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Abstract

This note points out a hitherto unrecognized identification issue in a class of rational expectations (RE) models with news shocks. We show that different degrees of anticipation (information flows) have strikingly different implications for the identifiability of the underlying structural model, irrespective of its non-fundamental time-series representation. In particular, under full shock anticipation equilibrium reduced forms behave as noisy perfect foresight state motions, which are non-identifiable. As a consequence, the underlying news shocks model fails to be (first-order) identified. The identification failure is illustrated with a New Keynesian model that can be solved analytically.

Keywords: Rational expectations; Perfect foresight; News shocks; Identification

JEL Classification: C1; E32

1 Introduction

A recent strand of literature has emphasized the role of expectations-driven fluctuations in macroeconomic aggregates in the presence of news shocks, i.e. an information structure under which forthcoming developments in the economy are (possibly imperfectly) anticipated (e.g. [3]; [5]). In this view, anticipation is not merely randomness unrelated to fundamentals, as it conveys a piece of (possibly noisy) information about the future, which is taken into account by forward-looking agents when optimally designing their intertemporal behavior.

The quantitative importance of news shocks in DSGE models has been the focus of a fairly large empirical literature (among others, [4]; [19]; [12]; [2]; [11]). Notably, several studies have assessed the reliability of the sVAR approach for the identification of news shocks models (e.g. [13]; [9]; [10]).

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This note contributes to the econometric analysis of news shocks models by investigating the identifiability implications of shock anticipation in a class of linear rational expectations (LRE) models, namely those displaying lagged expectations. The presence of lagged expectations typically arises under several microfoundations, like the presence of staggered-price setting under past information (e.g.[26]), information stickiness (e.g. [15]) or imperfect information in monetary policymaking (e.g. [16]). In this regard, we show that the degree to which information about future economic states is anticipated by rational agents has crucial implications for the identification of the underlying structural model, irrespective of its non-fundamental time-series representation. More specifically, we establish that, under full anticipation, equilibrium reduced forms of LRE models under lagged expectations follow (noisy) perfect foresight dynamics. Since the latter are non-identifiable, the corresponding news shocks model fails to be (first-order) identified.

We believe that our analysis of model identifiability under news shocks is important along two relevant dimensions. First, it points to the need for a more thoughtful understanding of the relationship between the modelling of information flows in forward-looking models and the econometric evaluation of the latter, as identification crucially hinges on the effective degree of shock anticipation. Second, the identification failure arises independently of the misalignment between the agents' and the econometrician's information sets, which is typically advocated as a source of severe consequences for the econometric analysis of news shocks models (e.g. [13])[1].

The note is organized as follows. Section 2 presents the definition of identifiability employed for the subsequent analysis. To provide clear insights into the identification failure explored in this note, section 3 introduces stylized model economies. The analysis is then applied to a New Keynesian monetary business cycle model with lagged expectations, that can be solved analytically (3.2). Section 4 concludes.

[1]See also [21] for a different analysis of the identification problem in news shocks frameworks.

2 Identification of statistical models: preliminaries

The identification problem of rational expectations models has a long history in econometrics (e.g. [20]; [23]; [18]; [24]). Following [7], let $(y, \mathcal{P})$ be a statistical model, where y is a (scalar) process and $\mathcal{P}$ the set of (possible) laws defined on y. Then:

Definition 1. *The statistical model $(y, \mathcal{P})$ is semi-parametric if and only if:*

$$\exists\, n \in \mathbb{N}_0 : \; \widetilde{\mathcal{P}} = \{\mathcal{P}_\theta,\, \theta \in \Theta\}\,, \quad \mathcal{P}_\theta \neq \emptyset, \quad \mathcal{P}_\theta \subset \mathcal{P}$$

where $\Theta \subseteq \mathbb{R}^n$ is a parameter space and $\bigcup_{\theta \in \Theta} \mathcal{P}_\theta = \mathcal{P}$.

Definition 2. *The value $\theta_0 \in \Theta$ is identifiable for the semi-parametric model $(y, \mathcal{P})$ if and only if:*

$$\forall\, \theta \in \Theta : \; \mathcal{P}_\theta \cap \mathcal{P}_{\theta_0} \neq \emptyset \Rightarrow \theta = \theta_0$$

The model is identifiable if and only if $\forall\, \theta_0 \in \Theta$, θ is identifiable.

When the cover $\widetilde{\mathcal{P}}$ is defined through the mean of the law of $y|\Omega_{t-1}$, where Ω_t denotes the information included in the model by the econometrician, identifiability is said *first-order identifiability.* This is the concept we will exploit in the following section.

3 Shock anticipation and identification

This section examines the identification of lagged expectations LRE models under news shocks in two stylized economic environments. While the lack of sophistication enables us to simply single out the link between information flows and identification, we demonstrate how the findings and conclusions reached in the following subsection extend to the more complex model studied in 3.2.

3.1 Analytical examples

A general (scalar) LRE model with lagged expectations on both current and future can be expressed as:

$$y_t = \sum_{i=1}^{K} \lambda_i E_{t-i} y_{t+1} + \sum_{i=1}^{K} \gamma_i E_{t-i} y_t + \phi x_t \tag{1}$$

where y_t is an endogenous variable of interest and x_t is an exogenous process (shock). $K \in \mathbb{N}_0$ is the order of lagged expectations. The structural parameters of interest are $(\lambda_i, \gamma_i, \phi) \in \mathbb{R}$.

Rational expectations in (1) are formed on the basis of the information sets I_{t-i}, $i \in \{1, \dots, K\}$, which collect available observations on all the endogenous and exogenous variables up to (and including) time $t-i$.

Let us consider two well-known (scalar) versions of, i.e. the Muth-type model ([17]):

$$y_t = \gamma_1 E_{t-1} y_t + \phi x_t, \quad \gamma_1, \phi \notin \{0, 1\} \tag{2}$$

and the Taylor-type model ([22]):

$$y_t = \gamma_1 E_{t-1} y_t + \lambda_1 E_{t-1} y_{t+1} + \phi x_t, \quad \lambda_1 \neq 0, \gamma_1 \neq 1, \phi \notin \{0, 1\} \tag{3}$$

in which $K = 1$. While widely used in the rational expectations literature[2], the two class of models (2) and (3) are particularly relevant for the purposes of the analysis since, under imperfect predictability of the x_t process, they both have no perfect foresight solution, i.e. particular solutions of the RE model corresponding to the case in which endogenous expectations coincide with the expected (endogenous) variables. By contrast, perfect foresight solutions become admissible in the presence of full shock anticipation.

[2]Direct generalizations of the Muth model have been studied, among others, in [1], [7] and [23]. An example of (3) can be found in [8].

In fact, the reduced forms of (2) and (3) are respectively (for a generic process x_t):

$$y_t = \frac{\gamma_1\phi}{1-\gamma_1}E_{t-1}x_t + \phi x_t \tag{4}$$

and:

$$y_t = \frac{1-\gamma_1}{\lambda_1}y_{t-1} + \phi\eta_t + \frac{\phi\gamma_1}{\lambda_1}\eta_{t-1} - \frac{\phi}{\lambda_1}x_{t-1} + \xi_{t-1} \tag{5}$$

where $\eta_t := x_t - E_{t-1}x_t$ is the revision process for x_t and $\xi_t := E_t y_{t+1} - E_{t-1}y_{t+1}$ is an arbitrary martingale difference sequence with respect to I_t. The following result is then straightforward:

Proposition 1. *Denote with y_t^* the perfect foresight solution, for which $E_{t-1}y_{t+h}^* = y_{t+h}^*$, $\forall t \in \mathbb{Z}$, $h \in \{1,2\}$. Then:*
i) y_t^* is the unique solution to (2) if and only if $\eta_t = 0$ a.s. $\forall t \in \mathbb{Z}$, while model (3) admits y_t^* as a solution only if η_t satisfies the first-order difference equation[3]*:*

$$\eta_t = -\frac{\gamma_1}{\lambda_1}\eta_{t-1}$$

ii) Let $x_t = \tilde{x}_{t-q} + \mu_t$, $q \geq 1$, $\tilde{x}_t \sim i.i.d.(0,1)$, $\mu_t \sim i.i.d.(0,1)$, with $\tilde{x}_t$ and μ_t mutually independent (partial anticipation). Then both models are first-order identifiable;
iii) Let $x_t = \tilde{x}_{t-q}$, $q \geq 1$, $\tilde{x}_t \sim i.i.d.(0,1)$ (full anticipation). Then both models are not (uniformly) first-order identifiable.

Proof. - See the Appendix. □

According to proposition 1, the way information on future realizations of the x_t process enters the economy (information flow) is crucial to the identification of the underlying structural model[4]. In fact, different degrees of anticipation have strikingly different im-

[3] For model (2), perfect foresight also requires that ξ_t be a uniformly zero process, which is an admissible choice.

[4] The information processes sub ii) and sub iii) - which are referred to as *i.i.d. news* in [14] - have appeared, with minor variations, across a large number of studies (e.g. [12], [19]; [25]). However, the result sub i) holds true also for moving average specifications of the news process of the form $x_t = D(L)\tilde{x}_t$, where $D(L) = \sum_{i=0}^{q} L^i$ and L denotes the lag operator (*correlated news*).

plications in terms of identifiability conditions, which ultimately depend on the nature of equilibrium reduced forms. Since full shock anticipation renders the perfect foresight solution an admissible one, the corresponding RE model is not (first-order) identified.

3.2 A New Keynesian model with lagged expectations

We apply our previous findings to a simple version of the prototypical New Keynesian monetary model with lagged expectations (e.g. [26]). Lagged expectations are easily introduced in the basic New Keynesian framework by assuming that aggregate consumption and/or investment decisions entail planning lags and are thus based on past information.

The linearized equilibrium representation of the model is described by the following equations:

$$g_t = E_{t-1}g_{t+1} - \sigma E_{t-1}(i_t - \pi_{t+1}); \quad \sigma > 0 \tag{6}$$

$$\pi_t = \beta E_{t-1}\pi_{t+1} + \kappa E_{t-1}g_t; \quad 0 < \beta < 1, \quad \kappa > 0 \tag{7}$$

$$i_t = \mu E_{t-1}\pi_t + x_t, \quad \mu > 1 \tag{8}$$

where g_t is the output gap, π_t is the inflation rate and i_t is the nominal interest rate[5].

Equations (6)-(7) represent the private sector block of the model. The first equation is the intertemporal Euler equation that arises from the representative agent's consumption choices with planning lags. The second equation is the aggregate supply function, as derived from firms' optimal price-setting problem under past information ([26]). The monetary authority affects the economy's equilibrium via the interest rule (8). In reality, central bankers are faced with considerable lack of information regarding current values of several macroeconomic variables (e.g.[16]); we embed this informational source of uncertainty by assuming that the central banker is unaware of the current level of inflation and hence employes the best forecast available, on the basis of past information.

The economy is perturbed by a fundamental shock to monetary policy, x_t. For

[5] All variables are expressed as log-deviations from a unique non-stochastic steady state

the purpose of the analysis, we consider three different scenarios: (i) Full anticipation: $E_{t-1}x_t = x_t$; (ii) Partial anticipation: $E_{t-1}x_t = x_t - \mu_t$, $\mu_t \sim i.i.d.(0,1)$; and (iii) independent white noise shock: $E_{t-1}x_t = 0$. The system (6)-(8) can then be written as:

$$y_t = \Gamma E_{t-1} y_t + \Lambda E_{t-1} y_{t+1} + \Phi x_t \tag{9}$$

where:

$$y_t' = (g_t \ \phi_t); \quad \Gamma = \begin{pmatrix} 0 & -\sigma\mu \\ \kappa & 0 \end{pmatrix}$$

$$\Lambda = \begin{pmatrix} 1 & \sigma \\ 0 & \beta \end{pmatrix}; \quad \Phi = \begin{pmatrix} -\sigma \\ 0 \end{pmatrix}$$

The reduced form of the model (9) is:

$$\left(I - \Lambda^{-1}(I-\Gamma)L\right) y_t = \left(\Phi + \Lambda^{-1}\Gamma\Phi L\right)\eta_t - \Lambda^{-1}\Phi x_{t-1} + \Xi_{t-1} \tag{10}$$

where L denotes the lag operator and the 2×1 vector $\Xi_t := E_t y_{t+1} - E_{t-1} y_{t+1}$ collects a pair of arbitrary martingale differences[6].

The following proposition characterizes the identifiability of the representation (9) under different degrees of shock anticipation:

Proposition 2. *Consider the New Keynesian model (9). Then:*
i) Under full anticipation or white noise shock, the model is not (first-order) identifiable;
ii) Under partial anticipation, the model is (first-order) identifiable.

Proof. - Follows along the same lines as the proof of proposition 1. □

[6]The multiplicity of arbitrary martingale differences follows from being the matrices $I - \Gamma$ and Λ both non-singular (see the Appendix).

Proposition 2 remarks that the transition from a standard LRE framework without anticipation to the corresponding one featuring news shocks dramatically alters the identifiability requirements for the underlying model. This feature clearly relates to the early work of [18], who emphasized how the identifiability properties of LRE models strongly rely upon the (assumptions behind the) process generating the forcing variable x_t. If the latter is only known to be not fully predictable in advance - i.e. $\eta_t \neq 0$ and $E_{t-1}x_t \neq 0$, $\forall t$ -, then first-order identification requires the absence of exact multicollinearities between $x_t, x_{t-1}, E_{t-1}x_t, E_{t-2}x_{t-1}$ (e.g. [18]); this is clearly the case under partial anticipation. Remarkably, under exact predictability of the exogenous process x_t (i.e. $\eta_t = 0$ $\forall t$), the identification of the structural parameters of the New Keynesian model is not achievable because of the dynamic behavior of the equilibrium reduced form (10), which follows a (noisy) perfect foresight state motion.

4 Conclusion

This note has explored the identification of a class of LRE models under alternative degrees of shock anticipation. It has been shown that model identifiability hinges critically on assumptions about the degree of shock anticipation (information flows). Given the close link between identifiability conditions and the consistency properties of estimators, exploring the practical relevance of this issue for the empirical evaluation of news shocks models may be a fruitful avenue for future research.

Appendix

Proof of proposition 1 - Part sub i) is trivial. To demonstrate points sub ii) and sub iii), let us consider given values $(\gamma_1^*, \lambda_1^*, \phi^*)$ for the structural parameters. First-order identifiability is defined in terms of the mean of the law $y_t|\Omega_{t-1}$, where $\Omega_{t-1} = \{y_s, x_s : s \leq t-1\} \cup \{x_t\}$[7]. Then:

ii) *Partial anticipation.* Under partial anticipation, the reduced form (4) boils down to:

$$y_t = \frac{\phi}{1-\gamma_1}\tilde{x}_{t-q} + \phi\mu_t$$

By definition 2, the structural parameters are first-order identifiable if and only if:

$$E(y_t|\Omega_{t-1}) = E^*(y_t|\Omega_{t-1}) \quad \Rightarrow \quad \begin{cases} \gamma_1 = \gamma_1^* \\ \phi = \phi^* \end{cases}$$

where the conditional expectations E^* is computed with respect to the pair (γ_1^*, ϕ^*). This is equivalent to:

$$\begin{cases} \frac{\phi}{1-\gamma_1} = \frac{\phi^*}{1-\gamma_1^*} \\ \phi = \phi^* \end{cases} \quad \Rightarrow \quad \begin{cases} \gamma_1 = \gamma_1^* \\ \phi = \phi^* \end{cases}$$

which is fulfilled for any choice of (γ_1^*, ϕ^*). For the reduced form (5), first-order identifiability requires (since $\eta_t = \mu_t$):

$$\begin{cases} \phi = \phi^* \\ \frac{1-\gamma_1}{\lambda_1} = \frac{1-\gamma_1^*}{\lambda_1^*} \\ \phi\frac{\gamma_1-1}{\lambda_1} = \phi^*\frac{\gamma_1^*-1}{\lambda_1^*} \\ \frac{\phi}{\lambda_1} = \frac{\phi^*}{\lambda_1^*} \end{cases} \quad \Rightarrow \quad \begin{cases} \gamma_1 = \gamma_1^* \\ \phi = \phi^* \end{cases}$$

which is fulfilled for any $(\gamma_1^*, \lambda_1^*, \phi^*)$.

iii) *Full anticipation.* By virtue of proposition 1, under full anticipation the only solution

[7] In other words, for the purposes of first-order identification, the information set exploited by the econometrician is assumed to contain all past variables and also the current exogenous one (e.g. [7]).

to (2) is the perfect foresight one, whereas the equilibrium reduced form of (3) is a linear noisy version of the latter, with the noise being an adapted process (i.e. $\xi_t = E_t\xi_t$.). Hence, it is sufficient to show that, given (2) and (3), the corresponding perfect foresight models are not first-order identifiable. For (2) it obtains:

$$y_t = \frac{\phi}{1-\gamma_1}x_t \tag{11}$$

whose identifiability condition is:

$$\frac{\phi}{1-\gamma_1} = \frac{\phi^*}{1-\gamma_1^*} \quad \Rightarrow \quad \begin{cases} \gamma_1 = \gamma_1^* \\ \phi = \phi^* \end{cases}$$

which is not fulfilled without additional restrictions. In the same vein, the perfect foresight version of (3) yields:

$$y_t = \frac{1-\gamma_1}{\lambda_1}y_{t-1} - \frac{\phi}{\lambda_1}x_{t-1} \tag{12}$$

and first-order identification of the model requires for this particular solution:

$$\begin{cases} \frac{1-\gamma_1}{\lambda_1} = \frac{1-\gamma_1^*}{\lambda_1^*} \\ \frac{\phi}{\lambda_1} = \frac{\phi^*}{\lambda_1^*} \end{cases} \quad \Rightarrow \quad \begin{cases} \gamma_1 = \gamma_1^* \\ \phi = \phi^* \end{cases}$$

Since this condition is not fulfilled, the structural form of the RE model (3) under news shocks fails to be (uniformly) identifiable.

Reduced form of the New Keynesian model - According to [7], the reduced form (10) only involves $n-m$ arbitrary martingale differences, where n is the number of equations and m is the number of zero roots α of the characteristic equation:

$$det\,(\Lambda + \alpha(I-\Gamma)) = 0 \tag{13}$$

Being $(I - \Gamma)$ invertible, the previous equation is equivalent to:

$$det\,(\Omega - \alpha I) = 0$$

where $\Omega := -(I - \Gamma)^{-1}\Lambda$. Hence, the characteristic equation (13) admits no zero root (i.e. $m = 0$) since $det(\Omega) \neq 0$.

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