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Jitendra Kuma[a], Anoop Chaturvedi[b] and Umme Afifa[c]

Abstract

Present paper studies the panel data auto regressive (PAR) time series model for testing the unit root hypothesis. The posterior odds ratio (POR) is derived under appropriate prior assumptions and then empirical analysis is carried out for testing the unit root hypothesis of Net Asset Value of National Pension schemes (NPS) for different fund managers. The unit root hypothesis for the model with linear time trend and linear time trend with augmentation term is carried out. The estimated autoregressive coefficient is far away from one in case of linear time trend only so, testing is not executed but in consideration of augmentation term, it is close to one. Therefore, we performed the unit root hypothesis testing using the derived POR. In all cases unit root hypothesis is rejected therefore all NPS series are concluded trend stationary.

JEL classification: **C11, C12, C22, C23, C39**

Keywords: *Panel data, Stationarity, Autoregressive time series, Unit root, Posterior odds ratio, New Pension Scheme, Net Asset Value*

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1. INTRODUCTION

Pension provision is a way of social security in many countries and covered by public scheme. This is often supplemented by occupational pension schemes as public schemes vary substantially among advanced economies. There are limited working population ranging from 10 to 25 percent Schwarz (2003) and in India there is at the most 15% working population who was getting pension through old pension scheme. Old pension plan was not able to facilitate common people as well as it is increasing load budget on government expenditure (Robert and Daniel 2001). Shah (2006) summarizes the pension reform and its goal with detailed comparison of new and old pension scheme.

Occupational pension schemes are implemented through defined benefit (DB) and defined contribution (DC) schemes. DB schemes provide more income after retirement without portability however DC schemes is well user friendly and permits to change the employer with the returns as contributed to the retirement fund. Main inconveniences under this, its dependency and relationship with influence on the design of retirement plans and labour markets Friedberg (2011). There are several studies on the pension fund industry to discuss the issues, performance, challenges and reforms to be required for pensions fund, please refer Black (1989), (Brown, Clark and Rauh 2011), (Dushi, Friedberg and Webb 2010) and Franzen (2010). The concern is also taken care by many institutions like Department for Works and Pensions (2010), Deutsche Bank Research (2011), Global Financial Stability Report (2005) and OECD (2005)). (Sane and Thomas 2013) have addressed well on all associate issues with NPS. Present NPS is designed as the participation of different institutions from the market and banking may take the initiative to explore the possibilities on the market and work as the benefits to the contributors may maximize.

Time series is the way of data analytics, where present observation is modeled based on past observation. If the dependency is linear, model is called Autoregressive Time Series Models and it is recorded in respect to different paces, categorization is termed as Panel Data Autoregressive Time Series Model. For better forecasting, the efficiency of the parameter/model is very important. If, parameters depend on time, series is called non stationary series. Main cause of non-stationarity of a series is time trend but this may also occur due to unit root (Dickey and Fuller 1979). A single equation method has low power when the root is close to unity. These tests also have low power in short term time span, see (Shiller and Perron 1995). There are several approaches which are explored to test the unit root hypotheses considering various type of model in reference to trend as well as error please refer (Schotman and Van Dijk 1991a, 1991b), (Kumar, Shukla and Chaturvedi 2012), (Kumar, Shukla and Tiwari 2014), (Kumar, Kumar and Chaturvedi 2012). There are several approaches in the literature on unit root test to extend the situations, where panel data are available. (Levin and Lin 1993a, 1993b) proposed to apply unit root test on a pooled cross-sectional data set, instead of single equation unit root test for each series.

The main motivation behind panel data unit root tests, as discussed by (Kim and Maddala 1998) is to increase the power of the test by increasing the sample size. Such panel data unit root tests exploit the cross sectional information and lead to increase in the power of the test. Panel data unit root tests have been widely applied in empirical studies. (Breitung and Meyer 1994) obtained the asymptotic normality of Dickey-Fuller test statistic for panel data with arbitrarily large cross sectional dimensions and small fixed time series dimensions. Pappell (1997) examined the stationarity of purchase power purity using panel data unit root tests. (Im, Pesaran and Shin 1997) criticized the Levin-Lin test as it assumes the same long-run multiplier across countries under the alternative hypothesis and proposed a new test based on the mean group approach. They observed that their *t*-bar statistic has higher power than Levin-Lin test by allowing for a greater heterogeneity across individuals. Wu (2000) applied the panel data unit root tests to obtain support for the mean reverting property of current account. (Breuer, McNown and Wallaca 2002) considered an alternative panel data unit root test that exploit the power of panel data analysis without imposing uniformity across the panel under either the null or the alternative hypothesis. (Levin, Lin and Chu 2002) analyzed the asymptotic and finite sample properties of panel data unit root test when the intercept and trend are allowed to vary across individuals.

(Bond, Nauges and Windmeijer 2005) studies the unit root tests for micro panels considering number of individuals is typically large and periods very small. Study was more based toward the identification of parameters of interest in context of unit root. Calculations of asymptotic local power and Monte Carlo evidence indicate that two simple *t*-tests based on ordinary least squares estimators perform particularly well. Kruiniger (2008) discussed panel data time series model considering stationary covariate to test the stationarity. He had discussed panel data unit root test procedures based on the

First difference MLE**.** One may also refer for more details about the panel data in this context from (Harris and Tzavalis 1999), (Maddala and Wu 1999), (Kruiniger and Tzavalis 2002), (Moon and Perron 2004), (DeWachter, Harris and Tzavalis 2007), (Moon and Perron 2008), Madsen (2010), (De Blander and Dhaene 2012).

Recently (Karavias and Tzavalis 2014) had study the asymptotic local power properties in reference to panel data unit root test for various fixed T and serially correlated error. They also studied the case considering the instrumental variable and found that variables are dominant in the case of test based on the within-groups estimator.

The classical unit root tests are based on the assumption that population is finite and parameters are constant. However Bayesian testing procedures are free from such assumption. Present paper dealt the Bayesian analysis of panel data time series model to test that series is difference stationary or trend stationary through unit root hypothesis. A posterior odds ratio is derived under appropriate assumptions for testing the unit root hypothesis. An empirical analysis is carried-out on the recorded Net Asset Value (NAV) of National Pension Scheme (NPS). We have tested the unit root hypothesis of recorded NAV considering linear time trend and linear time trend with augmentation term. The estimated value of AR coefficient is far away from one if trend is taken linear only, so we did not performed the testing. When linear trend is taken with augmentation term then AR coefficient is close to one. Therefore, we have tested the unit root hypothesis and get that series are trend stationary. The observed error variance is minimum when trend is taken by linear with augmentation term in comparison to linear only.

2. MODEL AND HYPOTHESIS:

Let $\{y_{it};\ i=1,2,\ldots,n;\ t=1,2,\ldots,T\}$ be panel data time series observations on each of *n* cross section. We assume that the time series follows the process

$$y_{it} = \mu_i + \delta_i t + u_{it} \qquad (i=1,2,\ldots,n;\ t=1,2,\ldots,T), \tag{1}$$

Where u_{it} is stochastic error term following AR (1) process

$$u_{it} = \rho u_{it-1} + \varepsilon_{it} \qquad (i=1,2,\ldots,n;\ t=1,2\ldots,T), \tag{2}$$

Further, ε_{it}'s are *iid* random variables, each following normal distribution with mean zero and variance τ^{-1}.

We can write the model (1) incorporating (2) as;

$$y_{it} = \rho y_{it-1} + \left[(1-\rho)\mu_i + \rho\delta_i\right] + (1-\rho)\delta_i t + \varepsilon_{it}\,; \quad (i=1,2,\ldots,n;\ t=1,2,\ldots,T) \tag{3}$$

Let us define

$$\phi_i = \mu_i + \frac{\rho}{(1-\rho)}\delta_i; \quad \beta_i = (1-\rho)\delta_i \quad (i=1,2\ldots,n;\ t=1,2,..,T)$$

Then model (3) can be expressed as;

$$(y_{it} - \phi_i) = \rho(y_{it-1} - \phi_i) + \beta_i t + \varepsilon_{it}\,; \qquad (i=1,2\ldots,n;\ t=1,2,..,T) \tag{4}$$

We are interested in testing the unit root hypothesis H_0: ρ=1against the alternative $H_1{:}\rho \in S$ with $S = \{a < \rho < 1; a > -1\}$. Under the null hypothesis of unit root, the model (3) reduces to

$$\Delta y_{it} = \delta_i + \varepsilon_{it}; \qquad (i=1,2\ldots,n;\ t=1,2,..,T) \qquad (5)$$

Where Δ is the difference operator defined as;

$$\Delta y_{it} = y_{it} - y_{it-1}$$

We may write above model (4) and (5) incorporating augmentation term. Under the alternative hypothesis, model is

$$(y_{it} - \phi_i) = \rho(y_{it-1} - \phi_i) + \beta_i t + \sum_{j=1}^{k_i} \theta_{ij} \Delta y_{i\,t-j} + \varepsilon_{it}; \ (i=1,2,\ldots,n;\ t=1,2\ldots,T) \qquad (6)$$

and under the null hypothesis of unit root, model reduces to

$$\Delta y_{it} = \delta_i + \sum_{j=1}^{k_i} \theta_{ij} \Delta y_{i\,t-j} + \varepsilon_{it}; \qquad (i=1,2,\ldots,n;\ t=1,2\ldots,T \qquad (7)$$

Notice that the *i*-th equation includes an augmentation term of order k_i.

For writing the models (4), (5) (6) and (7) in matrix notations, considering l_T be a $T\times 1$ vector with all elements 1, I_n is the identity matrix of order n and $l_n = (1,2,\ldots,T)'$. Further, we define

$$\Delta y_i = (\Delta y_{i1}, \Delta y_{i2}, \ldots, \Delta y_{it})', \qquad \Delta y = \left(\Delta y_1', \Delta y_2', \ldots, \Delta y_n'\right)'$$

$$Z = \left[(I_n \otimes l_T) \quad (I_n \otimes l_n)\right], \qquad \alpha_i = (1-\rho)\phi_i$$

$$\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)', \qquad \beta = (\beta_1, \beta_2, \ldots, \beta_n)'$$

$$\gamma = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \qquad \delta = (\delta_1, \ldots, \delta_n)'$$

$$k=\sum_{i=1}^{n}k_i \ ,$$

$$\theta_i=\begin{pmatrix}\theta_{i1}\\ \theta_{i2}\\ \vdots\\ \theta_{i\,k_i}\end{pmatrix}, \qquad X_i=\begin{pmatrix}\Delta y_{i\ 0} & \Delta y_{i\ -1} & \cdots & \Delta y_{i\ 1-k_i}\\ \Delta y_{i\ -1} & \Delta y_{i\ 0} & \cdots & \Delta y_{i\ 2-k_i}\\ \vdots & \vdots & \ddots & \vdots\\ \Delta y_{i\ T-1} & \Delta y_{i\ T-2} & & \Delta y_{i\ T-k_i}\end{pmatrix},$$

$$\theta=\begin{pmatrix}\theta_1\\ \theta_2\\ \vdots\\ \theta_n\end{pmatrix} \qquad X=\begin{pmatrix}X_1' & 0 & \cdots & 0\\ 0 & X_2' & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & X_n'\end{pmatrix} \qquad (8)$$

Using the above notations, the model with linear time trend can be represented as:

Under H_0: $$\Delta y=(I_n\otimes l_T)\delta+\varepsilon \qquad (9)$$

Under H_1: $$y=\rho y_{-1}+Z\gamma+\varepsilon \qquad (10)$$

and model with linear time trend and augmentation term as:

Under H_0: $$\Delta y=(I_n\otimes l_T)\delta+X\theta+\varepsilon \qquad (11)$$

Under H_1: $$y=\rho y_{-1}+Z\gamma+X\theta+\varepsilon \qquad (12)$$

3. POSTERIOR ODDS RATIO

In this section, we derive the posterior odds ratio for the unit root hypothesis. We assume the following prior distributions for the parameters of the model

$$\delta\sim N\left(0,\frac{1}{\vartheta\tau}I_n\right)$$

$$\alpha \sim N\left((1-\rho)y_0, \quad \frac{(1+\rho)}{(1-\rho)\tau} I_n \right)$$

$$\beta \sim N\left(0, \frac{(1-\rho)^2}{\vartheta\tau} I_n \right)$$

$$p(\tau) \propto \frac{1}{\tau}; 0 < \tau < \infty$$

$$p(\rho) = \frac{1}{1-a}; \; a < \rho < 1; \, p(\theta) \propto 1 (13)$$

Then prior distribution for γ is given by

$$\gamma \sim N\left((1-\rho)\phi_0, \frac{1}{\tau} V(\rho)^{-1} \right) \tag{14}$$

Where y_0 is the vector of initial observations,

$$\phi_0 = \begin{pmatrix} y_0 \\ 0 \end{pmatrix}, \qquad V(\rho) = \begin{pmatrix} \frac{(1+\rho)}{(1-\rho)} I_n & 0 \\ 0 & \frac{\vartheta}{(1-\rho)^2} I_n \end{pmatrix} \tag{15}$$

We also assume that the prior probability in favor of H_0 is $P(\rho = 1) = p_0$ and prior probability in favor of H_1 is $P(\rho \in S) = 1 - p_0$. Let we define

$$R = I - (\vartheta + \tau)^{-1} \left(I_n \otimes l_T l_T{}' \right)$$

$$\eta = \Delta y' R \Delta y$$

$$G(\rho) = Z'Z + V(\rho)$$

$$\eta(\rho) = (1-\rho)^2 y_0{}' y_0 + (y - \rho y_{-1})'(y - \rho y_{-1}) - \left(Z'(y - \rho y_{-1}) + (1-\rho)\phi_0 \right)' G(\rho)^{-1} \left(Z'(y - \rho y_{-1}) + (1-\rho)\phi_0 \right)$$

$$\Sigma = I_{nT} - X(X'X)^{-1} X',$$

$$A = (I_n \otimes l_T)' \Sigma (I_n \otimes l_T) + \vartheta I_n,$$

$$\varsigma = \Delta y'\left(\Sigma - \Sigma(I_n \otimes l_T)A^{-1}(I_n \otimes l_T)'\Sigma\right)\Delta y$$
$$B(\rho) = Z'\Sigma Z + V(\rho)$$
$$\varsigma(\rho) = (y - \rho y_{-1})'\Sigma(y - \rho y_{-1}) - \left(Z'(y - \rho y_{-1}) + (1-\rho^2)\phi_0\right)' B(\rho)\left(Z'(y - \rho y_{-1}) + (1-\rho^2)\phi_0\right) \tag{16}$$

Theorem-1: *The posterior odds ratio, denoted by β_{01}, for testing the unit root hypothesis for the PAR (1) model with linear time trend, with prior odds ratio ($p_0/1-p_0$), is given by:*

$$\beta_{01} = \frac{p_0}{1-p_0}\frac{1-a}{(\vartheta+T)^{-1}\eta^{\frac{nT}{2}}}\left[\int_a^1 \frac{(1+\rho)^{n/2}}{(1-\rho)^{3n/2}|G(\rho)|^{\frac{1}{2}}(\eta(\rho))^{\frac{nT}{2}}}d\rho\right]^{-1} \tag{17}$$

Theorem-2*: A PAR(1) panel data time series model with linear time trend and augmentation term is difference stationary or trend stationary equivalent to H_0:ρ=1 against the alternative H_1: $\rho \in S$ with $S = \{a < \rho < 1; a > -1\}$ with prior odds ratio $p_0/1-p_0$ can be tested by the posterior odds ratio is given by:*

$$\beta_{01} = \frac{p_0}{1-p_0}\frac{1-a}{|A|^{1/2}\varsigma^{\frac{nT-k}{2}}}\left[\int_a^1 \frac{(1+\rho)^{n/2}}{(1-\rho)^{3n/2}|B(\rho)|^{\frac{1}{2}}(\varsigma(\rho))^{\frac{nT-k}{2}}}d\rho\right]^{-1} \tag{18}$$

4. NUMERICAL SIMULATION

New pension scheme was started with motive to get benefit from the market as market has more opportunity of profit with several plans. NPS fund is regulated by Pension Fund Regulatory Development Authority (PFRDA) and it is recognizing the Bank and other institutions who may participate in NPS fund activities. We have taken the time series of recorded daily NAV for the period February 01, 2010 to December 31, 2015. For analysis purpose, series is converted into monthly average and tested the Unit root hypothesis using the derived theorem, considering panel in respect to Scheme. Panel data time series are permitting to record the data from panels and do the analysis. We have recorded the time series of daily Net Asset Value (NAV) and considered the panel as Scheme in respect to bank. We are interested to test that series is difference stationary or trend stationary equivalently the unit root hypothesis H_0: ρ=1 against the alternative H_1: $\rho \in S$ with $S = \{a < \rho < 1; a > -1\}$.

Let $\{y(i,j,k)_t\}$ is a recorded NAV of i^{th} Bank, j^{th} Tier and k^{th} Schemes, *i=1(SBI),2(ICICI),3(KM),4(UTI)* stands for bank; *j =1(Tier-1),2(Tier-2)* stands for tier and *k=1(E),2(C),3(G)* stands for schemes. We have taken schemes as panel in respect to different Banks and Model is given below:-

$$NAV(i,j)_{k,t} = \text{intercept}(i,j)_k + \text{trend}(i,j)_k t + \rho(i,j) NAV(i,j)_{k,t-1} + \varepsilon(i,j)_{k,t} \quad (19)$$

The study of above model also explored for the model with augmentation term

$$NAV(i,j)_{k,t} = \text{intercept}(i,j)_k + \text{trend}(i,j)_k t + \rho(i,j) NAV(i,j)_{k,t-1} + \sum_{j=1}^{k} \theta_{AUG} \Delta NAV(i,j)_{k,t-j} + \varepsilon(i,j)_{k,t} \quad (20)$$

Using the derived Theorem-1 and Theorem-2 for the model (4) and (6) considering augmentation term of order 1 and 2. We have tested the unit root hypothesis H_0:ρ=1 on the basis of estimated value of Posterior Odds Ratio (β_{01}), $\hat{\rho}$ and $SE(\hat{\rho})$ against the alternative H_1:$\rho \in S$ with $S = \{a < \rho < 1; a > -1\}$ equivalently series is trend stationary considering (i) linear time trend and (ii) linear time trend with augmentation term. The findings are given in table-1 for tier I and tier II.

Table-1: Posterior Odds Ratio for the PAR series

		Tier- 1				Tier-ll			
		ICICI	**KM**	**SBI**	**UTI**	**ICICI**	**KM**	**SBI**	**UTI**
Only Linear Time Trend	$\hat{\rho}$	0.8274	0.8276	0.8237	0.845	0.8342	0.8365	0.8266	0.8381
	$SE(\rho)$	1.5902	1.5059	1.4803	1.517	1.33891	1.27488	1.40364	1.39804
	$\hat{\sigma}^2$	24.849	20.216	20.165	23.761	16.359	15.056	17.233	17.741
Linear Time Trend With Augmentation order 1	β_{01}	5.46	0.212	0.131	0.734	0.0995	0.176	0.121	0.184
	$\hat{\rho}$	0.93994	0.9337	0.93462	0.94463	0.9303	0.93615	0.93091	0.93782
	$SE(\rho)$	0.56511	0.5148	0.4963	0.51456	0.43556	0.42621	0.45497	0.43414
	$\hat{\sigma}^2$	8.697	6.831	6.67	7.971	5.303	4.977	5.529	5.461
Linear Time Trend With Augmentation order 2	β_{01}	5.56E-143	5E-154	2.8E-171	2.2E-154	1.7E-172	4.1E-184	3.3E-173	8.9E-177
	$\hat{\rho}$	0.94366	0.9391	0.94275	0.94725	0.9347	0.94209	0.93895	0.94302
	$SE(\rho)$	0.56837	0.5134	0.49314	0.51476	0.43552	0.42249	0.45229	0.42947
	$\hat{\sigma}^2$	8.5290545	6.702592	6.503682	7.852787	5.205614	4.864284	5.3998	5.322138

We got optimum posterior odds ratio in the case of augmentation term with order 2 and for the analysis purpose optimum results of maximum likelihood estimates of autoregressive regression coefficients and their variances are listed on table-2 for all fund managers under study of both tiers-

Table-2: Maximum likelihood estimates of AR coefficients and their variances

	TIER-I											
	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_3$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\theta}_{11}$	$\hat{\theta}_{21}$	$\hat{\theta}_{31}$	$\hat{\theta}_{12}$	$\hat{\theta}_{22}$	$\hat{\theta}_{32}$
ICICI	0.670 [33.576]	0.607 [32.753]	0.554 [31.698]	0.008 [0.008]	0.008 [0.007]	0.006 [0.005]	0.002 [3.211]	0.465 [69.088]	0.580 [36.020]	0.006 [3.277]	-0.349 [69.737]	-0.310 [36.913]
KM	0.676 [26.702]	0.701 [30.513]	0.635 [27.527]	0.008 [0.006]	0.009 [0.006]	0.007 [0.004]	-0.014 [2.491]	0.566 [41.069]	0.579 [21.805]	0.006 [2.562]	-0.397 [41.743]	-0.327 [22.012]
SBI	0.561 [19.817]	0.631 [24.804]	0.626 [25.567]	0.007 [0.004]	0.008 [0.005]	0.007 [0.004]	-0.015 [2.654]	0.562 [34.172]	0.548 [13.176]	-0.014 [2.707]	-0.379 [34.651]	-0.322 [13.503]
UTI	0.637 [29.365]	0.53 [25.584]	0.518 [25.871]	0.007 [0.005]	0.007 [0.005]	0.006 [0.004]	-0.003 [2.948]	0.541 [78.57]	0.553 [26.309]	0.023 [2.95]	-0.4 [80.488]	-0.311 [26.838]
TIER-II												
ICICI	0.583 [15.113]	0.709 [19.231]	0.648 [18.366]	0.009 [0.003]	0.009 [0.004]	0.007 [0.003]	0.002 [2.01]	0.437 [29.014]	0.605 [14.752]	0.025 [2.025]	-0.339 [29.432]	-0.315 [15.19]
KM	0.575 [14.129]	0.587 [16.795]	0.567 [16.777]	0.007 [0.003]	0.007 [0.003]	0.006 [0.002]	0.006 [1.816]	0.541 [33.27]	0.613 [14.658]	-0.011 [1.867]	-0.408 [32.781]	-0.364 14.341
SBI	0.561 [19.817]	0.631 [24.804]	0.626 [25.567]	0.007 [0.004]	0.008 [0.005]	0.007 [0.004]	-0.015 [2.654]	0.562 [34.172]	0.548 [13.176]	-0.014 [2.707]	-0.379 [34.651]	-0.322 [13.503]
UTI	0.546 [14.393]	0.587 [17.676]	0.626 [19.512]	0.007 [0.003]	0.007 [0.003]	0.006 [0.003]	0.003 [2.091]	0.56 [33.086]	0.539 [10.462]	0.033 [2.102]	-0.444 [33.832]	-0.3 [10.702]

The variance covariance matrix of regression coefficient is calculated for all banks under all three cases but we have reported the result only for the model with optimum variance i.e., model of augmentation term with order 2, which is listed on Appendix tables from A1-1.1 to A1-1.4 and from A2-2.1 to A2-2.4 for tier-I and tier-II respectively.

Here, it is noted that the unit root hypothesis is not performed for the model with linear time trend because estimated value of autoregressive coefficient is too less. However it is close to one, when we have considered the linear time trend with augmentation term. We have tested the unit root hypothesis considering the augmentation term of order 1 and 2 and get that series is trend stationary on both cases. The error variance is smallest in case of augmentation term of order 2 on all cases. The POR is less than one therefore the unit root hypothesis is rejected on all cases and we conclude that NAV series are trend stationary on both cases followed by linear time trend also.

5. CONCLUDING REMARKS

Present article has explored the unit root hypothesis under Bayesian framework using the Posterior Odds Ratio. Based on theorem, we got that selected series is trend stationary with and without consideration of augmentation term with the intercept trend. Work may be extended for the model with time effect and covariate taken into account of structure break as well as non-normal errors also.

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Appendix

Proof of the Theorem 1

Proof:

The likelihood function under the unit root hypothesis $H_0 : \rho = 1$ is by

$$p(y|\delta,\tau) = \frac{\tau^{\frac{nT}{2}}}{(2\pi)^{\frac{nT}{2}}} \exp\left[-\frac{\tau}{2}\left\{(\Delta y - (I_n \otimes l_T)\delta)'(\Delta y - (I_n \otimes l_T)\delta)\right\}\right] \qquad \text{(A1.1)}$$

Combining the likelihood function (A1.1) with the prior distribution (14) for the parameters δ and τ we obtain

$$p(y|H_0) = \int p(y/\delta,\tau).p(\delta/\tau).p(\tau)d\delta d\tau$$

$$= \int_0^\infty \int_{R^n} \frac{\tau^{\frac{nT+n}{2}-1} \vartheta^{\frac{n}{2}}}{(2\pi)^{\frac{nT+n}{2}}} \exp\left[-\frac{\tau}{2}\left((\Delta y - (I_n \otimes l_T)\delta)'(\Delta y - (I_n \otimes l_T)\delta) + \vartheta\delta'\delta\right)\right] d\delta d\tau$$

$$= \int_0^\infty \int_{R^n} \frac{\tau^{\frac{nT+n}{2}-1} \vartheta^{\frac{n}{2}}}{(2\pi)^{\frac{nT+n}{2}}} \exp\left[-\frac{\tau}{2}\left(\Delta y'\Delta y + \delta'(I_n \otimes l_T)'(I_n \otimes l_T)\delta - 2\delta'(I_n \otimes l_T)'\Delta y + \vartheta\delta'\delta\right)\right] d\delta d\tau$$

(A1.2)

Now we observe that

$$(I_n \otimes l_T)'(I_n \otimes l_T) = TI_n \qquad \text{(A1.3)}$$

Further we write

$$\hat{\delta} = \frac{1}{\vartheta + T}(I_n \otimes l_T \Delta y$$

Then

$$\delta'(I_n \otimes l_T)'(I_n \otimes l_T)\delta + \vartheta\delta'\delta - 2\delta'(I_n \otimes l_T)\Delta y$$

$$= (\vartheta + T)\left[\delta'\delta - 2\delta'\hat{\delta}\right]$$

$$= (\vartheta + T)(\delta - \hat{\delta})'(\delta - \hat{\delta}) - \frac{1}{\vartheta + T}\Delta y'(I_n \otimes l_T)(I_n \otimes l_T)'\Delta y$$

$$= (\vartheta + T)(\delta - \hat{\delta})'(\delta - \hat{\delta}) - \frac{1}{\vartheta + T}\Delta y'\left(I_n \otimes l_T l_T'\right)\Delta y \qquad \text{(A1.4)}$$

Utilizing (A1.3) and (A1.4) we have

$$p(y|H_0) = \int_0^\infty \int_{R^n} \frac{\tau^{\frac{nT+n}{2}-1} \vartheta^{\frac{n}{2}}}{(2\pi)^{\frac{nT+n}{2}}} \exp\left[-\frac{\tau}{2}\left\{\Delta y'\Delta y + (\vartheta + T)(\delta - \hat{\delta})'(\delta - \hat{\delta})'\right.\right.$$

$$-\frac{1}{(\vartheta+T)}\Delta y'\left(I_n\otimes l_T l_T{}'\right)\Delta y\Big\}\Big]d\delta d\tau$$

$$=\int_0^\infty \frac{\tau^{\frac{nT}{2}-1}\vartheta^{\frac{n}{2}}}{(2\pi)^{\frac{nT}{2}}(\vartheta+T)^{\frac{n}{2}}}\exp\left[-\frac{\tau}{2}\eta\right]\int_{R^n}\frac{(\vartheta+T)^{\frac{n}{2}}}{(2\pi)^{\frac{n}{2}}}\exp\left[-\frac{\tau(\vartheta+T)}{2}(\delta-\hat{\delta})'(\delta-\hat{\delta})\right]d\delta d\tau$$

$$=\int_0^\infty \frac{\tau^{\frac{nT}{2}-1}\vartheta^{\frac{n}{2}}}{(2\pi)^{\frac{Tn}{2}}(\vartheta+T)^{\frac{n}{2}}}\exp\left[-\frac{\tau}{2}\eta\right]d\tau$$

$$=\frac{\vartheta^{\frac{n}{2}}\Gamma(\frac{nT}{2})}{(\vartheta+T)^{\frac{n}{2}}\pi^{\frac{nT}{2}}\eta^{\frac{nT}{2}}} \qquad \text{(A1.5)}$$

Under the alternative hypothesis H_1, the likelihood function is given by

$$p(y|\gamma,\rho,\tau)=\frac{\tau^{\frac{nT}{2}}}{(2\pi)^{\frac{nT}{2}}}\exp\left[-\frac{\tau}{2}\left((y-\rho y_{-1}-Z\gamma)'(y-\rho y_{-1}-Z\gamma)\right)\right] \qquad \text{(A1.6)}$$

Combining the likelihood function with prior distributions we get

$$p(y|H_1)=\int_a^1\int_0^\infty\int_{R^{2n}}\frac{\tau^{\frac{nT}{2}+n-1}|V(\rho)|^{\frac{1}{2}}}{(2\pi)^{\frac{nT}{2}+n}(1-a)}\exp\left[-\frac{\tau}{2}\{y-\rho y_{-1}-Z\gamma)'(y-\rho y_{-1}-Z\gamma)\right.$$

$$\left.+(\gamma-(1-\rho)\phi_0)'V(\rho)(\gamma-(1-\rho)\phi_0)\}\right]d\gamma d\tau d\rho$$

$$=\int_a^1\int_0^\infty\int_{R^{2n}}\frac{\tau^{\frac{nT}{2}+n-1}|V(\rho)|^{\frac{1}{2}}}{(2\pi)^{\frac{nT}{2}+n}(1-a)}\exp\left[-\frac{\tau}{2}\{(y-\rho y_{-1})'(y-\rho y_{-1})+(1-\rho)^2\phi'_0 V(\rho)\phi_0\right.$$

$$\left.\gamma'(Z'Z+V(\rho))\gamma-2\gamma Z'(y-\rho y_{-1})-2(1-\rho)\gamma'V(\rho)\phi_0\}\right]d\gamma d\tau d\rho \qquad \text{(A1.7)}$$

Now we observe that

$$\gamma'(Z'Z+V(\rho))\gamma-2\gamma'Z'(y-\rho y_{-1})-2(1-\rho)\gamma'V(\rho)\phi_0$$

$$=(\gamma-\hat{\gamma})'G(\rho)(\gamma-\hat{\gamma})-\hat{\gamma}'G(\rho)\hat{\gamma} \qquad \text{(A1.8)}$$

Further we have

$$|V(\rho)| = \frac{(1+\rho)^n \vartheta^n}{(1-\rho)^{3n}}, \quad V(\rho)\phi_0 = \frac{(1+\rho)}{(1-\rho)}\phi_0$$

$$(1-\rho)^2 \phi_0' V(\rho)\phi_0 = (1-\rho^2)\phi_0'\phi_0 = (1-\rho^2) y_0 y_0 \tag{A1.9}$$

Utilizing (A1.7), (A1.8) and (A1.9), we obtain

$$p(y|H_1 = \int_a^1 \int_0^\infty \frac{\tau^{\frac{nT}{2}-1}(1+\rho)^{\frac{n}{2}} \vartheta^{\frac{n}{2}}}{(2\pi)^{\frac{nT}{2}}(1-a)(1-\rho)^{\frac{3n}{2}} |G(\rho)|^{\frac{1}{2}}} \exp\left[-\frac{\tau}{2}\eta(\rho)\right] \times$$

$$\int_{R^{2n}} \frac{\tau^n |G(\rho)|^{\frac{1}{2}}}{(2\pi)^n} \exp\left[-\frac{\tau}{2}\{(\gamma-\hat{\gamma})' G(\rho)(\gamma-\hat{\gamma})\}\right] d\gamma d\tau d\rho$$

$$= \int_a^1 \frac{(1-\rho)^{\frac{n}{2}} \vartheta^{\frac{n}{2}}}{(2\pi)^{\frac{nT}{2}}(1-a)(1-\rho)^{\frac{3n}{2}} |G(\rho)|^{\frac{1}{2}}} \int_0^\infty \tau^{\frac{nT}{2}-1} \exp\left[-\frac{\tau}{2}\eta(\rho)\right] d\tau d\rho$$

$$= \frac{\vartheta^{\frac{n}{2}} \Gamma(\frac{nT}{2})}{(2\pi)^{\frac{nT}{2}}(1-a)} \int_a^1 \frac{(1-\rho)^{\frac{n}{2}}}{(1-\rho)^{\frac{3n}{2}} |G(\rho)|^{\frac{1}{2}} (\eta(\rho))^{\frac{nT}{2}}} d\rho \tag{A1.10}$$

From (A1.5) and (A1.10) the posterior odds ratio for the unit root hypothesis is given by

$$\beta_{01} = \frac{p_0}{1-p_0} \frac{(1-a)}{(T+\vartheta)^{\frac{n}{2}} \eta(\rho)^{\frac{nT}{2}}} \left[\int_a^1 \frac{(1+\rho)^{\frac{n}{2}}}{(1-\rho)^{\frac{3n}{2}} |G(\rho)|^{\frac{1}{2}} \eta(\rho)^{\frac{nT}{2}}} d\rho\right]^{-1} \tag{A1.11}$$

Hence, we follow the theorem.

Proof of the Theorem 2

Under the unit root hypothesis $H_0 : \rho = 1$, the likelihood function for the model with augmentation term

$$p(y|\delta,\theta,\tau) = \frac{\tau^{\frac{nT}{2}}}{(2\pi)^{\frac{nT}{2}}} \exp\left[-\frac{\tau}{2}\left\{(\Delta y - (I_n \otimes l_T)\delta - X\theta)'\right.\right.$$

$$\left.\left.(\Delta y - (I_n \otimes l_T)\delta - X\theta)\right\}\right] \tag{A2.1}$$

Combining the likelihood function with the prior distributions of parameters (14) and (15), we get

$$p(y|H_0)=\int_0^\infty\int_{R^n}\int_{R^k}\frac{\tau^{\frac{nT+n}{2}-1}\vartheta^{\frac{n}{2}}}{(2\pi)^{\frac{nT+n}{2}}}\exp\Big[-\frac{\tau}{2}\Big\{(\Delta y-(I_n\otimes l_T)\delta-X\theta)' \qquad \text{(A2.2)}$$

$$(\Delta y-(I_n\otimes l_T)\delta-X\theta)+\vartheta\delta'\delta\Big\}\Big]d\theta d\delta d\tau$$

Let

$$\tilde{\theta}=(X'X)^{-1}X'(\Delta y-(I_n\otimes l_T)\delta) \qquad \text{(A2.3)}$$

Then, we can write (A2.2) after integrating with respect to θ as

$$p(y/H_0)=\int_0^\infty\int_{R^n}\frac{\tau^{\frac{nT+n-k}{2}-1}\vartheta^{\frac{n}{2}}}{(2\pi)^{\frac{nT+n-k}{2}}|X'X|^{\frac{1}{2}}}\exp\Big[-\frac{\tau}{2}\Big\{\Delta y'\Sigma\Delta y+$$

$$\delta'(I_n\otimes l_T)'\Sigma(I_n\otimes l_T)\delta$$

$$-2\delta'(I_n\otimes l_T)'\Sigma\Delta y+\vartheta\delta'\delta\Big\}\Big]d\delta d\tau \qquad \text{(A2.4)}$$

Here, we observe that

$$\delta'(I_n\otimes l_T)'\Sigma(I_n\otimes l_T)\delta+\vartheta\delta'\delta-2\delta'(I_n\otimes l_T)'\Sigma\Delta y$$

$$=(\delta-\tilde{\delta})'A(\delta-\tilde{\delta})-\Delta y'\Sigma(I_n\otimes l_T)A^{-1}(I_n\otimes l_T)'\Sigma\Delta y$$

Where;

$$\tilde{\delta}=A^{-1}(I_n\otimes l_T)'\Sigma\Delta y$$

Then

$$p(y|H_0)=\int_0^\infty\frac{\tau^{\frac{nT-k}{2}-1}\vartheta^{\frac{n}{2}}}{(2\pi)^{\frac{nT-k}{2}}|A|^{\frac{1}{2}}|X'X|^{\frac{1}{2}}}\exp\Big[-\frac{\tau}{2}\Big\{\Delta y'(\Sigma-\Sigma(I_n\otimes l_T)A^{-1}(I_n\otimes l_T)'\Sigma)\Delta y\Big\}\Big]d\tau$$

(A2.5)

$$=\frac{\vartheta^{\frac{n}{2}}\Gamma\left(\frac{nT-k}{2}\right)}{\pi^{\frac{nT-k}{2}}|A|^{\frac{1}{2}}|X'X|^{\frac{1}{2}}[\varsigma]^{\frac{nT-k}{2}}} \qquad \text{(A2.7)}$$

Further, under H_1, the likelihood function is given by

$$p(y|y_0,\gamma,\rho,\theta,\tau)=\frac{\tau^{\frac{nT}{2}}}{(2\pi)^{\frac{nT}{2}}}\exp\left[-\frac{\tau}{2}\{(y-\rho y_{-1}-Z\gamma-X\theta)'(y-\rho y_{-1}-Z\gamma-X\theta)\}\right]$$

(A2.8)

Combining the likelihood function with the prior distribution leads to

$$p(y|H_1)=\int_a^1\int_0^\infty\int_{R^{2n}}\int_{R^k}\frac{\tau^{\frac{nT}{2}+n-1}\vartheta^{\frac{n}{2}}|V(\rho)|^{\frac{1}{2}}}{(2\pi)^{\frac{nT}{2}+n}(1-a)}$$
$$\exp\left[-\frac{\tau}{2}\left\{(y-\rho y_{-1}-Z\gamma-X\theta)'(y-\rho y_{-1}-Z\gamma-X\theta)\right.\right.$$
$$\left.\left.+(\gamma-(1-\rho)\phi_0)'V(\rho)(\gamma-(1-\rho)\phi_0)\right\}\right]d\theta d\gamma d\tau d\rho$$

(A2.9)

Writing $\hat{\theta}=(X'X)^{-1}X'(y-\rho y_{-1}-Z\gamma)$ and integrating (A2.9) with respect to θ,γ and τ we get,

$$P(y/H_1)=\frac{\vartheta^{\frac{n}{2}}\Gamma\left(\frac{nT-k}{2}\right)}{\pi^{\frac{nT-k}{2}}(1-a)|X'X|^{\frac{1}{2}}}\int_a^1\frac{(1+\rho)^{\frac{n}{2}}}{(1-\rho)^{\frac{3n}{2}}|B(\rho)|^{\frac{1}{2}}[\varsigma(\rho)]^{\frac{nT-k}{2}}}d\rho \qquad \text{(A2.10)}$$

Utilizing (A2.7) and (A2.10), we obtain the theorem. ■

The posterior odds ratio derived in the above theorem can be used for testing the unit root hypothesis. For obtaining the operational value of *a*, one may follow the procedure discussed by Schotman and Van Dijk (1991).

Variance Covariance Matrix $(\hat{\Sigma})$ with Augmentation term order 2

Table: A1-1.1- ICICI TIER-I
Variance Covariance Matrix $(\hat{\Sigma})$

0.323	-3.092	-3.025	-2.994	-0.044	-0.041	-0.030	-0.167	-0.127	-0.053	-0.159	-0.295	-0.317
-3.092	33.576	28.950	28.658	0.343	0.390	0.282	1.223	1.219	0.505	1.219	2.820	3.033
-3.025	28.950	32.753	28.033	0.409	0.316	0.276	1.564	-2.679	0.494	1.490	-0.449	2.967
-2.994	28.658	28.033	31.698	0.405	0.378	0.200	1.548	1.180	-0.343	1.475	2.730	2.499
-0.044	0.343	0.409	0.405	0.008	0.006	0.004	0.021	0.017	0.007	0.017	0.040	0.043
-0.041	0.390	0.316	0.378	0.006	0.007	0.004	0.021	-0.046	0.007	0.020	-0.041	0.040
-0.030	0.282	0.276	0.200	0.004	0.004	0.005	0.015	0.012	-0.025	0.015	0.027	-0.014
-0.167	1.223	1.564	1.548	0.021	0.021	0.015	3.211	0.066	0.027	0.205	0.152	0.164
-0.127	1.219	-2.679	1.180	0.017	-0.046	0.012	0.066	69.088	0.021	0.063	-21.198	0.125
-0.053	0.505	0.494	-0.343	0.007	0.007	-0.025	0.027	0.021	36.020	0.026	0.048	-14.774
-0.159	1.219	1.490	1.475	0.017	0.020	0.015	0.205	0.063	0.026	3.277	0.145	0.156
-0.295	2.820	-0.449	2.730	0.040	-0.041	0.027	0.152	-21.198	0.048	0.145	69.737	0.289
-0.317	3.033	2.967	2.499	0.043	0.040	-0.014	0.164	0.125	-14.774	0.156	0.289	36.913

Table: A2-2.1- ICICI-TIER-II

Variance Covariance Matrix $\left(\hat{\Sigma}\right)$

0.190	-1.599	-1.814	-1.781	-0.021	-0.024	-0.018	-0.085	-0.065	-0.021	-0.089	-0.158	-0.184
-1.599	15.113	15.290	15.009	0.140	0.198	0.151	0.654	0.549	0.174	0.660	1.330	1.551
-1.814	15.290	19.231	17.027	0.199	0.194	0.171	0.817	-1.099	0.197	0.850	-0.023	1.759
-1.781	15.009	17.027	18.366	0.195	0.221	0.133	0.802	0.611	-0.173	0.835	1.481	1.497
-0.021	0.140	0.199	0.195	0.003	0.003	0.002	0.006	0.007	0.002	0.006	0.017	0.020
-0.024	0.198	0.194	0.221	0.003	0.004	0.002	0.011	-0.018	0.003	0.011	-0.013	0.023
-0.018	0.151	0.171	0.133	0.002	0.002	0.003	0.008	0.006	-0.011	0.008	0.015	-0.001
-0.085	0.654	0.817	0.802	0.006	0.011	0.008	2.010	0.029	0.009	0.108	0.071	0.083
-0.065	0.549	-1.099	0.611	0.007	-0.018	0.006	0.029	29.014	0.007	0.031	-8.333	0.063
-0.021	0.174	0.197	-0.173	0.002	0.003	-0.011	0.009	0.007	14.752	0.010	0.017	-6.278
-0.089	0.660	0.850	0.835	0.006	0.011	0.008	0.108	0.031	0.010	2.025	0.074	0.086

-0.158	1.330	-0.023	1.481	0.017	-0.013	0.015	0.071	-8.333	0.017	0.074	29.432	0.153
-0.184	1.551	1.759	1.497	0.020	0.023	-0.001	0.083	0.063	-6.278	0.086	0.153	15.190

Table: A1-1.2- KM TIER-I

Variance Covariance Matrix $\left(\hat{\Sigma}\right)$

0.264	-2.519	-2.696	-2.560	-0.034	-0.034	-0.024	-0.119	-0.050	-0.054	-0.109	-0.240	-0.239
-2.519	26.702	25.771	24.466	0.266	0.323	0.233	1.047	0.478	0.520	0.985	2.296	2.282
-2.696	25.771	30.513	26.190	0.345	0.296	0.250	1.220	-1.544	0.556	1.112	0.549	2.442
-2.560	24.466	26.190	27.527	0.327	0.328	0.183	1.158	0.486	-0.008	1.056	2.333	1.845
-0.034	0.266	0.345	0.327	0.006	0.004	0.003	0.010	0.006	0.007	0.007	0.031	0.031
-0.034	0.323	0.296	0.328	0.004	0.006	0.003	0.015	-0.030	0.007	0.014	-0.012	0.031
-0.024	0.233	0.250	0.183	0.003	0.003	0.004	0.011	0.005	-0.016	0.010	0.022	0.001
-0.119	1.047	1.220	1.158	0.010	0.015	0.011	2.491	0.023	0.025	0.206	0.109	0.108
-0.050	0.478	-1.544	0.486	0.006	-0.030	0.005	0.023	41.069	0.010	0.021	-15.283	0.045
-0.054	0.520	0.556	-0.008	0.007	0.007	-0.016	0.025	0.010	21.805	0.022	0.050	-8.863
-0.109	0.985	1.112	1.056	0.007	0.014	0.010	0.206	0.021	0.022	2.562	0.099	0.098
-0.240	2.296	0.549	2.333	0.031	-0.012	0.022	0.109	-15.283	0.050	0.099	41.743	0.218
-0.239	2.282	2.442	1.845	0.031	0.031	0.001	0.108	0.045	-8.863	0.098	0.218	22.012

Table: A2-2.2- KM TIER-II

Variance Covariance Matrix $\left(\hat{\Sigma}\right)$

0.178	-1.508	-1.653	-1.656	-0.020	-0.019	-0.015	-0.081	-0.049	-0.040	-0.073	-0.134	-0.156
-1.508	14.129	13.967	13.996	0.143	0.157	0.129	0.598	0.416	0.338	0.595	1.129	1.316
-1.656	13.996	15.340	16.777	0.189	0.172	0.113	0.751	0.457	-0.153	0.675	1.240	1.426
-0.020	0.143	0.188	0.189	0.003	0.002	0.002	0.007	0.006	0.005	0.004	0.015	0.018
-0.019	0.157	0.146	0.172	0.002	0.003	0.002	0.008	-0.022	0.004	0.008	-0.028	0.016

-0.015	0.129	0.141	0.113	0.002	0.002	0.002	0.007	0.004	-0.005	0.006	0.011	-0.006
-0.081	0.598	0.749	0.751	0.007	0.008	0.007	1.816	0.022	0.018	0.079	0.061	0.071
-0.049	0.416	-0.937	0.457	0.006	-0.022	0.004	0.022	33.270	0.011	0.020	-11.314	0.043
-0.040	0.338	0.370	-0.153	0.005	0.004	-0.005	0.018	0.011	14.658	0.016	0.030	-5.786
-0.073	0.595	0.674	0.675	0.004	0.008	0.006	0.079	0.020	0.016	1.867	0.054	0.063
-0.134	1.129	0.530	1.240	0.015	-0.028	0.011	0.061	-11.314	0.030	0.054	32.781	0.117
-0.156	1.316	1.442	1.426	0.018	0.016	-0.006	0.071	0.043	-5.786	0.063	0.117	14.341

Table: A1-1.3- SBI TIER-I

Variance Covariance Matrix $\left(\hat{\Sigma}\right)$

0.243	-2.067	-2.325	-2.378	-0.026	-0.031	-0.025	-0.117	-0.058	-0.056	-0.104	-0.223	-0.225
-2.067	19.817	19.759	20.211	0.181	0.265	0.211	0.874	0.496	0.474	0.858	1.898	1.911
-2.325	19.759	24.804	22.728	0.252	0.259	0.238	1.117	-1.479	0.533	0.997	0.339	2.149
-2.378	20.211	22.728	25.567	0.258	0.305	0.201	1.143	0.571	-0.077	1.019	2.183	1.700
-0.026	0.181	0.252	0.258	0.004	0.003	0.003	0.009	0.006	0.006	0.005	0.024	0.024
-0.031	0.265	0.259	0.305	0.003	0.005	0.003	0.015	-0.012	0.007	0.013	0.002	0.029
-0.025	0.211	0.238	0.201	0.003	0.003	0.004	0.012	0.006	-0.001	0.011	0.023	0.012
-0.117	0.874	1.117	1.143	0.009	0.015	0.012	2.654	0.028	0.027	0.179	0.107	0.108
-0.058	0.496	-1.479	0.571	0.006	-0.012	0.006	0.028	34.172	0.013	0.025	-12.858	0.054
-0.056	0.474	0.533	-0.077	0.006	0.007	-0.001	0.027	0.013	13.176	0.024	0.051	-5.011
-0.104	0.858	0.997	1.019	0.005	0.013	0.011	0.179	0.025	0.024	2.707	0.096	0.096
-0.223	1.898	0.339	2.183	0.024	0.002	0.023	0.107	-12.858	0.051	0.096	34.651	0.206
-0.225	1.911	2.149	1.700	0.024	0.029	0.012	0.108	0.054	-5.011	0.096	0.206	13.503

Table: A2-2.3- SBI TIER-II

Variance Covariance Matrix $\left(\hat{\Sigma}\right)$

0.205	-1.679	-2.005	-2.040	-0.022	-0.024	-0.020	-0.086	-0.057	-0.046	-0.075	-0.188	-0.188
-1.679	15.489	16.458	16.745	0.144	0.193	0.167	0.673	0.464	0.376	0.648	1.546	1.543
-2.005	16.458	21.631	19.992	0.216	0.197	0.199	0.843	-1.142	0.449	0.740	0.472	1.842
-2.040	16.745	19.992	22.105	0.220	0.234	0.167	0.858	0.563	0.035	0.752	1.878	1.415
-0.022	0.144	0.216	0.220	0.003	0.003	0.002	0.005	0.006	0.005	0.001	0.020	0.020
-0.024	0.193	0.197	0.234	0.003	0.004	0.002	0.010	-0.005	0.005	0.009	0.003	0.022
-0.020	0.167	0.199	0.167	0.002	0.002	0.003	0.009	0.006	-0.002	0.007	0.019	0.012
-0.086	0.673	0.843	0.858	0.005	0.010	0.009	2.207	0.024	0.019	0.143	0.079	0.079
-0.057	0.464	-1.142	0.563	0.006	-0.005	0.006	0.024	27.144	0.013	0.021	-10.079	0.052
-0.046	0.376	0.449	0.035	0.005	0.005	-0.002	0.019	0.013	10.041	0.017	0.042	-3.783
-0.075	0.648	0.740	0.752	0.001	0.009	0.007	0.143	0.021	0.017	2.274	0.069	0.069
-0.188	1.546	0.472	1.878	0.020	0.003	0.019	0.079	-10.079	0.042	0.069	27.273	0.173
-0.188	1.543	1.842	1.415	0.020	0.022	0.012	0.079	0.052	-3.783	0.069	0.173	10.234

Table: A1-1.4- UTI TIER-I

Variance Covariance Matrix $\left(\hat{\Sigma}\right)$

0.265	-2.629	-2.421	-2.448	-0.032	-0.029	-0.024	-0.136	-0.012	-0.035	-0.129	-0.201	-0.232
-2.629	29.365	24.021	24.286	0.258	0.287	0.236	1.057	0.123	0.347	0.974	1.994	2.306
-2.421	24.021	25.584	22.366	0.294	0.210	0.217	1.243	-2.642	0.320	1.176	-0.267	2.124
-2.448	24.286	22.366	25.871	0.297	0.267	0.159	1.256	0.115	-0.317	1.189	1.857	1.756
-0.032	0.258	0.294	0.297	0.005	0.004	0.003	0.015	0.002	0.004	0.014	0.024	0.028
-0.029	0.287	0.210	0.267	0.004	0.005	0.003	0.015	-0.062	0.004	0.014	-0.063	0.025
-0.024	0.236	0.217	0.159	0.003	0.003	0.004	0.012	0.001	-0.016	0.012	0.018	-0.006
-0.136	1.057	1.243	1.256	0.015	0.015	0.012	2.948	0.006	0.018	0.171	0.103	0.119
-0.012	0.123	-2.642	0.115	0.002	-0.062	0.001	0.006	78.570	0.002	0.006	-29.444	0.011
-0.035	0.347	0.320	-0.317	0.004	0.004	-0.016	0.018	0.002	26.309	0.017	0.027	-10.567
-0.129	0.974	1.176	1.189	0.014	0.014	0.012	0.171	0.006	0.017	2.950	0.098	0.113

-0.201	1.994	-0.267	1.857	0.024	-0.063	0.018	0.103	-29.444	0.027	0.098	80.488	0.176
-0.232	2.306	2.124	1.756	0.028	0.025	-0.006	0.119	0.011	-10.567	0.113	0.176	26.838

Table: A2-2.4- UTI TIER-II

Variance Covariance Matrix $\left(\hat{\Sigma}\right)$

0.184	-1.534	-1.713	-1.813	-0.020	-0.020	-0.018	-0.078	-0.018	-0.036	-0.072	-0.138	-0.164
-1.534	14.393	14.249	15.078	0.131	0.168	0.148	0.603	0.146	0.300	0.586	1.148	1.366
-1.713	14.249	17.676	16.835	0.185	0.157	0.165	0.728	-1.202	0.335	0.668	0.260	1.525
-1.813	15.078	16.835	19.512	0.195	0.199	0.142	0.770	0.173	-0.172	0.707	1.356	1.222
-0.020	0.131	0.185	0.195	0.003	0.002	0.002	0.005	0.002	0.004	0.003	0.015	0.018
-0.020	0.168	0.157	0.199	0.002	0.003	0.002	0.009	-0.021	0.004	0.008	-0.020	0.018
-0.018	0.148	0.165	0.142	0.002	0.002	0.003	0.008	0.002	0.000	0.007	0.013	0.008
-0.078	0.603	0.728	0.770	0.005	0.009	0.008	2.091	0.007	0.015	0.081	0.059	0.070
-0.018	0.146	-1.202	0.173	0.002	-0.021	0.002	0.007	33.086	0.003	0.007	-12.418	0.016
-0.036	0.300	0.335	-0.172	0.004	0.004	0.000	0.015	0.003	10.462	0.014	0.027	-4.065
-0.072	0.586	0.668	0.707	0.003	0.008	0.007	0.081	0.007	0.014	2.102	0.054	0.064
-0.138	1.148	0.260	1.356	0.015	-0.020	0.013	0.059	-12.418	0.027	0.054	33.832	0.123
-0.164	1.366	1.525	1.222	0.018	0.018	0.008	0.070	0.016	-4.065	0.064	0.123	10.702