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Quantity Competition vs. Price Competition Under Optimal Subsidy in a Mixed Duopoly

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Abstract. This paper reconsiders the literature on the irrelevance of privatization in mixed markets, addressing both quantity and price competition in a duopoly with differentiated products. By allowing for partially privatizing a state-controlled firm, we explore competition under different timings of firms' moves and derive the conditions under which an optimal subsidy allows to achieve maximum efficiency. We show that, while the ownership of the controlled firm is irrelevant when firms play simultaneously, it matters when firms compete sequentially, requiring the leader to be publicly-owned for an optimal subsidy to restore the first-best allocation, irrespective of the mode of competition. The paper also focuses on the extent to which a subsidy is needed to attain the social optimum, highlighting the equivalence between a price (quantity) game with public leadership or simultaneous moves and a quantity (price) game with private leadership.

JEL codes: H21, H44, L13

Keywords: Cournot, Bertrand, privatization, optimal subsidy

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1 Introduction

This paper contributes to the growing literature which advocates the use of subsidies in mixed markets. A number of papers discuss the effectiveness of production subsidies, which are chosen by a government on a welfare-maximizing basis, in restoring the first-best allocation, pointing out the absence of consequences from privatization when governments undertake such subsidization policies. The irrelevance of privatization was first highlighted by White (1996) who addressed simultaneous competition among one public and a number of private firms, proving that the optimal subsidy and the market variables, which yield maximum social welfare at equilibrium, are identical before and after privatization.[1] Poyago-Theotoky (2001), in a framework with sequential competition in quantities, extended by Myles (2002) to general demand and cost specifications, states that the irrelevance result holds even when the public firm moves as the leader in the competition game. The analysis, however, relies on the assumption that firms compete sequentially in the mixed market and simultaneously in the privatized market, so that it does not prove the irrelevance of privatization, since it violates the *ceteris paribus* assumption on the order of firms' moves needed for a correct comparison between ante-privatization and post-privatization markets. Indeed, as shown by Fjell and Heywood (2004) who model competition under the same demand and cost assumptions as in Poyago-Theotoky (2001), when the public firm keeps the leadership after privatization, the irrelevance result does not hold anymore. An explanation for this result is that, while an optimal subsidy succeeds in implementing the first best in a mixed market irrespective of whether the public firm plays simultaneously against the private rivals or acts as a leader, it fails to do so in a private market *à la* Stackelberg.

In the light of the above arguments, the Poyago-Theotoky theorem should be interpreted as a result establishing an equivalence between the outcomes of a simultaneous game and a sequential game with public leadership, rather than a result of irrelevance of privatization. Indeed, what is demonstrated in that work is that the amount of subsidy needed to recover the social optimum when a public firm operates in the market is the same regardless of whether the public firm plays simultaneously with the private rivals or assumes the role of the leader in the competition game. However, when competition is sequential, a subsidy is shown to work effectively yielding the first best, provided that the leader is a public firm, as underlined by Fjell and Heywood (2004). In this sense, previous literature reveals that privatization is *not* irrelevant when firms' actions are sequential, since public ownership of the leader is required for an optimal subsidy to restore efficiency.

This paper starts from this point and investigates duopolistic competition under optimal subsidies in different scenarios in which firms play simultaneously or sequentially, with respect to quantities or prices. The analysis is carried out with the aim of identifying the key features related to the different timing or the different mode of competition, which lead to a result of irrelevance of the own-

[1] The same result has been obtained by Hashimzade et al. (2007) in a setting with price competition and a setting with differentiated products.

ership of a state-controlled firm, or a result of equivalence of market outcomes across games. For this purpose, we assume that private leadership can also characterize the considered sequential settings, thus extending the analyses of previous literature, exclusively confined to games with simultaneous moves and public leadership. Moreover, instead of assuming the existence of a publicly-owned firm competing with a private one in a pure mixed market, and then evaluating the opportunity to privatize that firm as in most of the studies cited above, we assume that one firm is controlled by the government which chooses the firm ownership structure associated to maximum welfare. In other words, the government's choice regards the optimal degree of privatization of its controlled firm which encompasses both the choice to fully privatize a market and the preference for a pure mixed one, covered by our model as extreme cases.[2] The assumption of partial privatization introduced in our framework allows us to model this government's choice at a pre-play stage of the competition game, and to verify the existence of an irrelevance result by keeping constant the order of firms' moves.[3] For any assumed order of moves, our analysis aims to capture the frictions which prevent an optimal subsidy from achieving efficiency objectives and shed light on how those frictions can be overcome by orienting appropriately the ownership of the controlled firm. Moreover, by focusing on the extent to which a subsidy is provided in order to yield the first best, the paper identifies some equivalence results, implying that the optimal subsidies and the market outcomes coincide at equilibrium, which allow to assess the extent to which the toughness and efficiency of competition depend on the timing or the mode of competition.

The results obtained are the following. We start from the analysis of simultaneous moves for each mode of competition, quantity or price, and use these frameworks as benchmark models which prove the irrelevance of both full and partial privatization of the state-controlled firm.[4] This irrelevance is then shown not to exist when firms compete sequentially in quantities or prices: in such contexts, indeed, public ownership of the leader or the follower, respectively in a game in which the controlled firm moves earlier or later, is required for an optimal subsidy to restore efficiency. We also focus on the equivalence between games with public leadership and games with simultaneous moves, extending the results that Poyago-Theotoky (2001) and Myles (2002) obtain in a quantity setting to the case of price competition and to product differentiation. Finally,

[2]While public firms are pure welfare-maximizers, and private firms are pure profit-maximizers, firms with a mixture of public and private ownership are assumed to maximize social welfare, to some extent, and their own profits.

[3]Under partial privatization, we show that a result of irrelevance applies when a given outcome at the market stage, and the associated optimal subsidy, are sustained as a subgame perfect equilibrium regardless of the ownership of the controlled firm. Partial privatization was first addressed by Matsumura (1998) and then extended to a number of competitive settings, including a product differentiation framewok by Fujiwara (2007) and a quantity setting under optimal subsidy by Tomaru (2006). The latter examines competition with simultaneous moves and homogeneous products, demonstrating that the irrelevance result survives the introduction of partial privatization.

[4]See Hashimzade et al. (2007) for a generalization of the irrelevance result under simultaneous moves.

and more importantly, we establish an equivalence between quantity (price) public leadership/simultaneous moves and price (quantity) private leadership.

The paper is organized as follows: Section 2 presents the model, while Section 3 discusses the main results and draws some conclusions.

2 The model

Two technologically identical firms are assumed to compete in quantities or prices, facing a linear demand on a market with differentiated products. One firm is private and is denoted as firm 2, while the ownership structure of the other one, the *ex-ante* public firm denoted as firm 1, is defined following the decision upon its optimal ownership structure by a welfare-maximizing government. As standard in the literature on partial privatization, the government optimally chooses whether to retain full ownership of firm 1, rather than share its ownership with the private sector or fully privatize it. The different alternatives are captured by the parameter α attached to firm 1's profit, with $\alpha \in (0.1)$ ranging from full nationalization ($\alpha = 0$) to full privatization ($\alpha = 1$), and entailing partial privatization in all the intermediate cases. The government selects the optimal degree of privatization for its firm at the first stage of a game which describes simultaneous or sequential competition against the private firm at the last stage(s).[5] A further stage, which captures the subsidy's choice of the government, is considered as an intermediate stage of this game. Consistently with the objective of deriving, for any given order of moves, the firm's optimal ownership structure sustaining a market outcome under optimal subsidy, we assume that the government first decides upon firm 1's degree of ownership and then chooses the optimal subsidy to give both firms, which are assumed to compete in quantities or prices at the market stage.

2.0.1 Quantity competition

We assume the inverse linear demand $p_i = 1 - q_i - \gamma q_j$ ($i = 1, 2$) which derives from a quadratic utility function, where the parameter γ (with $\gamma \in (0, 1)$) captures the degree of product substitutability (goods are independent, weak substitutes or perfect substitutes according to whether $\gamma = 0$, $0 < \gamma < 1$ or $\gamma = 1$). Moreover, we assume that constant marginal costs c and null fixed costs

[5] In this paper we take as given the order of firms' moves. Conversely, in a number of works on mixed markets the endogenous choice of firms' moves is determined by solving an observable delay game. In these works the public firm is found to play simultaneously with the private firm at equilibrium, or to act as a leader or as follower, the results depending on the mode of competition (for quantity competition see the seminal work by Pal (1998) *inter alia*, for price competition see Bàrcena-Ruiz (2007)), on the number of private firms (Pal, 1998), on the presence of foreign firms (Lu, 2006; Matsumura, 2003), the existence of free-entry markets (Ino and Matsushima, 2010) and, finally, the managerial firm structure (Nakamura and Inoue, 2009). Within this literature, see Tomaru and Kiyono (2010) for an analysis under increasing marginal costs and Tomaru and Saito (2010) as the only work examining endogenous timing in a market with subsidized firms.

are sustained by firm 1 and firm 2, and that both firms receive an undifferentiated subsidy on production.[6] In this paragraph, we first address simultaneous competition, then we consider sequential competition with the state-controlled firm in the role of leader (case of public leadership indexed by PL), finally we solve a sequential game with the private firm in the role of leader (case of private leadership indexed by $\Pr L$).

Simultaneous moves in quantities

Given the following profit functions of the two firms:
$\pi_1 (q_1, q_2, s) = (1 - q_1 - \gamma q_2 - c) q_1 + sq_1$
$\pi_2 (q_1, q_2, s) = (1 - q_2 - \gamma q_1 - c) q_2 + sq_2$

and the consumer surplus $CS (q_1, q_2) = \left((1-\gamma) \left(q_1^2 + q_2^2\right) + \gamma (q_1 + q_2)^2 \right) /2$,

we define the social welfare function as the sum of consumers' surplus and the aggregate profits of subsidized firms, net of the social cost of subsidies:[7]

$$W (q_1, q_2) = CS (q_1, q_2) + \sum_{i=1}^{2} \pi_i (q_1, q_2, s) - s (q_1 + q_2) \tag{1}$$

At the last stage of the game, firm 1 maximizes the following weighted average of social welfare and its own profits:
$G_1 (q_1, q_2, s, \alpha) = \alpha W (q_1, q_2) + (1-\alpha) \pi_1 (q_1, q_2, s)$.

The First Order Condition (FOC) $\partial G_1 (q_1, q_2, s, \alpha) / \partial q_1 = 0$ is satisfied at the following firm 1 quantity:

$$q_1^* (q_2, s, \alpha) = \frac{1 - c + s (1-\alpha) - \gamma q_2}{2-\alpha} \tag{2}$$

At the same game stage, firm 2 maximizes its own profits by choosing that quantity which satisfies the condition $\partial \pi_2 (q_1, q_2, s) / \partial q_2 = 0$. As a result, the following reaction function is obtained:

$$q_2^* (q_1, s) = \frac{1 - c + s - \gamma q_1}{2} \tag{3}$$

[6] In contrast to Fjell and Heywood (2004), Poyago-Theotoky (2001) and White (1996) who discuss the irrelevance of privatization under the assumption of quadratic cost function, our analysis relies on the assumption of constant and equal marginal costs between the two firms, which is also nested in the analyses of Myles (2002) and Hashimzade et al. (2007) respectively in a quantity and a price setting with general cost functions. The introduction of product differentiation in a framework with constant marginal costs allows us to easily compare quantity and price competition. Moreover, the focus of the present analysis on markets in which an optimal subsidy succeeds in restoring the first best, makes differences in firms' cost structures less relevant. Indeed, as underlined by White (1996), an effective subsidy equalizes total production between public and private firms, thus causing, under convex costs, a redistribution of firms' costs at equilibrium, with effects similar to the cost identity assumed *a priori* in our model.

[7] Notice that social welfare is not directly affected by the subsidy s which conversely impacts both firms' profits.

The solution of the system of the two reaction functions in (2) and (3) yields the following optimal quantities:

$$q_1^* (s, \alpha) = \frac{(2-\gamma)(1-c) + s(2(1-\alpha)-\gamma)}{(2-\gamma)(2+\gamma) - 2\alpha} \tag{4}$$

$$q_2^* (s, \alpha) = \frac{(2-\gamma-\alpha)(1-c) + s(2-\alpha-\gamma(1-\alpha))}{(2-\gamma)(2+\gamma) - 2\alpha} \tag{5}$$

By substituting (4) and (5) in the social welfare function in (1) and by maximizing it with respect to s, we obtain the optimal subsidy chosen by the government, denoted by s^C in this simultaneous Cournot game:

$$s^C = \frac{1-c}{1+\gamma} \tag{6}$$

A result of neutrality of full and partial privatization of firm 1 is highlighted in the following remark.

Remark 1 In a quantity game with simultaneous moves, the optimal subsidy s^C is independent of α and allows, whatever α, to achieve the highest welfare $W^{SO} = (1-c)^2 / (1+\gamma)$, with SO denoting the social optimum. Firm 1's ownership structure is therefore irrelevant with respect to the objective of implementing the first best, as highlighted in the neutrality theorems of White (1996), Tomaru (2006) and Hashimzade et al. (2007). The first-best allocation entails the optimal quantities $q_i^{SO} = (1-c)/(1+\gamma)$ and the market-clearing prices $p_i^{SO} = c$ $(i = 1, 2)$.

Sequential moves with the state-controlled firm in the role of leader (Quantity Public Leadership)

Under quantity public leadership, firm 1 takes as given the reaction function $q_2^* (q_1, s) = R_2 (q_1, s)$ of the private firm in (3) moving at the last stage of the game. The objective function of the government is therefore expressed as a function of q_1 only and is the following:
$G_1 (q_1, s, \alpha) = \alpha W (q_1, R_2 (q_1, s, \alpha)) + (1-\alpha) \pi_1 (q_1, R_2 (q_1, s, \alpha))$.

We maximize $G_1 (q_1, s, \alpha)$ finding firm 1's optimal quantity, then we substitute it in firm 2's reaction function, thus obtaining the following solutions:

$$q_1^* (s, \alpha) = \frac{(1-c)(2\gamma-4+\alpha\gamma)+s(2\gamma-4+\alpha(4-\gamma))}{4(\gamma^2-2)+\alpha(2-\gamma)(\gamma+2)}$$

$$q_2^* (s, \alpha) = \frac{(1-c)\left(\gamma(2+\gamma)-4+\alpha\left(2-\gamma^2\right)\right)+s(\gamma(2+\gamma)-4+2\alpha(1-\gamma))}{4(\gamma^2-2)+\alpha(2-\gamma)(\gamma+2)}$$

The solution of the FOC $\partial W (s, \alpha) / \partial s = 0$ yields the optimal subsidy $s_{PL}^C (\alpha)$ (see Appendix a) for its expression).

Solving for the optimal degree of privatization at the first stage of the game, we obtain $\alpha^* = 1$. At the Subgame Perfect Nash Equilibrium (SPNE), the optimal subsidy is:

$$s_{PL}^C = \frac{1-c}{1+\gamma} \tag{7}$$

with the market variables coinciding with the efficient outcomes and social welfare achieving its maximum.

Sequential moves with the state-controlled firm in the role of follower (Quantity Private Leadership)

Under quantity private leadership, the private firm takes as given the reaction function $q_1^*(q_2, s, \alpha) = R_1(q_2, s, \alpha)$ of the state-controlled firm in (2) moving at the last stage of the game. By maximizing $\pi_2(R_1(q_2, s, \alpha), q_2)$, and then substituting the solution in firm 1's reaction function, we obtain the following quantities:

$$\begin{aligned} q_1^*(s,\alpha) &= \frac{\left(4-\gamma^2-2\alpha-\gamma(2-\alpha)\right)(1-c)+s\left(4-\gamma^2-2\alpha(3-\alpha)-\gamma(2-\alpha(1+\gamma))\right)}{2(2-\gamma^2-\alpha)(2-\alpha)} \\ q_2^*(s,\alpha) &= \frac{(1-c)(2-(\alpha+\gamma))+s(\alpha\gamma+2-(\alpha+\gamma))}{2(2-\gamma^2-\alpha)} \end{aligned}$$

The first-order condition to the welfare-maximization problem with respect to s yields by the optimal subsidy $s_{\Pr L}^C(\alpha, \gamma)$, the expression of which is in Appendix b).

We substitute $s_{\Pr L}^C(\alpha)$ in the social welfare function and solve its maximization problem with respect to α, thus obtaining $\alpha^* = 1$. At the SPNE, the welfare-maximizing subsidy is:

$$s_{\Pr L}^C = (1-c)(1-\gamma) \tag{8}$$

which allows to achieve the first-best allocation.

By comparing the above sequential settings, we can state the following remark.

Remark 2 In the sequential games with quantity competition, the optimal subsidy depends on α and coincides with $s_{PL}^C = (1-c)/(1+\gamma)$ and $s_{\Pr L}^C = (1-c)(1-\gamma)$, respectively in a PL and a $\Pr L$ game, at the subgame perfect equilibrium $\alpha^* = 1$. At this equilibrium social welfare is maximum, which allows us to state that the optimal subsidy yields the first best, provided that firm 1 is entirely public. Notice the equivalence $s_{PL}^C = s^C$, that is the result highlighted by Poyago-Theotoky (2001) and Myles (2002).

2.0.2 Price competition

We keep the assumptions on demand and costs of the quantity competition case and address price competition in simultaneous and sequential moves as in the previous framework.

Simultaneous moves in prices

Given the direct demand function $q_i = \dfrac{(1-\gamma) - p_i + \gamma p_j}{(1-\gamma^2)}$ $(i = 1.2)$, the profit functions of the two firms are:

$\pi_1(p_1, p_2, s) = (p_1 - c)\left(\frac{(1-\gamma)-p_1+\gamma p_2}{(1-\gamma^2)}\right) + s\left(\frac{(1-\gamma)-p_1+\gamma p_2}{(1-\gamma^2)}\right)$
$\pi_2(p_1, p_2, s) = (p_2 - c)\left(\frac{(1-\gamma)-p_2+\gamma p_1}{(1-\gamma^2)}\right) + s\left(\frac{(1-\gamma)-p_2+\gamma p_1}{(1-\gamma^2)}\right)$
The consumers' surplus is $CS(p_1, p_2) = \frac{p_1^2+p_2^2+2(1-p_1-p_2)-2\gamma(1-p_1)(1-p_2)}{2(1-\gamma^2)}$, so that social welfare is:

$$W(p_1, p_2) = CS(p_1, p_2) + \sum_{i=1}^{2} \pi_i(p_1, p_2, s) - s\left(\frac{2 - p_1 - p_2}{1+\gamma}\right) \tag{9}$$

Given $G_1(p_1, p_2, s, \alpha) = \alpha W(p_1, p_2) + (1-\alpha)\pi_1(p_1, p_2, s)$, at the last stage firm 1 maximizes this function choosing the following price:

$$p_1^*(p_2, s, \alpha) = \frac{(1-\alpha)(1-\gamma) + c(1-\alpha\gamma) - s(1-\alpha) + \gamma p_2}{2-\alpha} \tag{10}$$

At the same stage, the private firm maximizes its own profits and replies to the rival's choice by setting:

$$p_2^*(p_1, s) = \frac{1 + c - \gamma(1-p_1) - s}{2} \tag{11}$$

By solving the system of the reaction functions in (10) and (11), we obtain the following optimal prices:

$$p_1^*(s, \alpha) = \frac{2c(1-\alpha\gamma) + \gamma c + (2(1-\alpha) + \gamma)(1-\gamma-s)}{(2-\gamma)(\gamma+2) - 2\alpha} \tag{12}$$

$$p_2^*(s, \alpha) = \frac{c(2-\alpha) + \gamma c(1-\alpha\gamma) + (2 - \alpha + (1-\alpha)\gamma)(1-\gamma-s)}{(2-\gamma)(\gamma+2) - 2\alpha} \tag{13}$$

The subsidy maximizing the social welfare function $W(s, \alpha)$, which is obtained by substituting (12) and (13) in (9), is denoted by s^B in this simultaneous Bertrand game and is equal to:

$$s^B = (1-c)(1-\gamma) \tag{14}$$

The results of this setting are summarized in the following remark.[8]

Remark 3 The irrelevance of firm 1's ownership also emerges in a price game with simultaneous moves. Indeed, the optimal subsidy s^B is independent of α and succeeds, whatever α, in leading the optimal prices and the equilibrium quantities, and thus social welfare, to the efficient levels p_i^{SO}, q_i^{SO} and W^{SO}.

Sequential moves with the state-controlled firm in the role of leader (Price Public Leadership)

Under price public leadership, firm 1 takes as given the reaction function $p_2^*(p_1, s) = R_2(p_1, s)$ of the private firm in (11). The objective function of the government is:
$G_1(p_1, s, \alpha) = (\alpha W(p_1, R_2(p_1, s)) + (1-\alpha)\pi_1(p_1, R_2(p_1, s)))$.

By maximizing $G_1(p_1, s, \alpha)$ with respect to p_1, and by substituting firm 1's optimal price in the rival's reaction function, we obtain the following solutions:

$$p_1^*(s,\alpha) = \frac{(1-\gamma)(2(2+\gamma)-\alpha(4+\gamma))+c(2(2+\gamma)-\gamma(2\gamma+3\alpha))-s(2(1+\gamma)(2-\gamma)-\alpha(4+\gamma(1-2\gamma)))}{4(2-\gamma^2)-\alpha(2-\gamma)(2+\gamma)}$$
$$p_2^*(s,\alpha) = \frac{(1-\gamma)\left(4-\gamma^2-2\alpha\gamma+2\gamma-2\alpha\right)+c\left(2(2+\gamma)-\gamma^2(1+\gamma)-\alpha\left(2+\gamma^2\right)\right)}{4(2-\gamma^2)-\alpha(2-\gamma)(2+\gamma)} -$$
$$-\frac{s\left(4+\gamma(2+\gamma)(1-\gamma)+\alpha\left(\gamma^3-2(1+\gamma)\right)\right)}{4(2-\gamma^2)-\alpha(2-\gamma)(2+\gamma)}$$

By solving the FOC $\partial W(s,\alpha)/\partial s = 0$, we obtain the optimal subsidy $s_{PL}^B(\alpha)$ (see Appendix c) for its expression).

We obtain $\alpha^* = 1$ as the solution to the welfare-maximization problem. At this equilibrium the optimal subsidy is:

$$s_{PL}^B = (1-c)(1-\gamma) \tag{15}$$

with a first-best allocation achieved at equilibrium.

Sequential moves with the state-controlled firm in the role of follower (Price Private Leadership)

Under price private leadership, the private firm takes as given firm 1's reaction function $p_1^*(p_2, s, \alpha) = R_1(p_2, s, \alpha)$. By maximizing $\pi_2(R_1(p_2, s, \alpha), p_2)$ and substituting the optimal private firm's quantity in firm 1's reaction function, we obtain the following solutions:

[8] Our results extend to partial privatization those obtained by Hashimzade et al. (2007) in a price setting with a pure welfare-maximizing firm.

$$p_1^*(s,\alpha) = \frac{(1-\gamma)\left(4-\gamma^2+2\gamma\right)+\alpha\gamma(5-2\alpha+\gamma(2-\gamma))-2\alpha(3-\alpha)+c(4+\gamma(2+\gamma)(1-\gamma))}{2(2-\alpha-\gamma^2)(2-\alpha)} -$$
$$-\frac{\alpha c\left(2+\gamma\left(5-2\alpha-\gamma^2\right)\right)-s(6\alpha-\alpha(2\alpha-\gamma(1-\gamma))-4-\gamma(2+\gamma)(1-\gamma))}{2(2-\alpha-\gamma^2)(2-\alpha)}$$
$$p_2^*(s,\alpha) = \frac{(1-\gamma)(2+\gamma-\alpha(1+\gamma))-\alpha c\left(1+\gamma^2\right)+c(1+\gamma)(2-\gamma)+s(1+\gamma)(\alpha+\gamma-2)}{2(2-\alpha-\gamma^2)}$$

The optimal subsidy, denoted by $s^B_{\Pr L}(\alpha)$ in such a framework, is the one which satisfies the condition $\partial W(s,\alpha)/\partial s = 0$ (see Appendix d) for its expression).

Finally, the search for the optimal α reveals that $\alpha^* = 1$. At the SPNE, the following welfare-maximizing subsidy restores the first best:

$$s^B_{\Pr L} = \frac{1-c}{1+\gamma} \tag{16}$$

By comparing the above sequential settings, we formulate the following remark.

Remark 4 In the sequential games with price competition, the optimal subsidy depends on α and coincides with $s^B_{PL} = (1-c)(1-\gamma)$ and $s^B_{\Pr L} = (1-c)/(1+\gamma)$, respectively in a PL and a $\Pr L$ game, at the subgame perfect equilibrium $\alpha^* = 1$. At this equilibrium social welfare is maximum, which reveals that the optimal subsidy yields the first best as log as firm 1 is entirely public. Notice the equivalence $s^B_{PL} = s^B$ which holds under price competition.

3 The results

In this section we discuss the results presented in the previous section. By endogenizing the optimal ownership structure of the state-controlled firm, we have identified the conditions under which a welfare-maximizing subsidy succeeds in maximizing allocative efficiency, for any assumed order of moves. These conditions are established in the following proposition.

Proposition 1 *When firms compete simultaneously, a welfare-maximizing subsidy is always effective and yields the first-best allocation, irrespective of firm* 1*'s ownership structure (for any* α*) and the mode of competition. In sequential games, by contrast, the optimal subsidy requires firm* 1 *to be entirely owned by the public sector (*$\alpha^* = 1$*), namely to maximize pure welfare, in order to succeed in implementing the first best.*

Proof: It follows from Remarks 1-4.

In the sequel we discuss the results of the above proposition. Indeed, in a quantity simultaneous game, maximum efficiency is achieved when the following conditions are met:

$$R_1\left(q_2^{SO}, s, \alpha\right) = q_1^{SO} \tag{17}$$
$$R_2\left(q_1^{SO}, s\right) = q_2^{SO} \tag{18}$$

which amount to requiring that the reaction functions of both firms at the product market stage cross the efficient point SO, as shown in Figure 1a.[9] In other words, the above conditions require that each firm reacts by producing the efficient quantity to the independent rival's decision to produce the same quantity. Condition (17) regarding firm 1 is satisfied when $\alpha = 1$ or, alternatively, when $s = s^C$ - that is, efficiency by firm 1 is attained when this firm is entirely public or, for any degree of privatization $\alpha \in [0, 1[$, when s^C is provided to that firm. By contrast, condition (18) on firm 2's efficiency can be met only through the optimal subsidy s^C. The above considerations imply the irrelevance of firm 1's ownership with respect to the objective of achieving allocative efficiency as long as a subsidy s^C is provided to both firms. Indeed, when firm 1 is privatized ($\alpha = 0$), the two competing firms have the same profit-maximizing objective and are both oriented towards efficiency by the subsidy s^C, the latter acting as a cost reduction and causing a parallel-out shift of firm 2's reaction function until it crosses SO. The same subsidy s^C induces firm 1's efficiency even when it is semi-public ($0 < \alpha < 1$), that is interested to some extent in social welfare besides profits. In this case, an increasing concern of firm 1 for social welfare induces on the one hand that firm to expand its output, on the other hand it makes the firm less sensitive to the subsidy, namely less willing to translate the subsidy into an output expansion.[10] The latter two effects exactly compensate each other, making irrelevant the differences in firm objectives (or in ownership structure) at equilibrium, and requiring to restore efficiency the same subsidy s^C as the one needed for the private firm.[11] Last, the subsidy turns out to be irrelevant with respect to firm 1's optimal behavior when it is fully public ($\alpha = 1$), case in which condition (17) is met whatever subsidy applies, and s^C is functional to induce firm 2's efficiency only.

The same argument explains the irrelevance of firm 1's ownership when an optimal subsidy is provided in the price simultaneous game. In this case the conditions ensuring the achievement of the social optimum are:

$$R_1\left(p_2^{SO}, s, \alpha\right) = p_1^{SO} \tag{19}$$

$$R_2\left(p_1^{SO}, s\right) = p_2^{SO} \tag{20}$$

which require that the reaction functions of both firms at the product market stage cross the efficient point SO (see Figure 1b[12]). In other words, the above conditions require that each firm react by setting the efficient price to the

[9] In Figure 1a, firm 1's reaction function R_1 is depicted for $\alpha = 1$, case in which it is independent of s, while firm 2's reaction function R_2 is represented both as a function of a generic subsidy s and at the optimal subsidy.

[10] The sensitiveness of the equilibrium output towards subsidy, measured by $\frac{\partial q_1^*}{\partial s} = (1-\alpha)/(2-\alpha)$, decreases as α increases: $\frac{\partial}{\partial \alpha}\left(\frac{\partial q_1^*}{\partial s}\right) = -1/(2-\alpha)^2 < 0$).

[11] While a subsidy per unit of output shifts a reaction function in such a quantity setting outwards, a change of α causes it to rotate around a point which coincides with the efficient one SO when the subsidy is provided in the optimal amount s^C.

[12] In Figure 1b, firm 1's reaction functions R_1 is depicted for $\alpha = 1$, thus being independent of s, while firm 2's reaction function R_2 is represented both as a function of a generic subsidy and at the optimal subsidy.

independent rival's decision to set the same price. Conditions (19) and (20) are met by providing to both firms the same subsidy s^B which causes both firms's efficiency when $\alpha \in [0, 1[$ and is the one needed to regulate the private firm's when $\alpha = 1$.

While in simultaneous games a subsidy provided indiscriminately to the two firms succeeds in inducing efficient behavior by both of them, despite a potential heterogeneity of objectives, in sequential games a unique subsidy fails to do so. Indeed, in a game with public (private) leadership, the achievement of maximum efficiency depends on the possibility to let firm 1 (firm 2) choose, at the first stage of the game, the efficient quantity (price) on the reaction function of the private (state-controlled) firm, and the latter to reply efficiently at the second stage. Due to the sequentiality of moves, a subsidy per unit of output has a different impact at the margin on the behavior of the two firms so that, in contrast to the simultaneous case, the same subsidy cannot correct the inefficiencies caused by both firms. In other words, the subsidy which satisfies conditions (18) and (20), thus ensuring the private firm's efficiency in the PL games, does not satisfy conditions (17) and (19) regarding firm 1 since the behavior of the latter is also affected by the way the subsidy impacts the rival's decision at the last stage. Likewise, the subsidy which would satisfy conditions (17) and (19) for firm 1's efficiency in the $\Pr L$ games, would not satisfy (18) and (20) regarding the private firm, the behavior of which would also be affected by the optimal reaction of firm 1 at the following stage. In such circumstances it turns out to be optimal to align the two firms's objectives on welfare maximization by weighing the corrective subsidy according to firm 2's incentives only, and inducing pure welfare maximization by firm 1 setting $\alpha^* = 1$.[13] Firm 2's efficiency is achieved in the Cournot and the Bertrand games with public leadership respectively through the subsidies s^C_{PL} and s^B_{PL}, which comply conditions (18) and (20) and coincide respectively with s^C and s^B, while firm 1's efficiency is achieved, and (19) and (20) satisfied, by imposing $\alpha^* = 1$. The latter condition guarantees firm 1's efficiency in each game with private leadership in which, moreover, the provision of the subsidies $s^C_{\Pr L}$ and $s^B_{\Pr L}$, respectively in the Cournot and in the Bertrand setting, lets conditions (17) and (19) regarding firm 2's efficiency to be met.

The above discussion introduces the following proposition.

Proposition 2 *Under both quantity and price competition, the optimal subsidy under simultaneous moves coincides with the optimal subsidy under public leadership. Formally:* $s^C_{PL} = s^C = (1-c)\,/\,(1+\gamma)$ *and* $s^B_{PL} = s^B = (1-c)\,(1-\gamma)$.

Proof: It descends from Remark 2 and Remark 4

[13] Tackling this question, our analysis reveals how the impossibility to restore the social optimum in the presence of sequential moves and private firms does not reflect the ineffectiveness of a subsidy to remedy low production or cost inefficiencies, as underlined by Fjell and Heywood (2004, pg. 415), but rather on the impossibility through an undifferentiated subsidy to align firms' conduct on the efficient outcome.

This equivalence result, obtained by Poyago-Theotoky (2001) and Myles (2002) in a quantity setting, is extended to a price setting in this paper. It derives from the fact that the optimal subsidy in a game with public leadership is determined according to the private firm's incentives, so that it coincides with the subsidy driving the private firm towards efficiency in markets with simultaneous moves, in both cases affecting the optimal reply of a private simultaneous player to any given rival's choice. Figures 1a and 2a depict the mechanism at work. Let us denote firm 2's reaction function calculated at the generic subsidy s by $R_2(\cdot, s)$, where 'dot' stands for q_1 or p_1 according to quantity or price competition. The equilibria under simultaneous moves in the quantity and the price game are identified respectively by points C and B on the $R_2(\cdot, s)$ function, while the equilibria under public leadership are identified by points PL on the same curve: these points converge at point SO when an optimal subsidy applies by shifting $R_2(\cdot, s)$ upwards and downwards, respectively in a quantity and a price game, until they coincide with $R_2(\cdot, s^*)$, with $s^* = s^C = s^C_{PL}$ and $s^* = s^B = s^B_{PL}$ in the two games. The measure of this shift, namely the higher production or the lower price induced by the optimal subsidy, is clearly independent of the *ex-ante* firm 1's choice, which differs depending on whether this firm acts as a simultaneous player or the leader in the market, without affecting the private firm's optimal behavior.

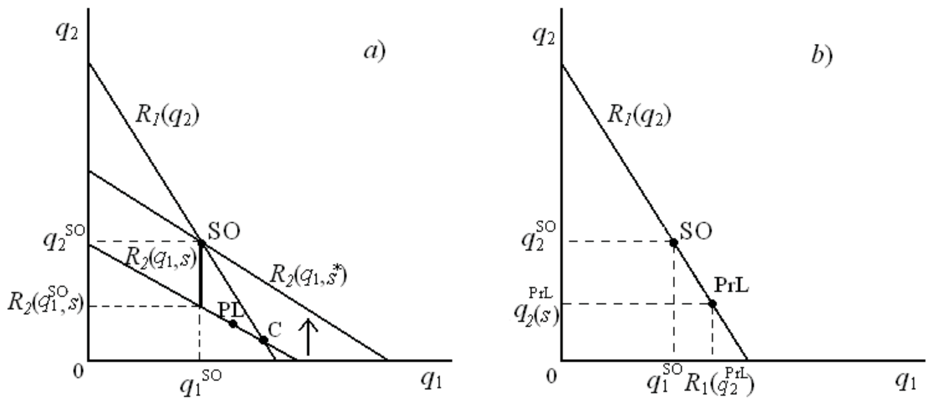

Figure 1. The quantity competition case: the games under simultaneous moves and public leadership (a); the game under private leadership (b).

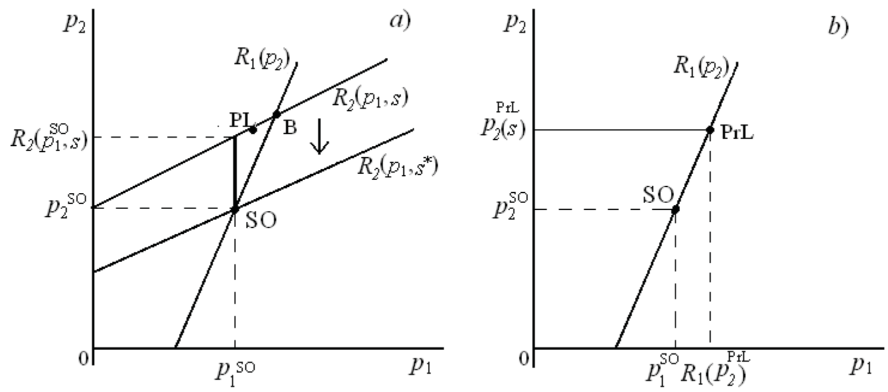

Figure 2. The price competition case: the games under simultaneous moves and public leadership (a); the game under private leadership (b).

We will now focus on the equilibria under private leadership. In Figures 1b and 2b, respectively for a quantity and a price game, these equilibria are represented by points $\Pr L$ when evaluated at a generic subsidy. When an optimal subsidy applies, these points coincide with the efficient ones. Both $\Pr L$ and SO lie on firm 1's reaction functions since the latter, represented at the subgame perfect equilibrium $\alpha^* = 1$, are independent of s.

In order to compare the outcomes under optimal subsidy across all the games, it is worth considering the extent to which the optimal subsidies are provided in both the quantity and the price settings. Indeed, while the same subsidy is provided at equilibrium under simultaneous moves and public leadership, a subsidy of a smaller magnitude is required under private leadership in the quantity competition case ($s^C_{\Pr L} < s^C = s^C_{PL}$). In this case, the provision of a subsidy is finalized to discipline the behavior of a private firm which anticipates the more aggressive reaction of a firm maximizing welfare at equilibrium, and exploits its position of first-mover to expand its production, consistently with the aim of maximizing profits under strategic substitutability. This increased aggressiveness reduces the behavioral differences between the two firms and thus the distortion from the social optimum with respect to the games with simultaneous moves or public leadership. A similar argument applies to the price competition case to demonstrate that the optimal subsidy under private leadership is of a greater magnitude than the equivalent subsidy under simultaneous moves or public leadership ($s^B_{\Pr L} > s^B = s^B_{PL}$). Indeed, under price competition the subsidy regulates the behavior of a private leader that anticipates the aggressive reaction of a follower maximizing welfare at equilibrium, and under strategic complementarity takes advantage of being the first-mover by setting a price that is higher than in the two other cases. This choice widens the firms' behavioral differences and the distortion from the social optimum, thus requiring a higher subsidy in order to achieve the first-best solution.

A comparison across all the games allows us to introduce the second equivalence result which is stated in the following proposition.

Proposition 3 *The optimal subsidy required in a quantity (price) game with private leadership to achieve efficiency is equivalent to that required in a price (quantity) game with simultaneous moves or public leadership. Formally:* $s^B = s^B_{PL} = s^C_{\Pr L} = (1-c)(1-\gamma)$ *and* $s^C = s^C_{PL} = s^B_{\Pr L} = (1-c)/(1+\gamma)$.

Proof: It descends from (6-7-8) and (14-15-16).

Proposition 3 basically states that the lower (greater) optimal subsidy under quantity (price) private leadership coincides with the optimal subsidies characterizing the more (less) efficient price (quantity) competition in sequential moves and public leadership. We focus attention on both the equivalence results in the following paragraphs.

The equivalence between quantity public leadership/simultaneous moves and price private leadership

In this paragraph we demonstrate the equivalence $s^C = s^C_{PL} = s^B_{\Pr L}$, namely we show that the same subsidy $(1-c)/(1+\gamma)$ restores the first best under both public leadership/simultaneous moves in quantities and private leadership in prices, by inducing the same output expansion by the private firm.

In a price game with private leadership, firm 1's reaction function evaluated at the SPNE $\alpha^* = 1$ is $R_1(p_2) = c(1-\gamma) + \gamma p_2$, which is clearly independent of s. When a generic subsidy s is provided, the private firm sets the price $p_2^{\Pr L}(s) = (1 + c(1+2\gamma) - s(1+\gamma))/(2(1+\gamma))$, which is represented as the ordinate of point $\Pr L$ on the $R_1(p_2)$ curve in Figure 2b. A subsidy on production disciplines firm 2's behavior, inducing it to set $p_2^{SO} = c$ through an output expansion. Therefore, the price reduction needed for firm 2 to behave efficiently is measured by the difference $\Psi(s) = p_2^{\Pr L}(s) - p_2^{SO} = (1 - c - s(1+\gamma))/(2(1+\gamma))$, which shrinks to zero at the optimal subsidy $s^B_{\Pr L}$, leading point $\Pr L$ to coincide with the social optimum SO. By setting the efficient price at the first stage, the private firm enables firm 1 to react to the rival's efficient choice by setting the efficient price at the second stage. We now evaluate $\Psi(s)$ in terms of quantities and demonstrate that it coincides with the additional quantity needed for a private simultaneous-mover/follower to behave efficiently in a quantity game. Indeed, by substituting $p_2^{\Pr L}(s)$ and $p_1 = R_1\left(p_2^{\Pr L}\right)$ in the direct demand function $q_2 = ((1-\gamma) - p_2 + \gamma p_1)/\left(1-\gamma^2\right)$, we obtain $q_2^{\Pr L}(s) = (1 - c + s(1+\gamma))/(2(1+\gamma))$, which is the quantity produced at equilibrium by the private firm when it sets the price $p_2^{\Pr L}(s)$. Since the efficient quantity $q_2^{SO} = (1-c)/(1+\gamma)$ is associated to p_2^{SO} when the optimal subsidy applies, the difference $q_2^{SO} - q_2^{\Pr L}(s)$ denoted by $\Phi(s)$, and measuring the output expansion associated to the price reduction $\Psi(s)$, is equal to $\Phi(s) = (1 - c - s(1+\gamma))/(2(1+\gamma))$.

We turn now to consider a quantity game with simultaneous moves or public leadership. At the generic subsidy s, the reaction function of the private firm is $R_2(q_1, s) = (1 - c + s - \gamma q_1)/2$ and is depicted in Figure 1a, where the same reaction function is represented, and denoted by $R_2(q_1, s^*)$, when it is evaluated at the optimal subsidy $s^* = s^C = s^C_{PL}$. The quantity expansion needed for the private firm to be efficient when it acts as a simultaneous player or as the follower, is measured by the vertical shift of firm 2's reaction function, namely by the difference $\widetilde{\Phi}(s) = q_2^{SO} - R_2\left(q_1^{SO}, s\right) = (1 - c - s(1+\gamma))/(2(1+\gamma))$, where $q_2^{SO} = (1-c)/(1+\gamma)$ and $R_2\left(q_1^{SO}, s\right) = (1 - c + s(1+\gamma))/(2(1+\gamma))$.[14] We have therefore obtained $\Phi(s) = \widetilde{\Phi}(s)$, which proves that the same subsidy $s^C = s^C_{PL} = s^B_{\Pr L}$ induces an equal output expansion in the considered price and quantity settings.

We explain this equivalence result in what follows. Indeed, the equivalence $\Phi(s) = \widetilde{\Phi}(s)$, mirroring the equivalence among subsidies, proves that, irrespective of the mode of competition, a subsidized private firm has to produce the same additional output in order to achieve efficiency when the quantity produced by a public firm is kept constant at its efficient level. Indeed, both $\Phi(s)$ and $\widetilde{\Phi}(s)$ are calculated keeping $q_1 = q_1^{SO}$: this is behind the construction of the quantity difference $\Phi(s)$ and, moreover, follows from the calculus of $\widetilde{\Phi}(s)$, which is associated to a movement on the function $R_1(p_2)$ entailing $q_1 = q_1^{SO}$.[15]

The equivalence between price public leadership/simultaneous moves and quantity private leadership

In this paragraph we demonstrate the equivalence $s^B = s^B_{PL} = s^C_{\Pr L}$, namely we show that the same subsidy $(1-c)(1-\gamma)$ restores the first best under both private leadership in quantities and public leadership/simultaneous moves in prices, by inducing the same price reduction by the private firm.

In a quantity game with private leadership, the reaction function of the public firm evaluated at $\alpha^* = 1$ is $R_1(q_2) = 1 - c - \gamma q_2$, which is clearly independent of s. When a generic subsidy s is provided, the private firm produces the quantity $q_2^{\Pr L}(s) = (1 - \gamma - c(1-\gamma) + s)/\left(2\left(1-\gamma^2\right)\right)$, which is represented as the ordinate of point $\Pr L$ on the $R_1(q_2)$ curve in Figure 1b. The output expansion needed for firm 2 to behave efficiently and produce $q_2^{SO} = (1-c)/(1+\gamma)$ is measured by the difference $\widehat{\Phi}(s) = q_2^{SO} - q_2^{\Pr L}(s) = (1 - \gamma - c(1-\gamma) + s)/\left(2\left(1-\gamma^2\right)\right)$, which shrinks to zero at the optimal subsidy $s^C_{\Pr L}$. The efficient production by the private firm at the first stage also induces the public firm to produce the efficient output at the second stage, so that the first best is achieved when $s^C_{\Pr L}$ applies, with point $\Pr L$ coinciding with the social optimum SO. Let us now evaluate the difference $\widehat{\Phi}(s)$ in terms of

[14] Notice that at the optimal subsidy $\widetilde{\Phi}(s^*) = 0$, which implies that points C and PL coincide with the social optimum SO.

[15] This reflects a property of the reaction function $R_1(p_2)$, namely a characteristic of the public firm's optimal behavior in a price competition framework: for any given price set by the private rival, the public firm always sets that price at which the competitive quantity is produced.

prices, which allows us to show that it coincides with the price reduction required for a private simultaneous-mover/follower to behave efficiently in a price game. Indeed, by substituting $q_2^{\Pr L}(s)$ and $q_1 = R_1\left(q_2^{\Pr L}\right)$ in the inverse demand function $p_2 = 1 - q_2 - \gamma q_1$, we obtain the price $p_2^{\Pr L}(s) = (1 + c - \gamma(1-c) - s)/2$ that the private firm sets at equilibrium when it produces the optimal quantity $q_2^{\Pr L}(s)$. Since the efficient price $p_2^{SO} = c$ is associated to q_2^{SO} when the optimal subsidy applies, the difference $p_2^{\Pr L}(s) - p_2^{SO}$ denoted by $\widetilde{\Psi}(s)$, and measuring the price reduction associated to the output expansion $\widehat{\Phi}(s)$, is equal to $\widetilde{\Psi}(s) = ((1-c)(1-\gamma) - s)/2$.

Now we examine a price game with simultaneous moves or public leadership. The reaction function of the private firm, $R_2(s, p_1) = (1 + c - s - \gamma(1-p_1))/2$, is drawn in Figure 2a. The latter is represented in the same figure by $R_2(p_1, s^*)$, namely as a function of the optimal subsidy $s^* = s^B = s^B_{PL}$. The price reduction needed for the private firm to be efficient when it acts as a simultaneous player or the follower is measured by the vertical shift of firm 2's reaction function, namely by the difference $\widehat{\Psi}(s) = R_2\left(p_1^{SO}, s\right) - p_2^{SO} = ((1-c)(1-\gamma) - s)/2$, where $R_2\left(p_1^{SO}, s\right) = (1 - \gamma + c(1+\gamma) - s)/2$ and $p_2^{SO} = c$.[16] We have therefore obtained $\widetilde{\Psi}(s) = \widehat{\Psi}(s)$, which proves that the same subsidy $s^B = s^B_{PL} = s^C_{\Pr L}$ induces an equal price reduction in the considered price and quantity settings.

Also in this case we point out how the equivalence $\widetilde{\Psi}(s) = \widehat{\Psi}(s)$ represents the same price reduction required for a private firm to behave efficiently when the price set by the public firm is kept constant at its efficient level. Indeed, both $\widetilde{\Psi}(s)$ and $\widehat{\Psi}(s)$ are evaluated by keeping $p_1 = p_1^{SO}$: this underlies the calculus of $\Psi(s)$ and, moreover, characterizes the public firm's reaction function $R_1(q_2)$ when it is interpreted in the space (p_1, p_2), where $\widetilde{\Psi}(s)$ is measured on the vertical axis.[17]

3.1 Concluding remarks

The present paper examines simultaneous and sequential competition between a state-controlled firm and a private one, when both are subsidized by the government. Our findings contribute to the existing literature on mixed markets under optimal subsidies, by deriving the ownership structure of the controlled firm required for a subsidy to maximize allocative efficiency in a range of competitive settings which include quantity and price competition, both explored under different timing assumptions. By describing the forces shaping firms' reactions to a welfare-maximizing subsidy, the model highlights the circumstances under which firm ownership is irrelevant, or rather, it can be properly oriented in order to achieve maximum efficiency. The analysis has been carried out distinguishing the results which state an equivalence of subsidies and market outcomes

[16] At the optimal subsidy the following equality holds $\widetilde{\Psi}(s^*) = 0$, implying that point B and point PL coincide with the social optimum SO.

[17] An inspection of the public firm's reaction function $R_1(q_2)$ indeed reveals that for any given quantity set by the private rival, the public firm always produces that output at which the competitive price clears the market.

from the results of irrelevance of privatization or partial privatization, which have been considered as equivalent in former works. The study, moreover, by focusing on the extent to which a non-distortionary subsidy is provided in the considered scenarios, allows to assess the relative efficiency of quantity vs. price competition and to draw attention to the order of firms' moves as relevant variables in the design of a subsidy policy. The analysis under more general demand and costs,[18] as well as the analysis of the effects of distortionary subsidies, are left to future research.

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[18] While the result related to the irrelevance of firm ownership, as well the result of equivalence in Proposition 2, are robust to alternative cost specifications (Myles, 2002; Hashimzade et al., 2007), we speculate that the equivalence result in Proposition 3 does not hold for general costs.

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Appendix

The optimal subsidy under Cournot public leadership:

$$s^{C}_{PL}\left(\alpha\right)=\frac{(1-c)\left(\alpha\gamma\left(\alpha\gamma\left(\gamma^2+3-7\gamma\right)-2\gamma\left(\gamma^2+4-8\gamma\right)+20\alpha-48\right)+4\alpha(12-5\alpha)+\gamma^2\left(\gamma^2-12\gamma+8\right)-32(1-\gamma)\right)}{\gamma^2(32-4\gamma-5\gamma^2)-32+4\alpha(12-5\alpha)+\alpha\gamma^2(2\gamma(\gamma+4)+\alpha(15-4\gamma)-40)}$$

The optimal subsidy under Cournot private leadership:

$$s^{C}_{\Pr L}\left(\alpha\right)=$$
$$\frac{(1-c)\left(32-\gamma\left(32+8\gamma+\gamma^3-12\gamma^2\right)-\alpha\left(80-76\alpha+32\alpha^2-5\alpha^3\right)+\alpha\gamma\left(72+\gamma\left(\gamma^2+16-20\gamma\right)-\alpha\left(60+3\alpha^2-22\alpha\right)-\gamma\alpha(12-3\alpha+2\alpha\gamma-11\gamma)\right)\right)}{32+\gamma^2(4\gamma-32+5\gamma^2)-\alpha(80-76\alpha+32\alpha^2-5\alpha^3)-\alpha\gamma(+8-2\gamma(38-3\gamma-6\gamma^2)-2\alpha(8-5\alpha+\alpha^2)+\alpha\gamma(66+2\alpha\gamma^2-\gamma(2+9\gamma)-3\alpha(8-\alpha)))}$$

The optimal subsidy under Bertrand public leadership:

$$s^{B}_{PL}\left(\alpha\right)=$$
$$\frac{(1-\gamma)(1-c)\left(32(1+\gamma)-\gamma^2\left(\gamma^2+12\gamma+8\right)-4\alpha(12-5\alpha)-\alpha\gamma\left(4(12-5\alpha)+\alpha\gamma\left(\gamma^2+7\gamma+3\right)-2\gamma\left(\gamma^2+8\gamma+4\right)\right)\right)}{32-4\alpha(12-5\alpha)+\gamma(1-\gamma)(32+\gamma(16-3\gamma^2-4\gamma))+\alpha\gamma(2\gamma(2+\gamma)(6-3\gamma^2+5\gamma)+\alpha\gamma(\gamma(\gamma+3\gamma^2-15)-11)-4(12-5\alpha))}$$

The optimal subsidy under Bertrand private leadership:

$$s^{B}_{\Pr L}\left(\alpha\right)=$$
$$\frac{(1-c)(1-\gamma)\left(32(1+\gamma)-\gamma^2\left(\gamma^2+12\gamma+8\right)+\alpha\left(5\alpha^3+76\alpha-80-32\alpha^2\right)+\alpha\gamma\left(\gamma\left(\gamma^2+20\gamma+16\right)+\gamma\alpha(2\alpha\gamma+3\alpha-11\gamma-12)+\alpha\left(3\alpha^2-22\alpha+60\right)\right.\right.}{32(1+\gamma)+\gamma^2(\gamma(3\gamma^2+\gamma-20)-16)+\alpha(5\alpha^3+76\alpha-32\alpha^2-80)-\alpha\gamma(72+2\gamma(\gamma+1)(\gamma^2-\gamma-14)-\alpha(3\alpha^2-22\alpha+60)-2\gamma\alpha(\alpha(2+\gamma)-9-7}$$