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How avoiding overreaction to public information? Some insights on central bank communication practices

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Abstract

It is argued in literature that transparency may be detrimental to welfare. Morris and Shin (2002) suggest reducing the precision of public information or withholding it. The latter seems to be unrealistic. Thus, the issue is not whether central bank should disclose or not its information, but how the central bank should disclose it. We consider a static coordination game in which the private sector receives n semi-public information plus their specific information, and we analyse the impact on the private sector's welfare. The paper consists of three parts: (1) By making assumption that no costs are attached to the provision of private information, we determined the conditions under which the central bank faces a trade-off between enhancing commonality and the use of more precise, but fragmented information. Such intermediate transparent strategies may prevent the bad side of public information from overpowering the good side of it. (2) The latter result is found even in presence of positive externalities. (3) Introducing costs to that framework in equilibrium shows that strategic substitutability between semi-public and private precisions is a very likely outcome.

JEL codes: D82, D83, E58

Keywords: Transparency, Central bank Communication, semi public information, private information, static coordination game

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1 Introduction

Communication and information disclosure are central issues to the theory and practice of central banking. Econometric studies show that communication exerts a substantial impact on asset prices (Andersson et al. (2006), Kohn and sack (2004), Ehrmann and Fratzscher (2007a)...). Though earlier empirical researches argue that improved transparency of monetary policy and the associated communication have been effective, the question remains if a central bank should reveal more information to the public, thereby making its communication more explicit and forward looking.

This recognition initiated an extensive research program in how to design an optimal communication strategy by central banks. Morris and Shin (2002) and Amato et al. (2002), were the first to spark a debate on the value of transparency[1]. They studied a simple coordination game with imperfect common knowledge. It rests on the presumption that economic agents hold two signals that differ in nature, namely, they receive both private and public information about economic fundamentals. With respect to this theoretical framework, private information can be interpreted as insider information or simply as individual interpretation of commonly accessible information. Thus, private information will differentiate potentially within market participants. It can represent any information that individual has observed, such as news received through private discussions (Stasavage, 2002). As for the second type of signals, it is commonly shared by all agents[2]. The public signal can represent information gleaned from newspapers articles or other sources that report on central bank procedures (Stasavage, 2002, p. 5). Both types of information are faulty signals of the true fundamental state of economy. From a social welfare perspective, their central result states that agents may put too much weight on public information relative to private signals. In that sense, more precise public information plays two roles: it conveys fundamental information, but also it acts as a focal point for coordination. Cornand (2006) brought experimental evidence that the focal potential of public information cannot be ignored. Subjects particularly overweigh the public information when they receive both public and private signals. If private agents overreact to public information, then a policy of limited transparency may be warranted.

In this paper, we investigate the welfare effects of fragmented information in the presence of private signal. We consider the same beauty contest[3] in Morris and Shin (2007a)[4]. This means that public information is common only among agents belonging to the same group. Such a modelling of the informational structure is consistent with the idea that fragmented information may reduce eventual detrimental effects of the release of public information on social wel-

[1] It is a necessary condition for an efficient communicatiopn policy.

[2] Examples of public signals, inflation report, macroecnomic announcement...

[3] A Keynesian beauty contest is a concept developed by John Maynard Keynes to explain price fluctuations in equity markets.

[4] The information structure in Morris and Shin (2007a) assumes that agents receive a common signal and n semi-public signals, while ours includes n semi-public information and a private signal

fare, as agents overreact to the public information when it is fully disseminated. Although our analysis is simple, we believe it is important, as it gives a robust contribution to the fact that introducing certain opacity (i.e. fragmented information) may lead to superior results. The coordination game's approach doesn't mean advocating the lack of transparency, but rather identifying the mechanism for information disclosure to prevent a situation of overreaction. We outline proposals regarding information policy dissemination that central banks could follow within a context of monetary policy. If public announcements may be detrimental to welfare, then introducing a certain degree of uncertainty about their interpretation may reduce their focal potential and improves outcomes. Particularly, the setting challenges an older view that central banks should either provide as much information as possible or shut down, the public signal entirely. Fragmented public information which is considered as a form of partial revelation avoids such bang-bang outcomes and increases the plausibility of the results.

We dress two important issues:

- Showing that fragmented public information in presence of private one decreases private agents' overreaction ;
- Determine the conditions under which a trade-off happens when the central bank chooses between releasing entirely its public information but with some noise and disclosing n semi-public information with high precision, by establishing different cases of payoff functions.

The remainder of this paper is as follows. In section 2, we dress the advantages and the drawbacks from disseminating fully public information. In section 3, we describe the model, it develops a short stylized model of the reception of two types of signals, and we characterize the equilibrium set. In section 4, we present the welfare outcomes. We check also if substitutability is likely to occur even with that informational structure. Finally, some remarks and discussion will be offered in section 5.

2 Good and dark side of public information: Literature review

Generally, one might expect that better public information improves market functioning which means that financial markets become better at predicting the outcome of unrealized fundamentals. The coordination game approach shows, however, that increased transparency may lead to non optimal results, and then hamper market functioning. The purpose of this section is to shed light on the effects of public information in Morris and Shin's (2002) framework in order to motivate our subsequent framework.

The main feature of Morris and Shin framework is that public information is perceived as playing a dual role: on the one hand, it provides information about relevant fundamentals to financial markets. The central bank's assessment will be of importance to financial market participants, as it will affect future policy actions. On the other hand, public information may serve as a coordinating device for the beliefs. Decision by investors, are thus, based both on their specific information and their beliefs about other agents' beliefs. The damaging effect of public information comes from the fact that agents put more weight on public signal caused by the coordination motive. According to Svensson (2006), however, this negative impact occurs only when the precision of public information is below a certain threshold. Beyond such optimum, more transparency is undesirable[5]. There are two exceptions, for which transparency is dangerous, as underlined by van der Cruijsen et al.[6] (2010, p. 4), "(1) each agent puts more weight on the coordination motive than on the motive to bring actions in line with economic fundamentals, and (2) the noise in the public signal is at least eight times higher than the noise of the private signal. This is unlikely because central banks spend a lot of resources on collecting and interpreting data." Morris and Shin (2005) explored another model in which public information is endogenous, and gave rise to the result of possible negative effects of public information. Providing a lot of information to steer market expectations might be undesirable because it could lower the informativeness of financial markets and prices and, therefore, worsen public information. Woodford (2005) argues, however, that the damaging effect of public information is due to the fact that "beauty contest" term disappears at the aggregate level of the welfare[7]. Even in the presence of investment complementarities, Angeletos and Pavan (2004) think that welfare is enhanced. Nevertheless, an empirical support of the Morris and Shin's hypothesis was found by Ehrmann and Fratzscher (2007b). Now, what possible solutions are suggested in the literature in order to reduce agents' overreaction to the public information?

- **Partial announcement**: Walsh (2007) and Cornand and Heinemann (2008)[8] propose an original definition of transparency based on the degree of information dissemination: in this new framework, the central bank may decide to reveal its information, not to all agents, but only to a part of the population. Walsh (2006) uses this definition in terms of information dissemination while Cornand and Heinemann (2008) also retain the definition of transparency in terms of information more or less noisy. The authors show, based on the results of Morris and Shin, in situations such as public information is not desirable because of its effect of coordinating expectations on an equilibrium that moves away from the optimum, a more precise public signal but revealed only to a portion of the population is

[5]In a reply to Svensson (2006), Morris, Shin and Tong (2006) argue that if public signal is correlated with the private signal, then quantitative evaluation supports their original results", adopted from Ueda (2010, p 1).

[6]van der Cruijsen et al. (2010) found an optimal intermediate level of transparency, this result was checked also in Trabelsi (2012) in case of OECD countries.

[7]He found that when the "beauty contest" term is conserved in the welfare function, any in-crease of the public information precision is always beneficial to the social welfare.

[8]Cornand and Heinemann (2008) investigate the optimal number of private agents to be in-formed by a central bank.

higher from a welfare perspective than a signal of low precision revealed to all agents. The fact that information is not revealed to the general population reduces the incentives of individuals to overreact to the public signal and therefore reduces the deleterious effect associated with higher-order beliefs. Reducing the number of agents who receive the public signal is, according to these authors, an effective way to avoid the negative effects resulting from the coordination of expectations. Ultimately, if the public signal is very accurate, then the central bank has the interest to reveal it to all agents. In contrast, information with low precision should make the object of a partial publication, i.e. a limited number of individuals.

- **Public information with idiosyncratic noise**: Heinemann and Illing (2002) suggest that the central bank should release information to each agent privately with some idiosyncratic noise. This solution may avoid commonality. Using general from of the informational structure, allowing both public and private signals to be imperfectly correlated, respectively, Arato and Nakamura (2011) show that am-biguous announcements may be beneficial to welfare.

- **Fragmentation**: The idea of fragmented information put by Morris and Shin (2007a) goes back to Issing (2005, p 72) who stressed the challenges the central banker faces in communicating with the public: "Striking the balance between the need for clear and simple messages and the need to adequately convey complexity is a constant challenge for central bank communication". Because simplicity is a great virtue in its ability to generate common understanding, there would be a trade-off, as pointed by Morris and Shin (2007a). How to establish fragmented information in real world? The cheap talks used by central banks, for example, speeches by governors may be considered as a fragmented way of communication. It doesn't lead to common framework across private agents. Different interpretations by the agents lead to the fact that public signals become private ones.

However, although these strategies are interesting, we can derive some notable limits:

1. The results obtained in Cornand and Heinemann (2008) and Arato and Nakamura (2011) are keen dependent on the loss function (which is the same as in Morris and Shin (2002)). Indeed, the welfare function used in Morris and Shin (2002) is controversial since the detrimental effect of transparency is driven by the relative relevance of coordination and stabilization at the social level.

2. From practical point of view, limiting the publicity as recommended by Cornand and Heinemann (2008) would be hard to implement in real world given the widespread development of media.

3. The critique on the loss function of Morris and Shin (2002) seems to be solved in Morris and Shin (2007a). However, the new informational

structure is not comparable to the benchmark one. So, the overreaction problem was ignored in their paper.

All theses issues are taken into account in our paper and give a supplementary motivation of our subsequent framework.

3 The set up

There are many small agents, who have to decide on an underlying unknown state θ, but also try to guess other individuals' beliefs in the economy. Following Morris and Shin (2002, 2007a), the decision rule for an agent j is given by:

$$a_j = (1-r)\, E_j(\theta) + rE_j(\bar{a}) \tag{1}$$

Where $\bar{a}$ is the average action in the population, such that $\bar{a} = \int a_j dj$, r is a parameter that lies between zero and one, it measures the degree of strategic complementarities, called also the "beauty contest" term. The optimal action for an individual j is thus a function of two things: the view about the state θ, and the average expectation formed by all individuals.

According to Morris and Shin (2007a), the Central Bank publishes its information in a fragmented way. Thus, we argue that information used by agents are available in the form of n semi-public signals, observed each by $1/n$ of the population, and a private signal that is specific to each agent in the economy (See the following representation). These take the form of :

A semi-public signal:

$$Z_i = \theta + \eta_i, i = 1, 2, ..., n \tag{2}$$

And a private signal

$$x_j^i = \theta + \varepsilon_j^i \tag{3}$$

Both $\eta_i$and ε_j^i are i.i.d normally distributed with zero mean and variances $\sigma_\eta^2$and σ_ε^2, respectively. We define the relative precision of the semi-public signal as $\gamma = \frac{1}{\sigma_\eta^2}$ and $\beta = \frac{1}{\sigma_\varepsilon^2}$as of the private signal. As interpreted by Geraats (2007, p.42), the noises η_i and ε_j^i express the difficulty the private sector has in interpreting the central bank's communication. When $\sigma_\eta^2 = \sigma_\varepsilon^2 = 0$, the signals Z_i and x_j^i communicate without any noise. There's no more information asymmetry and there's perfect transparency about the central bank's objective.

Following Morris and Shin (2002), actions are linear[9] function of signals:

$$a_j^i = \lambda Z_i + (1-\lambda)\, x_j^i \tag{4}$$

The superscript i denotes the group to which the agent j belongs. Applying (2) and (3) on (1) then gives:

[9]To guarantee equilibrium uniqueness.

$$a_j^i = \frac{\gamma Z_i + \beta\left(1-\frac{r}{n}\right)x_j^i}{\gamma + \beta\left(1-\frac{r}{n}\right)} \tag{5}$$

With

$$\lambda_{eq} = \frac{\gamma}{\gamma + \beta\left(1-\frac{r}{n}\right)} \tag{6}$$

This implies that the average action over all agents is given by:

$$\bar{a} = \frac{\gamma \bar{Z} + \beta\left(1-\frac{r}{n}\right)\theta}{\gamma + \beta\left(1-\frac{r}{n}\right)} \tag{7}$$

Proof. See Appendix A □

From equation (7), when the semi-public information is imprecise ($\gamma \to 0$) or the number of fragmented information is unlimited ($n \to \infty$) or the private information is extremely precise ($\beta \to \infty$), the coordinating role of the semi-public signal is ignored. If the semi-public information is very precise ($\gamma \to \infty$) or the private information is imprecise ($\beta \to 0$), then the private sector will ignore its own information and coordinate on the semi public information.

$\lim()$	$\gamma \to 0$	$\gamma \to \infty$	$\beta \to 0$	$\beta \to \infty$	$n \to \infty$
$\bar{a}$	θ	$\bar{Z}$	$\bar{Z}$	θ	θ

The weight assigned to the public signal in anticipating the fundamental state is given by the relative precision of that signal: $\frac{\gamma}{\gamma+\beta}$. That is by its informational content. Thus, agents assign greater weight to the public signal because it contains information on the higher order beliefs in addition to information on fundamentals. However, the weight assigned to the semi public signal is given by (6), which is always greater than the informational content of the signal.

The weight is an increasing function of the degree of complementarities $r$and the precision γ. It is decreasing in n. Clearly the more important the coordination motive is, the more likely that the agent acts closely to his estimation of average action ($\frac{\partial \lambda}{\partial r} > 0$). We find that in presence of many sectors, agents attach less weight to the semi public information ($\frac{\partial \lambda}{\partial n} < 0$).

Note that the limiting case where $n = 1$ leads to the same decision function as in Morris and Shin (2002) paper, where each agent has an individual private information and a common information. In that case, agents may prefer to coordinate on the same action even with poor quality of public signal. The unique equilibrium will be:

$$a_j = \frac{\gamma}{\gamma + \beta(1-r)} y + \frac{\beta(1-r)}{\gamma + \beta(1-r)} x_j \tag{8}$$

Again, the weight attached to the public information in (8), (the case of Morris and Shin (2002)) exceeds the informational content on fundamental (which is$\frac{\gamma}{\gamma+\beta}$). This reflects the disproportionate impact of the public signal on the coordination of agents' actions. But mostly exceeds $\frac{\gamma}{\gamma+\beta\left(1-\frac{r}{n}\right)}$ when $n \geq 2$. The overreaction to public information when information is fragmented is then weaker than when it is fully disseminated.

4 Welfare effects and policy implications

We now examine how the incentives of the private sector may be affected if the transparency components were subject to choice, depending also on the private sector's objectives.

4.1 The case of negative externalities

In the case of Morris and Shin (2002) (Actions get right[10] , first column of Table 1), which corresponds to a particular situation at the aggregate level of the first loss function when $n = 1$, this is only true for a very specific choice of parameter values, the precision of the public information is beneficial to welfare only when it exceeds a certain threshold, that implies an inverted U relationship between the loss and the precision of public information (See Figure 1). According to that objective function, it is always beneficial that the central bank establishes fragmented information than sending unique public information. The central bank will not face a trade-off between enhancing commonality and the use of more precise, but fragmented information, as this latter leads always to better outcomes.

[10]Expression used by Morris nad Shin (2005).

Table 1: The loss functions and their respective first derivatives

Loss functions	Actions get right: Case 1	Reducing heterogeneity: Case 2	Mixture: Case 3
	$\iint \left(a_j^i-\theta\right)^2 didj$	$\iiiint \left(a_j^i-a_h^k\right) didkdhdj$	$(1-r)\iint \left(a_j^i-\theta\right)^2 didj+\frac{r}{2}\iiiint \left(a_j^i-a_h^k\right) didkdhdj$
Varying r	$\frac{2\beta\frac{r}{n^2}}{\left[\gamma+\beta\left(1-\frac{r}{n}\right)\right]^3}\geq 0$	$\frac{4\beta\frac{r-1}{n^2}}{\left[\gamma+\beta\left(1-\frac{r}{n}\right)\right]^3}\leq 0$	$\frac{-\frac{\gamma}{n}}{\left[\gamma+\beta\left(1-\frac{r}{n}\right)\right]^2}\geq 0$
Varying γ	$\frac{\beta\left(1-\frac{r}{n}\right)\left(2\frac{r}{n}-1\right)-\gamma}{\left[\gamma+\beta\left(1-\frac{r}{n}\right)^2\right]^3}\leq 0$	$2\frac{\beta\left(1-\frac{r}{n}\right)\left(2\frac{r}{n}-\frac{1}{n}-1\right)-\gamma\left(1-\frac{1}{n}\right)}{\left[\gamma+\beta\left(1-\frac{r}{n}\right)^2\right]^3}\leq 0$	$\frac{-\left(1-\frac{r}{n}\right)}{\left[\gamma+\beta\left(1-\frac{r}{n}\right)\right]^2}\leq 0$
Varying n	$\frac{-2\beta\gamma\frac{r^2}{n^3}}{\left[\gamma+\beta\left(1-\frac{r}{n}\right)^2\right]^3}\leq 0$	$2\frac{\beta\gamma\frac{r}{n^3}(1-r)+\frac{\gamma}{n^2}\left[\gamma+\beta\left(1-\frac{r}{n}\right)\right]}{\left[\gamma+\beta\left(1-\frac{r}{n}\right)^2\right]^3}>0$	$\frac{\gamma\frac{r}{n^2}}{\left[\gamma+\beta\left(1-\frac{r}{n}\right)\right]^2}\geq 0$
Varying β	$-\frac{\gamma\left(1-\frac{r^2}{n^2}\right)+\beta\left(1-\frac{r}{n}\right)^3}{\left[\gamma+\beta\left(1-\frac{r}{n}\right)^2\right]^3}\leq 0$	$2\left(1-\frac{r}{n}\right)\frac{\gamma\left(\frac{2-r}{n}-1\right)-\beta\left(1-\frac{r}{n}\right)^2}{\left[\gamma+\beta\left(1-\frac{r}{n}\right)^2\right]^3}\leq 0$	$\frac{-\left(1-\frac{r}{n}\right)^2}{\left[\gamma+\beta\left(1-\frac{r}{n}\right)\right]^2}\leq 0$
Conclusion	No trade-off		Trade-off under conditions

Note: The cases coloured in red are the transparency components used by the central bank. A fragmented information is when $n\geq 2$.as $0\leq r\leq 1$,we have always $1-\frac{2r}{n}>0$.

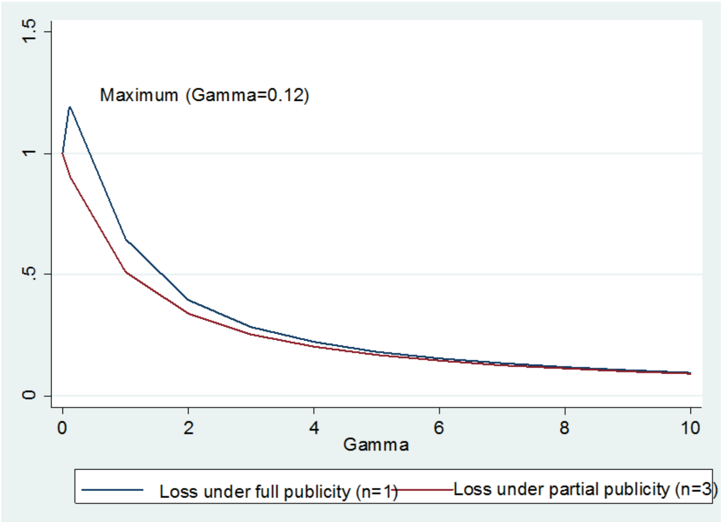

Figure 1: Loss against relative semi-public information: Case 1 Actions get right. Parameters used for calibration($\beta = 1, r = 0.7$)

4.2 The case of no externalities

If transparency components (precision γ, fragmentation measure n) vary into the same direction, this will result into an ambiguous effect on the loss function[11] , described in the last case (which is mainly our function of interest), as the marginal effect of the fragmentation measure has now the opposite sign $\frac{\partial E(L^{PS})}{\partial n} > 0$

The central bank has the choice between full publicity with low precision or to disseminate n semi-public information with an excellent quality. This trade-off is clearly shown in Figure 3. The parameters used for calibration are $r = 0.7$,$\beta = 1$. The point A describes the benchmark situation in which the central bank discloses all the information ($n = 1$) with precision GA(low). The corresponding loss is LA. We will illustrate the trade-off concept by comparing that situation to situations of more precise, but fragmented information (i.e.$n = 2$).

- At Point $B(GB > GA, n = 2)$: $LB > LA$: Full publicity with low precision is the best choice. There's no trade-off.
- At Point $C(GC > GA, n = 2)$: $LC < LA$: More accurate fragmented information is the best choice. There's no trade-off.

[11]Simulations are available and can be downloaded using this link: simulation_surréactionemna.xls. We tried to solve the optimization problem using LINGO software to find the optimal combination of n and γ , such that the loss function attains its minimum. We find always a local optimum.

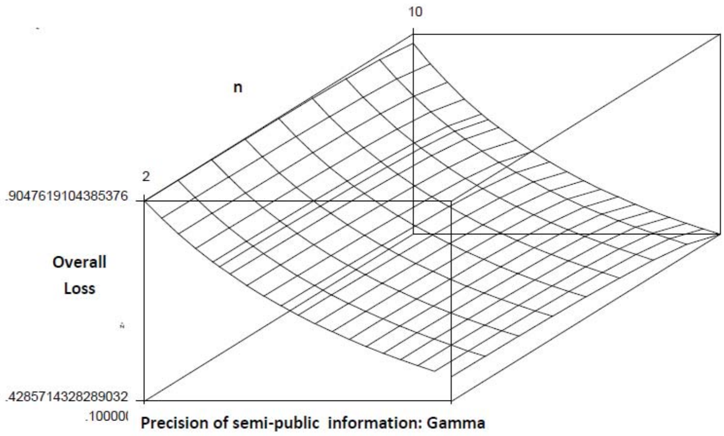

Figure 2: Overall loss function with transparency components: Case 3 mixture. Parameters used for calibration ($\beta = 1, r = 0.5$)

Now, let D be another point that corresponds to a precision $GD, n = 1$ and a corresponding loss LD.

- At point $D(GC > GD, n = 2)$: $LC = LD$: the central bank faces a trade-off, because both strategies yield the same outcome.

Proposition 1. With respect to the loss function described in case 3, it is better for the central bank to disseminate fragmented public information with high precision if the ratio (high to low) precision is greater than $\frac{1-\frac{r}{n}}{1-r}$.

Proof. See Appendix C □

We have to notice that if the coordination motive or the framgentation measure is suffciently high, then inaccurate tranparent strategy is optimal[12].

[12]We can rewrite $\frac{1-\frac{r}{n}}{1-r} = 1 + \frac{1-\frac{1}{n}}{\frac{1}{r}-1}$

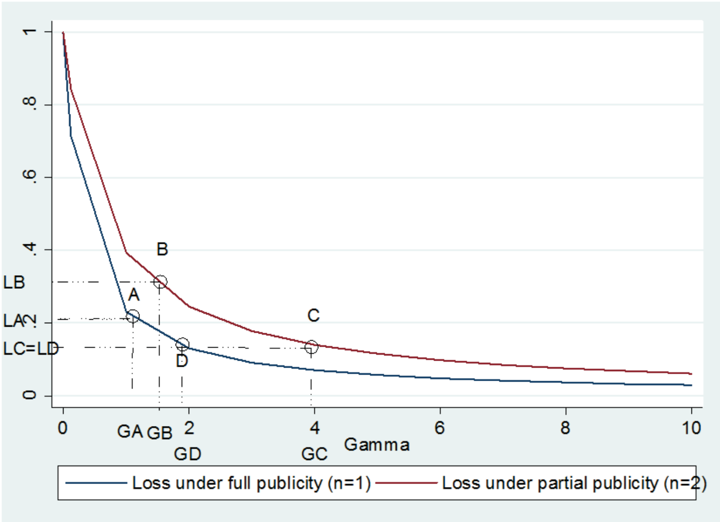

Figure 3: Loss varying with n. Case 3: Mixture. Parameters used for calibration $(\beta = 1, r = 0.7)$

In a related paper, Lindner (2007) argues that central bank should not face a trade-off, but good public information is a precondition for an efficient use of fragmented information. Unlike theoretical works using global games framework to study how central bank's transparency affects welfare, in which transparency is viewed as an exogenous increase in precision of public announcement, Lindner (2007) treats transparency as a strategic choice by the central bank, namely the central bank's policy is derived endogenously in his model.

4.3 The case of positive externalities

The above results (Third column of Table 1) hold even when complementary strategies generate positive externalities. "Positive externalities arise if agents benefit from being close to one another independently of the average dispersion" (Lindner, 2007, p 11). In such a case, the loss function is written as:

$$E\left(L^{PS}\right)_{pe} = (1-r)\iint \left(a_j^i - \theta\right)^2 didj + r \iiiint \left(a_j^i - a_h^k\right) didkdhdj \tag{9}$$

$$a_j^i = \frac{1-r}{1+r} E_j^i(\theta) + \frac{2r}{1+r} E_j^i(\bar{a}) \tag{10}$$

$$\lambda_{eq} = \frac{\gamma(1+r)}{\gamma(1+r) + \beta\left(1 + r - 2\frac{r}{n}\right)} \tag{11}$$

Proof. See Appendix B2 for alrernative method $\square$

Now, we derive conditions under which the central bank can either choose between both partial transparent strategies discussed previously:

Proposition 2. In presence of positive externalities, the condition under which it is equivalent for the central bank to choose either to disclose a less precise common signal or more accurate, but fragmented public information is $\frac{\gamma_2}{\gamma_1} = \frac{1+r-\frac{2r}{n}}{1-r}$. **Proof.** Let $E\left(L^{PS}\right)_{pe}/\delta_1, n=1 = \frac{1}{\frac{\gamma_1}{1-r}+\frac{\beta}{1+r}}$ and $E\left(L^{PS}\right)_{pe}/\delta_2, n \geq 2 = \frac{1}{\frac{\gamma_2}{1+r-2\frac{r}{n}}+\frac{\beta}{1+r}}$

The condition is determined such that $E\left(L^{PS}\right)_{pe}/\delta_1, n=1 = E\left(L^{PS}\right)_{pe}/\delta_2, n \geq 2$ $\square$

The welfare is higher with fragmented strategy if $\frac{\gamma_2}{\gamma_1} \geq \frac{1+r-\frac{2r}{n}}{1-r}$. The advantage is however smaller than in the case of no externalities[13].

4.4 Strategic substituability between the precision of the semi-public and the private information

Our analysis made the assumption that no costs are attached to the provision of private information. We start by assuming a linear cost[14] of improving the precision for a private agent. An assumption of a linear cost seems to be logical since there is a competition between a large numbers of private information providers. In such case, the cost of increasing the private precision would be the same for all agents, as underlined by Colombo and Femminis (2008).

The cost is given by the following expression:

$$C^{PS}(\beta) = c\beta, c > 0$$

$$E\left(T^{PS}\right) = E\left(L^{PS}\right) + C^{PS}(\beta) = \frac{1}{\frac{\gamma}{1-\frac{r}{n}}+\beta} + c\beta$$

An optimizing private sector is now faced with the first order condition:

$$\frac{\partial E\left(T^{PS}\right)}{\partial \beta} = \frac{-1}{\left(\frac{\gamma}{1-\frac{r}{n}}+\beta\right)} + c = 0$$

$$\beta^* = Max\left(0, \frac{1}{\sqrt{c}} - \frac{\gamma}{1-\frac{r}{n}}\right) \tag{12}$$

The second order condition is always met:

$$\frac{\partial^2 E\left(T^{PS}\right)}{\partial \beta^2} > 0 \Longleftrightarrow \frac{2\left(1-\frac{r}{n}\right)^3}{\left[\gamma+\beta\left(1-\frac{r}{n}\right)\right]^3} > 0 \tag{13}$$

[13] Note that $\frac{1+r-\frac{2r}{n}}{1-r} > \frac{1-\frac{r}{n}}{1-r}$

[14] We treat the case of non linear costs in appendix D.

By differentiating (12), we obtain:

$$\frac{\partial \beta^*}{\partial \gamma} = -\frac{1}{1-\frac{r}{n}} < 0 \tag{14}$$

From (14), an increase of the precision of public information leads strictly and unambiguously to a reduction of the private information. Note that (14) is a function of the coordination motive r and the fragmentation measure n. The ratio $\frac{1}{1-\frac{r}{n}}$ is larger than one. In fact, if the coordination motive is large, an increase of the precision of the public information leads agents to overpower that information and this situation crowds out from investing in their specific information. However, if the number of semi-public information is large, this will urge agents to invest in their own information. Now, we proceed at identifying the conditions of a trade-off under the hypothesis of costs attached to the provision of private information.

The loss function is given by:

$$E\left(T^{PS}\right)/c^* = \frac{1}{\frac{\gamma}{1-\frac{r}{n}}+\beta} + \frac{\beta}{\left(\frac{\gamma}{1-\frac{r}{n}}+\beta\right)^2}$$

We denote:

1. γ_1: low precision
2. γ_2: high precision

We calculate the expected loss function under full publicity and low precision, we get:

$$E\left(T^{PS}\right)/c^*, n=1 = \frac{1}{\frac{\gamma_1}{1-r}+\beta_1} + \frac{\beta_1}{\left(\frac{\gamma_1}{1-r}+\beta_1\right)^2}$$

Similarly, we calculate the expected loss under fragmented information with high precision:

$$E\left(T^{PS}\right)/c^*, n\geq 2 = \frac{1}{\frac{\gamma_2}{1-\frac{r}{n}}+\beta_2} + \frac{\beta_2}{\left(\frac{\gamma_2}{1-\frac{r}{n}}+\beta_2\right)^2}$$

We have:

$$E\left(T^{PS}\right)/c^*, n=1 \geq E\left(T^{PS}\right)/c^*, n\geq 2 \Longleftrightarrow \frac{\gamma_2}{1-\frac{r}{n}} - \frac{\gamma_1}{1-r} \geq \beta_1 - \beta_2$$

Given that $\beta_1 - \beta_2 \geq 0$, a necessary condition for the above inequalities to be hold is:

$$\frac{\gamma_2}{\gamma_1} \geq \frac{1-\frac{r}{n}}{1-r}$$

4.5 Trade-off in a more complex setting

In this section, we suggest an alternative way to illustrate the trade-off between enhancing commonality and the use of more precise, but fragmented information. Cornand and Heinemann (2008) claimed that even signals that are released in practice may evoke private information. We propose to consider a more general and complex structure, allowing signals to be imperfectly correlated, in the lines of Arato and Nakamura (2011). According to theses authors, authorities often announce their information ambiguously. Such a strategy could prevent an overreaction by the agents. We extend this structure by assuming that the central bank discloses n ambiguous semi public information. We assume that the informational structure is given by:

Semi public signal: $Z_j^i = \theta + \eta_z^i + \alpha_j^i$ $i = 1, 2, ..., n$ where $\alpha_j^i \rightarrow N\left(0, \frac{1}{\delta}\right)$

and a private signal: $x_j^i = \theta + \eta_x^i + \varepsilon_j^i$ $i = 1, 2, ..., n$ where $\eta_x^i \rightarrow N\left(0, \frac{1}{u}\right)$

δ drives the correlation of idiosyncratic noise of signals that are common among agents belonging to the same group.

We have

$$\rho_z = \rho\left(Z_j^i, Z_k^h\right) = \begin{cases} \frac{\delta}{\delta+\gamma} & if\ i = h \\ 0 & otherwise \end{cases}$$

We can also calculate the correlation between two private signals:

$$\rho_x = \frac{\beta}{\beta+u}$$

Appendix E shows the loss expression is given by:

$$E(L) = \frac{1}{\frac{\beta u}{u+\beta(1-r)} + \frac{\gamma\delta}{\gamma+\delta\left(1-\frac{r}{n}\right)}}$$

When $\delta \rightarrow \infty$ and $u \rightarrow \infty$, we got the expression given in (20). According to that loss function, increases in δ increases the agents' abilities to predict the induced beliefs of others, while an increase in the fragmentation measure would result in an opposite impact. Thus, there's equivalence between both instruments. The central bank would be then indifferent between both strategies if:

$$\frac{1}{\rho_{z_1}} - \frac{1}{\rho_{z_2}} = r\left(1 - \frac{1}{n}\right)$$

[15]

Where $\rho_{z_i} = \frac{\delta_i}{\delta_i+\gamma}$ $i = 1, 2$ such that $\delta_1 < \delta_2$

Proof. Let $E(L)/\delta_1, n = 1 = \frac{1}{\frac{\beta u}{u+\beta(1-r)} + \frac{\gamma\delta_1}{\gamma+\delta_1(1-r)}}$

and $E(L)/\delta_2, n \geq 2 = \frac{1}{\frac{\beta u}{u+\beta(1-r)} + \frac{\gamma\delta_2}{\gamma+\delta_2\left(1-\frac{r}{n}\right)}}$

[15] Note that $\frac{1}{\rho_{z_1}} - \frac{1}{\rho_{z_2}} = \frac{\delta_2-\delta_1}{\gamma}$

The condition is determined such that $E(L)/\delta_1, n = 1 = E(L)/\delta_2, n \geq 2$ □

Similarly to what discussed in previous sections, the higher the coordination motive (r) or the fragmentation measure (n) is, it turns optimal for the central bank to choose to disclose fully the information with less precision.

5 Conclusion

The public information could lead agents to make decisions more in line with fundamentals, but compared to the private information, it facilitates the coordination of agents. Not everyone agrees with the fact that disclosing all available information is optimal. If financial markets participants attach too much weight to central bank's views and don't take into account what they reflect as noisy signals, communication may be detrimental. We show that when the central bank communicates its information in a fragmented way, this reduces overreaction to public information. The result holds also in presence of positive externalities. In other term, we can summarize our findings as follow:

1. **The case without externalities:** it is better for the central bank to disseminate n semi-public information with high precision if the ratio of high to low precision is greater than $\frac{1-\frac{r}{n}}{1-r}$
2. **The case of negative externalities** (corresponds to the payoff function as in Morris and Shin (2002): There's no trade-off. Partial revelation is always better than full dissemination.
3. **The case of positive externalities**: it is better for the central bank to disseminate n semi-public information with high precision if the ratio of high to low precision is greater than $\frac{1+r-\frac{2r}{n}}{1-r}$

We find, also, that once the costs of providing private information are taken into account in our framework, a marginal increase in the precision of the semi-public signal induces the private sector to reduce the precision of its information. Nevertheless, some drawbacks related to fragmentation might be found again. The fragmented nature of speeches and /or testimonies, as commented by Gai and Shin (2003), may lead to a difficulty to reach and capture the desired picture by the market participants. This is not necessarily the case of other central bank's communication channels such as inflation report, minutes, votes that provide a clear informational platform in order to disseminate a coherent message to the audience. Finally, there's still scope of empirical verification of theoretical predictions discussed in this paper. One step in this direction would be referring to experimental economics. This will be examined in a forthcoming research.

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Appendix

A. Derivation of equation (5)

Using the fact that the best linear expectation of fundamental $\theta$is given by:

$$E_j^i(\theta) = \frac{\gamma Z_i + \beta x_j^i}{\gamma + \beta} \tag{15}$$

Recall that $i = 1, 2, ..., n$ designates the group to which individual j belongs.

And the average expected action:

$$E_j^i(\bar{a}) = E_j^i\left(\lambda \bar{Z} + (1-\lambda)\theta\right) = \lambda\left(\tfrac{n-1}{n} E_j^i(\theta) + \tfrac{1}{n} Z_i\right) + (1-\lambda) E_j^i(\theta)$$

$$E_j^i(\bar{a}) = \left(\frac{\gamma}{\gamma+\beta} + \frac{\lambda}{n}\frac{\beta}{\gamma+\beta}\right) Z_i + \left(1 - \left(\frac{\gamma}{\gamma+\beta} + \frac{\lambda}{n}\frac{\beta}{\gamma+\beta}\right)\right) x_j^i \tag{16}$$

Plugging (15) and (16) into (1) yields:

$$a_j^i = (1-r)\frac{\gamma Z_i + \beta x_j^i}{\gamma+\beta} + r\left[\left(\left(\frac{\gamma}{\gamma+\beta} + \frac{\lambda}{n}\frac{\beta}{\gamma+\beta}\right) Z_i + \left(1 - \left(\frac{\gamma}{\gamma+\beta} + \frac{\lambda}{n}\frac{\beta}{\gamma+\beta}\right)\right) x_j^i\right)\right] \tag{17}$$

Rearranging terms, we obtain (5).

B. Derivation of the loss function and the weight attached to the semi-public information in (6)

B.1 Case 1 : "Actions get right"

$$E_j^i\left[\left(a_j^i - \theta\right)^2\right] = \frac{\lambda^2}{\gamma} + \frac{(1-\lambda)^2}{\beta} = \frac{\gamma + \beta\left(1 - \frac{r}{n}\right)^2}{\left[\gamma + \beta\left(1 - \frac{r}{n}\right)\right]^2} \tag{18}$$

B.2 Case 2: "Reducing heterogenity"

$$E_j^i\left[E_h^k\left(a_h^k-a_j^i\right)^2\right]=2\left[\frac{\lambda^2}{\gamma}\left(1-\frac{1}{n}\right)+\frac{(1-\lambda)^2}{\beta}\right]=\frac{2\left[\gamma\left(1-\frac{1}{n}\right)+\beta\left(1-\frac{r}{n}\right)^2\right]}{\left[\gamma+\beta\left(1-\frac{r}{n}\right)\right]^2} \tag{19}$$

B.3 Case 3: The mixture consists of the weighted sum of losses in cases 1 (18) and 2 (19), we get then:

$$E\left(L^{PS}\right)=(1-r)\,E_j^i\left[\left(a_j^i-\theta\right)^2\right]+\frac{r}{2}E_j^i\left[E_h^k\left(a_h^k-a_j^i\right)^2\right]=\frac{1}{\frac{\gamma}{1-\frac{r}{n}}+\beta} \tag{20}$$

We propose now to find (6) by using the loss function in case 3:

Agents are faced with the minimization of their loss function given by:

$$E\left(L^{PS}\right)=(1-r)\,E_j^i\left[\left(a_j^i-\theta\right)^2\right]+\frac{r}{2}E_j^i\left[E_h^k\left(a_h^k-a_j^i\right)^2\right] \tag{21}$$

$$s.t\ a_j^i=\lambda Z_i+(1-\lambda)\,x_j^i$$

We get:

$$E\left(L^{PS}\right)=(1-r)\left[\frac{\lambda^2}{\gamma}+\frac{(1-\lambda)^2}{\beta}\right]+\frac{r}{2}2\left[\frac{\lambda^2}{\gamma}\left(1-\frac{1}{n}\right)+\frac{(1-\lambda)^2}{\beta}\right] \tag{22}$$

Differentiating with respect to λ, we obtain:

$$\frac{\partial E\left(L^{PS}\right)}{\partial\lambda}=2\frac{\lambda}{\gamma}\left(1-\frac{r}{n}\right)-2\frac{(1-\lambda)}{\beta}=0\Longleftrightarrow\lambda_{eq}=\frac{\gamma}{\gamma+\beta\left(1-\frac{r}{n}\right)}$$

We check for the second order condition:

$$\frac{\partial^2E\left(L^{PS}\right)}{\partial\lambda^2}=2\frac{1}{\gamma}\left(1-\frac{r}{n}\right)+2\frac{1}{\beta}>0$$

And the proof is complete □

C. Derivation of the loss function and the weight attached to the semi-public information in the case of positive externalities

We have

$$E_j^i\left[\left(a_j^i-\theta\right)^2\right]=\frac{\lambda^2}{\gamma}+\frac{(1-\lambda)^2}{\beta}=\frac{\gamma(1+r)^2+\beta\left(1+r-2\frac{r}{n}\right)}{\left[\gamma(1+r)+\beta\left(1+r-2\frac{r}{n}\right)\right]^2}$$

And

$$E_j^i\left[E\left(a_k^h-a_j^i\right)^2\right]=2\left[\frac{\lambda^2}{\gamma}\left(1-\frac{1}{n}\right)+\frac{(1-\lambda)^2}{\beta}\right]=2\frac{\gamma(1+r)^2\left(1-\frac{1}{n}\right)+\beta\left(1+r-2\frac{r}{n}\right)}{\left[\gamma(1+r)\left(1-\frac{1}{n}\right)+\beta\left(1+r-2\frac{r}{n}\right)\right]^2}$$

Then,

$$E\left(L^{PS}\right) = (1-r)\,E_j^i\left[\left(a_j^i-\theta\right)^2\right] + rE\left[E_h^k\left(a_h^k-a_j^i\right)^2\right] = \frac{(1+r)\left(1+r-2\frac{r}{n}\right)}{\left[\gamma(1+r)+\beta\left(1+r-2\frac{r}{n}\right)\right]} \tag{23}$$

Clearly, the loss function with positive externalities is decreasing in γ and $\beta$and increasing in n .

Now, we propose to find (11) by the following procedure:

$$E\left(L^{PS}\right)_{pe} = (1-r)\,E_j^i\left[\left(a_j^i-\theta\right)^2\right] + rE\left[E_h^k\left(a_h^k-a_j^i\right)^2\right] \tag{24}$$
$$s.t\ a_j^i = \lambda Z_i + (1-\lambda)\,x_j^i$$

We get:

$$E\left(L^{PS}\right)_{pe} = (1-r)\left[\frac{\lambda^2}{\gamma}+\frac{(1-\lambda)^2}{\beta}\right] + r\times 2\left[\frac{\lambda^2}{\gamma}\left(1-\frac{1}{n}\right)+\frac{(1-\lambda)^2}{\beta}\right] \tag{25}$$

Differentiating with respect to λ, we obtain:

$\frac{\partial E\left(L^{PS}\right)_{pe}}{\partial\lambda} = 2\frac{\lambda}{\gamma}\left(1+r-2\frac{r}{n}\right) - 2(1+r)\frac{(1-\lambda)}{\beta} = 0 \Longleftrightarrow \lambda_{eq} = \frac{\gamma((1+r)}{\gamma(1+r)+\beta\left(1+r-\frac{r}{n}\right)}$

We check for the second order condition:

$$\frac{\partial^2 E\left(L^{PS}\right)_{pe}}{\partial\lambda^2} = 2\frac{1}{\gamma}\left(1+r-\frac{r}{n}\right) + 2(1+r)\frac{1}{\beta} > 0$$

And the proof is complete □

C. Proof of proposition 1

Recall that: $E\left(L^{PS}\right) = \frac{1}{\frac{\gamma}{1-\frac{r}{n}}+\beta}$

We denote

1. γ_1: low precision
2. γ_2: high precision

We calculate the expected loss function under full publicity and low precision, we get:

$$E\left(L^{PS}/n=1,\gamma_1\right) = \frac{1}{\frac{\gamma_1}{1-r}+\beta}$$

Similarly, we calculate the expected loss under fragmented information with high precision:

$$E\left(L^{PS}/n\geq 2,\gamma_2\right) = \frac{1}{\frac{\gamma_2}{1-\frac{r}{n}}+\beta}$$

We have

$$E\left(L^{PS}/n=1,\gamma_1\right) \geq E\left(L^{PS}/n\geq 2,\gamma_2\right) \Longleftrightarrow \frac{\gamma_2}{\gamma_1} \geq \frac{1-\frac{r}{n}}{1-r}$$

D. Introducing non linear costs

Following Demertzis and Hoeberichts (2007), we assume that costs are positive and unbounded:

$$C^{PS}(\beta) = c\beta^{\kappa}, c < 0 \text{ and } \kappa > 1$$

$$E\left(T^{PS}\right) = E\left(L^{PS}\right) + C^{PS}(\beta) = \frac{1}{\frac{\gamma}{1-\frac{r}{n}} + \beta} + c\beta^{\kappa}$$

An optimizing private sector is now faced with the first order condition:

$$\frac{\partial E\left(T^{PS}\right)}{\partial \beta} = \frac{-1}{\left(\frac{\gamma}{1-\frac{r}{n}} + \beta\right)^2} + \kappa c \beta^{\kappa-1} = 0 \tag{26}$$

We check for the second order condition:

$$\frac{\partial^2 E\left(T^{PS}\right)}{\partial \beta^2} > 0 \Longleftrightarrow \frac{2\left(1-\frac{r}{n}\right)^3}{\left[\gamma+\beta\left(1-\frac{r}{n}\right)\right]^3} + \kappa(\kappa-1)c\beta^{\kappa-2} > 0$$

which is always satisfied.

Solving explicitly (26) is complex and doesn't give much insight. Then, we apply the implicit theorem function:

$$\frac{\partial \beta^*}{\partial \gamma} = -\frac{\frac{\partial^2 E\left(T^{PS}\right)}{\partial \beta \partial \gamma}}{\frac{\partial^2 E\left(T^{PS}\right)}{\partial \beta^2}} = -\frac{2\left(1-\frac{r}{n}\right)}{2\left(1-\frac{r}{n}\right)^3 + \kappa(\kappa-1)c\beta^{\kappa-2}\left[\gamma+\beta\left(1-\frac{r}{n}\right)\right]^3} < 0 \tag{27}$$

E. Equilbrium in a more complex setting

Recall that: $\begin{cases} a_j^i = \lambda Z_i + (1-\lambda)x_j^i \\ a_j^i = (1-r)E_j^i(\theta) + rE_j^i(\bar{a}) \end{cases}$

Given that $E_j^i(\bar{a}) = \lambda E_j^i\left(\bar{Z}\right) + (1-\lambda)E_j^i(x)$ such that:

$$\begin{cases} E_j^i(\theta) = \frac{\phi_z Z_j^i + \phi_x x_j^i}{\phi_z + \phi_x} \\ E_j^i\left(\bar{Z}\right) = \left(1-\frac{\rho_z}{n}\right)E_j^i(\theta) + \frac{\rho_z}{n}Z_j^i \\ E_j^i(x) = \frac{\phi_z(1-\rho_x)Z_j^i + \phi_x(1-\rho_z)x_j^i}{\phi_z+\phi_x} \end{cases}$$

We got finally: $\lambda_{eq} = \frac{\phi_z(1-r\rho_x)}{\phi_z(1-r\rho_x)+\phi_x\left(1-r\frac{\rho_z}{n}\right)}$

Where $\begin{cases} \phi_z = \frac{\gamma\delta}{\gamma+\delta} \\ \phi_x = \frac{\beta u}{\beta+u} \end{cases}$