., per hour weighted by the skill \( s_t \) embodied in the labor, \( e_t = s_t h_t \). The skill level will be treated as a stock of human capital, which can be augmented by investing \( i_t^s \) in education. The law of motion for skill accumulation is then

\[ s_{t+1} = i_t^s + (1 - \delta^s)s_t, \]

where \( 0 < \delta^s < 1 \) is the depreciation rate on capital.

The representative household saves by investing in physical capital, \( i_t^k \). As an owner of capital, the household receives before-tax interest income of \( r_t k_t \) from renting the capital to

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4Interestingly, this result does not change if we add logarithmic utility of leisure and labor supply choice.
the firms; \( r_t \) is the return to private capital, and \( k_t \) denotes physical capital stock in the beginning of period \( t \).

Household’s physical capital evolves according to the following law of motion:

\[
k_{t+1} = i_t^k + (1 - \delta^k)k_t,
\]
where \( 0 < \delta^k < 1 \) is the depreciation rate on capital.

Finally, the household owns all firms in the economy, and receives all profit \((\pi_t)\) in the form of dividends. The household’s budget constraint is

\[
c_t + i_t^k + i_t^s \leq (1 - \tau_t)[r_t k_t + w_t s_t h_t] + \pi_t + g_t,
\]
where similar to Guo and Lansing (1998),

\[
\tau_t = \eta \left( \frac{y_t}{y} \right)^\phi
\]
denotes the tax rate on total (capital and labor)income, i.e, \( y_t = r_t k_t + w_t s_t h_t \), and \( y \) is the per capita output on the economy's balanced growth path that is growing at a constant rate over time. In addition, \( 0 < \eta < 1 \) and \( 0 \leq \phi < 1 \), where \( \phi \) measures the progressivity of the tax system, and \( \eta \) is the average effective tax rate in steady state.

The representative household acts competitively by taking prices \( \{w_t, r_t\}_{t=0}^\infty \), the tax schedule \( \{\tau_t\}_{t=0}^\infty \) as given, and chooses allocations \( \{c_t, i_t^k, i_t^s, k_{t+1}, s_{t+1}, h_t\}_{t=0}^\infty \) to maximize Eq. (1) s.t. Eqs. (2)-(5), and initial conditions for physical and human capital stocks \( \{k_0, s_0\} \).

The optimality conditions from the household’s problem, together with the transversality

\[\text{[footnote]}\]

\[\text{[footnote]}\]

Even though technically physical and human capital stocks are state variables, and investment in physical and human capital are controls, by choosing how much to invest in the current period, the household indirectly "chooses" next period capital.
conditions (TVC) for physical and human capital are as follows:\(^6\)

\[
c_t : c_t^{-1} = \lambda_t \tag{6}
\]

\[
k_{t+1} : \lambda_t = \beta\lambda_{t+1} \left[ 1 - \delta + \left( 1 + \phi \right) \tau_{t+1} \right] r_{t+1} \tag{7}
\]

\[
s_{t+1} : \lambda_t = \beta\lambda_{t+1} \left[ 1 - \delta^s + \left( 1 + \phi \right) \tau_{t+1} \right] w_{t+1} h_{t+1} h_{t+1} \tag{8}
\]

\[
TVC : \lim_{t \to \infty} \beta^t c_t^{-1} k_{t+1} = 0 \tag{9}
\]

\[
TVC : \lim_{t \to \infty} \beta^t c_t^{-1} s_{t+1} = 0, \tag{10}
\]

where \(\lambda_t\) is the Lagrangian multiplier on the household’s budget constraint. The household equates marginal utility from consumption with the marginal cost imposed on its budget (the shadow price of consumption). Next, the Euler equation describes the optimal consumption allocations chosen in any two contiguous periods. Skill level\(^7\) is then chosen so that the marginal investment cost in education equals the marginal benefit of additional skill accumulation, measured in terms of additional future labor income generated. Note that the presence of progressive taxation (\(\phi > 0\)) decreases further the after-tax return to physical and human capital. The last two expression are the transversality (boundary) conditions imposed on the two types of capital.

### 2.3 Stand-in Firm

There is also a representative private firm in the model economy. It produces a homogeneous final product using a production function that requires physical capital \(k_t\) and efficiency units of labor \(e_t = s_t h_t\). Note that the firm cannot choose skill level and labor hours separately respectively. The production function is as follow

\[
y_t = Ah_t^\theta c_t^{1-\theta}, \tag{11}
\]

where \(A\) measures the level of total factor productivity, \(0 < \theta < 1\) denotes the productivity of physical capital (\(1 - \theta\) denotes the productivity of efficiency labor). The representative firm acts competitively by taking prices \(\{w_t, r_t\}_{t=0}^{\infty}\), income tax schedule \(\{\tau_t\}_{t=0}^{\infty}\), policy variable

\(^6\)Readers interested in a deeper mathematical presentation and a rigorous proof of the respective necessary conditions derived should consult the abstract setup presented in Blot and Chebbi (2000).

\(^7\)Throughout the paper "skill level" and "human capital stock" will be used interchangeably.
\( \{g_t\}_t^{\infty} \) as given, and chooses \( k_t, e_t, \forall t \) to maximize firm’s static profit:

\[
\pi_t = A k_t^\theta e_t^{1-\theta} - r_t k_t - w_t e_t. \tag{12}
\]

In equilibrium profit is zero. In addition, efficiency labor and capital receive their marginal products, i.e.

\[
r_t = \theta \frac{y_t}{k_t}, \tag{13}
\]

\[
w_t = (1 - \theta) \frac{y_t}{e_t}. \tag{14}
\]

### 2.4 Government

The government collects tax revenue from efficiency labor and capital income to finance government transfers. The government budget constraint is then

\[
\tau_t [r_t k_t + w_t e_t] = g_t. \tag{15}
\]

Government takes prices \( \{w_t, r_t\}_t^{\infty} \) and allocations \( \{k_t, e_t\}_t^{\infty} \) as given. Government transfers \( \{g_t\}_t^{\infty} \) will be residually determined: it will adjust to ensure the government budget constraint is balanced in every time period.

### 2.5 Decentralized Equilibrium and Balanced Growth Path

Given the initial conditions for the state variables \( k_0, s_0 \), a Decentralized Competitive Equilibrium (DCE) is defined to be a sequence of prices \( \{r_t, w_t\}_t^{\infty} \), allocations \( \{c_t, i_t^k, i_t^s, k_{t+1}, s_{t+1}, h_t, g_t\}_t^{\infty} \), income tax rate \( \{\tau_t\}_t^{\infty} \) such that (i) the representative household maximizes utility; (ii) the stand-in firm maximizes profit every period; (iii) government budget is balanced in each time period; (iv) all markets clear.

Given the initial conditions for the state variables \( k_0, s_0 \), a balanced growth path (BGP) is a set of sequences of prices \( \{r_t, w_t\}_t^{\infty} \), allocations \( \{c_t, i_t^k, i_t^s, k_{t+1}, s_{t+1}, h_t, g_t\}_t^{\infty} \), and income tax rate \( \{\tau_t\}_t^{\infty} \) satisfying the Decentralized Competitive Equilibrium definition such that the paths \( \{c_t, i_t^k, i_t^s, k_{t+1}, s_{t+1}, g_t\}_t^{\infty} \) grow at the same rate \( \gamma \), \( \{h_t\}_t^{\infty} \) and prices \( \{r_t, w_t\}_t^{\infty} \) remain constant, and the output-physical capital and output-human capital ratio is constant.
Next section proceeds to describe the specifics of Bulgaria’s tax regime, both before and after the adoption of proportional taxation, and then proceeds to carefully calibrates the model to Bulgarian data compute the BGP first, and then, after detrending the variables, to investigate the dynamic stability of the now stationary equilibrium system.

3 Bulgaria’s progressive income taxation regime

Until Dec. 31, 2007, Bulgaria applied progressive income taxation schedule on individual income, with the tax brackets for 2007 stipulated in Table 1 below.

<table>
<thead>
<tr>
<th>Annual taxable income (in BGN)</th>
<th>Tax owed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-2400</td>
<td>Zero-bracket amount</td>
</tr>
<tr>
<td>2400-3000</td>
<td>20% on the amount earned above BGN 2400</td>
</tr>
<tr>
<td>3000-7200</td>
<td>BGN 120 + 22% on the excess over BGN 3000</td>
</tr>
<tr>
<td>&gt; 7200</td>
<td>BGN 1044 + 24% on the excess over BGN 7200</td>
</tr>
</tbody>
</table>

*Source: Petkova (2012)*

As of 2008, a flat tax rate of 10% on personal income was introduced, which represented a substantial decrease in both the marginal and the average effective tax rate.

4 Data, model calibration and steady-state computation

The model is calibrated to Bulgarian data at annual frequency. The period under investigation is 1993-2014, where the 1993-2007 sub-period covers the progressive taxation regime, and 2008-14 is the proportional tax regime. As of 2008, a flat tax rate of 10% on personal income was introduced. Data on the household consumption and private fixed investment shares in output were obtained from the World Bank World Development Indicators (WDI) Database (2014). Private expenditure on human capital as a share of output, as well as government transfers as a share of output were then computed using data from the National
Statistical Institute (NSI). Finally, the long-term interest rate (LTIR) was obtained from Bulgarian National Bank (BNB) Statistics.

The discount factor was obtained as $\beta = \frac{1}{1 + \text{mean}(LTIR)} = 0.968$, which is a standard value in the literature. Next, following Ganev (2005), capital income share is set to its average value $\theta = 0.429$. Without any loss of generality, the level of steady-state output can be normalized to unity, $y = 1$. Next, using Ganev’s (2005) estimate that $k/y = 3.491$, and WDI’s data on average investment share in output $i^k/y = 0.165$, we can obtain the depreciation rate on physical capital, $\delta^k = 0.047$.\footnote{For 2008-14, $k/y = 5.41, i^k/y = 0.252, c/y = 0.7$.} Using NSI data on the total expenditure on education as a proxy for investment in skills, we obtain that the share of human investment in output is $i^s/y = 0.048$. Given that $i^s = \delta^s s$, we obtain $s/y = 7.61$ and $\delta^s = 0.006$. Lastly, the average effective tax rate $\eta$ is approximated by the average amount of tax actually paid, divided by total income, which produced $\eta = 0.14$ for the progressive tax regime, and $\eta = 0.11$ for the flat tax regime. Next, the (gross) degree of progressivity, $1 + \phi = 1.43$, was computed as the ratio of the marginal to the average tax rate. Average per capital growth rate during the progressive taxation regime was 1.58\% versus 0.95\% for the flat tax regime.\footnote{The lower growth during the period 2008-2014 was due to the global financial crisis in place at that time, which offset many of the positive effects of the tax reform.} Table 2 on the next page summarizes the values of all model parameters used:

<table>
<thead>
<tr>
<th>Param.</th>
<th>Value</th>
<th>Definition</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.968</td>
<td>Discount factor</td>
<td>Calibrated</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.429</td>
<td>Capital income share</td>
<td>Data Average</td>
</tr>
<tr>
<td>$\delta^k$</td>
<td>0.047</td>
<td>Depreciation rate of physical capital</td>
<td>Calibrated</td>
</tr>
<tr>
<td>$\delta^s$</td>
<td>0.006</td>
<td>Depreciation rate of human capital</td>
<td>Calibrated</td>
</tr>
<tr>
<td>$\eta$</td>
<td>${0.14; 0.11}$</td>
<td>Average effective income tax rate</td>
<td>Data average</td>
</tr>
<tr>
<td>$\phi$</td>
<td>${0.43; 0}$</td>
<td>Average tax progressivity (progressive/flat)</td>
<td>Data average</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>${0.0158; 0.0095}$</td>
<td>Avg. growth rate (progressive/flat)</td>
<td>Data average</td>
</tr>
</tbody>
</table>
Once model parameters were obtained, the steady-state ratios for the model calibrated to Bulgarian data were obtained. The results are reported in Table 3 below.

<table>
<thead>
<tr>
<th>Description</th>
<th>BG Data</th>
<th>Model (prog.)</th>
<th>Model (flat)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption-to-output ratio</td>
<td>0.672</td>
<td>0.787</td>
<td>0.700</td>
</tr>
<tr>
<td>Fixed investment-to-output ratio</td>
<td>0.165</td>
<td>0.165</td>
<td>0.252</td>
</tr>
<tr>
<td>Human capital investment-to-output ratio</td>
<td>0.048</td>
<td>0.048</td>
<td>0.048</td>
</tr>
<tr>
<td>Government transfers-to-output ratio</td>
<td>0.176</td>
<td>0.14</td>
<td>0.11</td>
</tr>
<tr>
<td>Physical capital-to-output ratio</td>
<td>3.491/5.410</td>
<td>3.491</td>
<td>5.410</td>
</tr>
<tr>
<td>Human capital-to-output ratio</td>
<td>N/A</td>
<td>7.610</td>
<td>7.610</td>
</tr>
<tr>
<td>Labor share in output</td>
<td>0.571</td>
<td>0.571</td>
<td>0.571</td>
</tr>
<tr>
<td>Capital share in output</td>
<td>0.429</td>
<td>0.429</td>
<td>0.429</td>
</tr>
</tbody>
</table>

In the next section, the non-stationary system is presented, and the stability of the detrended equilibrium system is discussed.

## 5 Stationary Equilibrium System

### 5.1 Detrending the original non-stationary system

\[
c_t^{-1} = \lambda_t
\]

\[
\lambda_t = \beta \lambda_{t+1} \left[ (1 - \delta_k) + \left( 1 - (1 + \phi) \tau_{t+1} \right) r_{t+1} \right]
\]

\[
\lambda_t = \beta \lambda_{t+1} \left[ (1 - \delta^s) + \left( 1 - (1 + \phi) \tau_{t+1} \right) w_{t+1} s_{t+1} \right]
\]

\[
r_t = \frac{\theta y_t}{k_t}
\]

\[
w_t = (1 - \theta) \frac{y_t}{s_t h_t}
\]

\[
g_t = \tau [r_t k_t + w_t s_t h_t]
\]

\[
A k_t^\theta (s_t h_t)^{1-\theta} = c_t + k_{t+1} - (1 - \delta_k) k_t + s_{t+1} - (1 - \delta^s) s_t
\]

After carefully calibrating the model parameters and the steady-state, the long-run growth
rate can be obtained as follows:

\[
\frac{c_t}{c_{t-1}} = 1 + \gamma = \beta \left[ 1 - \delta^k + \left( 1 - (1 + \phi) \tau_t \right) \frac{\theta y}{k_t} \right]
\]  

(23)

The balanced growth path rate is positively related to the discount factor (\(\beta\)), capital share parameter (\(\theta\)), and negatively related to the depreciation rate (\(\delta^k\)), capital-to-output ratio (\(k/y\)), degree of tax progressivity (\(\phi\)) and the average effective income tax rate (\(\tau\)).

Our model features a balanced growth path, where consumption (hence the Lagrangean multiplier), physical capital, skills, government transfers, output all grow at the same rate, \(\gamma\), while hours and prices are constant, so \(h_t = \bar{h} = 1\). Let the adjusted discount factor be denoted by \(\bar{\beta} = \beta/(1 + \gamma)\). Also, we will combine the two Euler equations to simplify the expression. Let \(\delta = \delta^k - \delta^s\). The detrended system then becomes (using upper bars to denote the made-stationary variables)

\[
\bar{c}^{-1}_t = \bar{\lambda}_t
\]  

(24)

\[
\bar{\lambda}_t = \bar{\beta}\bar{\lambda}_{t+1} \left[ (1 - \delta^k) + \left( 1 - (1 + \phi) \bar{\tau}_{t+1} \right) \bar{r}_{t+1} \right]
\]  

(25)

\[
\left( 1 - (1 + \phi) \bar{\tau}_t \right) \bar{r}_t = \left( 1 - (1 + \phi) \bar{\tau}_t \right) \bar{w}_t \bar{h} + \delta
\]  

(26)

\[
\bar{r}_t = \theta \frac{\bar{y}_t}{\bar{k}_t}
\]  

(27)

\[
\bar{w}_t = (1 - \theta) \frac{\bar{y}_t}{\bar{s}_t \bar{h}}
\]  

(28)

\[
\bar{g}_t = \tau_t \left[ \bar{r}_t \bar{k}_t + \bar{w}_t \bar{s}_t \bar{h} \right]
\]  

(29)

\[
A \bar{k}_t^\theta (\bar{s}_t)^{1-\theta} = \bar{c}_t + (1 + \gamma) \bar{k}_{t+1} - (1 - \delta^k) \bar{k}_t + (1 + \gamma) \bar{s}_{t+1} - (1 - \delta^s) \bar{s}_t
\]  

(30)

Next we proceed to analyze the dynamic stability of the now stationary equilibrium system.

### 5.2 Stability

The system is first log-linearized around its unique steady-state, and simplified until it can be sufficiently represented as a deterministic first-order linear difference system in consumption (a choice variable) and skills (the state variable):

\[
\begin{pmatrix}
\hat{c}_{t+1} \\
\hat{s}_{t+1}
\end{pmatrix} = \begin{pmatrix} I & J \\ K & L \end{pmatrix} \begin{pmatrix} \hat{c}_t \\ \hat{s}_t \end{pmatrix}
\]
where $I, J, K, L$ are functions of model parameters (for the analytical expressions, consult the Technical Appendix accompanying this paper). Let

$$Z = \begin{pmatrix} I & J \\ K & L \end{pmatrix}$$

In order to find the characteristic roots of $Z$ we set

$$\begin{vmatrix} I - \lambda & J \\ K & L - \lambda \end{vmatrix} = 0$$

The characteristic polynomial is then

$$\lambda^2 - (I + L)\lambda + (IL - JK) = 0$$

and thus the system has two distinct, real roots:

$$\lambda_1 = \frac{(I + L) + \sqrt{(I - L)^2 + 4JK}}{2}$$
$$\lambda_2 = \frac{(I + L) - \sqrt{(I - L)^2 + 4JK}}{2}$$

For the values of the model parameters for Bulgarian data, we obtain

$$I = 0.9629, J = 0.2001, K = -0.1969, L = 1.0628$$

and

$$\lambda_1 = 0.9840$$
$$\lambda_2 = 0.8841$$

Since the two characteristic roots are positive, but less than one, the system is globally stable (indeterminacy). We have two variables in the system and one initial condition (for skills). This means that the Bulgarian economy can reach the steady state with either high or low consumption. This finding could be rationalized with "animal spirits" explanation provided in Chen and Guo (2015) in the case of an AK endogenous growth model. However, here the transmission channel is slightly different. Households may become optimistic about the future after-tax return to capital and efficiency labor and cut on consumption today in order to invest more in both physical and human capital. Since the after-tax MPK is increasing
along the transition path due to the human capital accumulation, agents’ expectations are validated, so such a trajectory is also possible and becomes a "self-fulfilling equilibrium".

In contrast, for Bulgaria under the proportional (flat) tax regime (2008-2014) we obtain the following values:

\[ I = 0.9582, J = 0.1523, K = -0.3182, L = 1.1594 \]

and

\[ \lambda_1 = 1.0624 \]
\[ \lambda_2 = 0.8612 \]

Now the model exhibits saddle-path stability, with one stable and one unstable root. This means that the path above is no longer possible, as the initial conditions for physical and human capital \{k_0, s_0\} uniquely determine c_0 and the equilibrium path of the detrended model.\textsuperscript{10} Therefore, in the original endogenous growth model no endogenous growth fluctuations cannot occur.

### 6 Conclusions

We showed that in a endogenous growth model with human accumulation calibrated to Bulgarian data under the progressive taxation regime (1993-2007), the artificial economy exhibits equilibrium indeterminacy. These results are in line with the recent findings in Chen and Guo (2015) in the context of an AK endogenous growth model. Also, the findings are in contrast to Guo and Lansing (1988) who argue that progressive taxation works as an automatic stabilizer. Progressive taxation in our setup led to equilibrium indeterminacy. This indeterminacy result could explain, at least partially, why the economic performance under the progressive taxation regime in Bulgaria was not impressive.

\textsuperscript{10}Since the two types of capital are substitutes, the dynamics of physical capital is proportional to the dynamics of human capital series.
References


