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Compliance with the Institutional Wage in Dualistic Models

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ABSTRACT

This research extends simple two-sector models in order to inquire the impact of the extent of coverage or enforcement of minimum wage legislation in one of the sectors on the equilibrium outcome.

Two versions of institutional wage avoidance are presented. They may be seen as representing different institutional detection rules: one working through worker complaint, the other through firm sampling inspection (and enforcement) by the legal system. Both cases are modelled as enlargements of two dualistic models: Harris-Todaro (the wage in the other sector is market determined) and Bhagwati-Hamada (the wage in the other sector is institutionally fixed and coverage is complete).

Impact on population flows of changes in degree of coverage (compliance) is also confronted with the effect of a change in the institutional wage for each scenario.

JEL: O15, O17, O18, R23, J38, J42, J61, J62, F22, K42.

Keywords: Migration, Mobility, Minimum Wages, Segmented Labor Markets, Informal Sector, Regional Labor Markets, Dualistic Models, Coverage.

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CONTENTS

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I. Introduction.

One of the landmarks in the analysis of institutionally set wages is the Harris-Todaro model. The rationale behind it applies to a wide range of economic problems, whenever an artificial wage floor – higher than what the market would determine – affects (at least) one of two (or more) communicating labor markets. The scenario emerges when minimum wage laws are being practiced, whether they are legitimated by general state legislation or coerced in unionized sectors.

Traditional dualistic models [1] consider that in each sector either coverage is complete - or completely enforced - or it is totally absent. This was the framework considered by Todaro (1969) and Harris-Todaro (1970) - in which the wage in the rural sector is market determined. And by Bhagwati and Hamada (1974) - in which both sectors are subject to institutional rules, yet with different wage levels.

In the subsequent literature, degree of mobility across sectors was added to the framework, modelled in various ways - as in Mincer (1976), MacDonald and Solow (1985), Fields (1989) or, in general, in Martins (2008). These models yield equilibrium corner solutions which alternate between those cases and other outcomes, some with no unemployment [2]. Another way of enlarging the scope of the dualistic approach is to allow coverage to be incomplete in one of the sectors, due to illegal evasion, or to the existence of only partial unionisation in that sector. This is the aim of the present research. The issue is more relevant to economies which have a large uncovered sector, i.e., a lot of illegal immigrants willing to work for less than the minimum wage or when the economy is in a downturn and the number of sub-minimum workers is expected to increase. From a policy perspective, the paper analyses the consequences of minimum wage law enforcement on labor market outcomes.

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[1] See Martins (1996) for a survey of some analytical implications of similar structures to the ones considered.

[2] See Martins (1996) for a summary.

Theoretical approaches that have dealt with the problem of the existence of an informal sector - for example Bental, Ben-Zion and Wenig (1985), Ginsburgh, Michel, Schioppa and Pestieau (1985) and De Gijsel (1985) - consider scenarios where illegal behavior is oriented towards tax evasion and not necessarily minimum wage legislation avoidance, and/or are concerned with implications for standard macroeconomic policy effectiveness, or law enforcement measures.

In the Harris-Todaro (H.-T.) approach, we could argue that the size of the informal sector is implicitly fixed by the rural sector labor demand. Instead, we allow for coexistence of formal and informal jobs in one of the sectors - which we will call the partially covered sector, the first sector - and insert a parameter that will represent the degree of law enforcement of the institutional minimum wage in that sector or region. Compliance with minimum wage legislation by the firm was modelled by Ashenfelter and Smith (1979), Grenier (1982) and Chang and Ehrlich (1985); these authors address the problem of how the incentives for the firm to comply change with penalties and the probability of detection; with reference to these, we model a scenario where degree of law enforcement or coverage corresponds to (their) probability of detection, being the penalty the full payment of extra income needed to attain the minimum wage.

Jones (1987) presents a dualistic model where efficiency wages in the primary sector create unemployment in that sector; he then introduces minimum wage legislation affecting the low wage, secondary sector and analyses the consequences of partial compliance; unemployment is generated in the primary sector and not in the secondary (partially covered) sector. Instead, we focus on the effects of non-compliance in scenarios of institutionally set wages (e.g., unionized sectors).

Hence, we put forward two versions of the stochastic dynamics of expected wage formation in the partially covered sector; the two equilibrium definitions yield the same extreme point results (i.e., the cases where there is no enforcement at all and where there is total enforcement respectively), but we can interpret the second type as applying to a higher degree of segregation between formal and informal employment in the partially covered sector. The first case could correspond to a situation where workers are inspected or detection comes from workers' complaints; in the second case, firms are inspected by the legal system and once caught they must straighten up all their wage payments, but may adjust their employment decisions.

Two kinds of dualistic situations are considered - one in which the second sector is completely informal (in H.-T. lines); another in which the second sector is subject to institutional wage setting, eventually different from the first sector's, and where coverage or law enforcement is complete (as in Bhagwati and Hamada, B.-H., approach). With the two kinds of equilibrium, this implies that we establish four different models.

In each of the structures, we analyze the impact of the change of the degree of coverage or of law enforcement on population flows, mean wage in the partially covered sector, equilibrium expected wage, total unemployment, local or sector unemployment and unemployment rates, wage and/or wage bills.

Finally, we inquire whether increasing the degree of law enforcement acts on population flows similarly to rising the institutional wage in the several structures.

Our analysis is kept very simple - applying to the general long-run labor market equilibrium assessment of two sectors or two geographic areas. Degree of law enforcement is exogenous and acts directly on wage payment practices. We only consider one homogeneous input, labor, and ignore complications from detection and/or penalties of infractors. Search issues are discarded as well.

In section II, two versions of partial coverage dynamics in a dualistic scenario are presented, being the first sector the one where coverage is incomplete; requirements for internal solutions when the second sector is either completely covered by institutional wage setting rules or completely uncovered are also inspected. In section III, implications of a change in the degree of coverage in the first sector are analyzed in a setting where the wage in the second sector is market determined (i.e., in H.-T. tradition); both versions of partial coverage modelling are studied; the impact of a change in the partially covered sector institutional wage is also confronted with the impact of the change of the degree of coverage. Section IV reproduces the comparative statics of section III but for a scenario where the second sector wage is institutionally determined (i.e., in B.-H. type of model). In section V, we discuss possible extensions, namely of workers being expected utility rather than expected income maximizers. The exposition ends with a brief summary in section VI.

II. Partial One-Sector Coverage in Dualistic Models.

This section translates general notation and briefly presents the benchmark dualistic models enlarged by the research – sub-section i.

The two alternative extensions that simulate partial coverage - the existence of an informal sector within a covered region –, Type A and Type B, are forwarded in sub-sections ii and iii respectively. The general dynamic mechanism is stated, followed by the corresponding equilibrium condition, with qualifications of the possible wage range of the different employment status. In sub-section iv, some comments to the two structures are advanced.

i. Notation and Environment

1. Let us start by the usual assumption in the two-sector minimum wage model. There are two sectors - or two regions - and a fixed exogenous labor supply, $\bar{L}$. This total labor supply decides whether to go into sector 1 or 2. Denote by $\bar{L}_i$ local/industry supply in region/sector i. Then:

$$(1) \qquad \bar{L}_1 + \bar{L}_2 = \bar{L}$$

In sector i, the aggregate demand function is given by:

$$(2) \qquad L_i = L^i(W_i) \quad , \quad i = 1, 2$$

A non-positive slope – that is, $L^i(W_i)' = dL^i(W_i)/dW_i \leq 0$ – is always assumed. Denote the corresponding inverse demand function by:

$$(3) \qquad W_i = W^i(L_i) \quad , \quad i = 1, 2$$

The wage elasticity of demand of sector i at a particular point of labor demand will be denoted by $\varepsilon^i = L^i(W_i)' \, W_i / L^i(W_i) = W^i(L_i) / [W^i(L_i)' \, L_i]$.

2. Effective employment in region 2 will obey its demand function, $L_2^e = L_2 = L^2(W_2)$. In sector 1, where coverage is incomplete, employment, L_1^e, may differ from demand. Unemployment in region i will be defined as

$$(4) \qquad U_i = \bar{L}_i - L_i^e$$

with total unemployment in the economy being:

$$(5) \qquad U = U_1 + U_2 = \bar{L} - (L_1^e + L_2)$$

and local unemployment rate:

$$(6) \qquad u_i = U_i / \bar{L}_i$$

3. Assume further that:

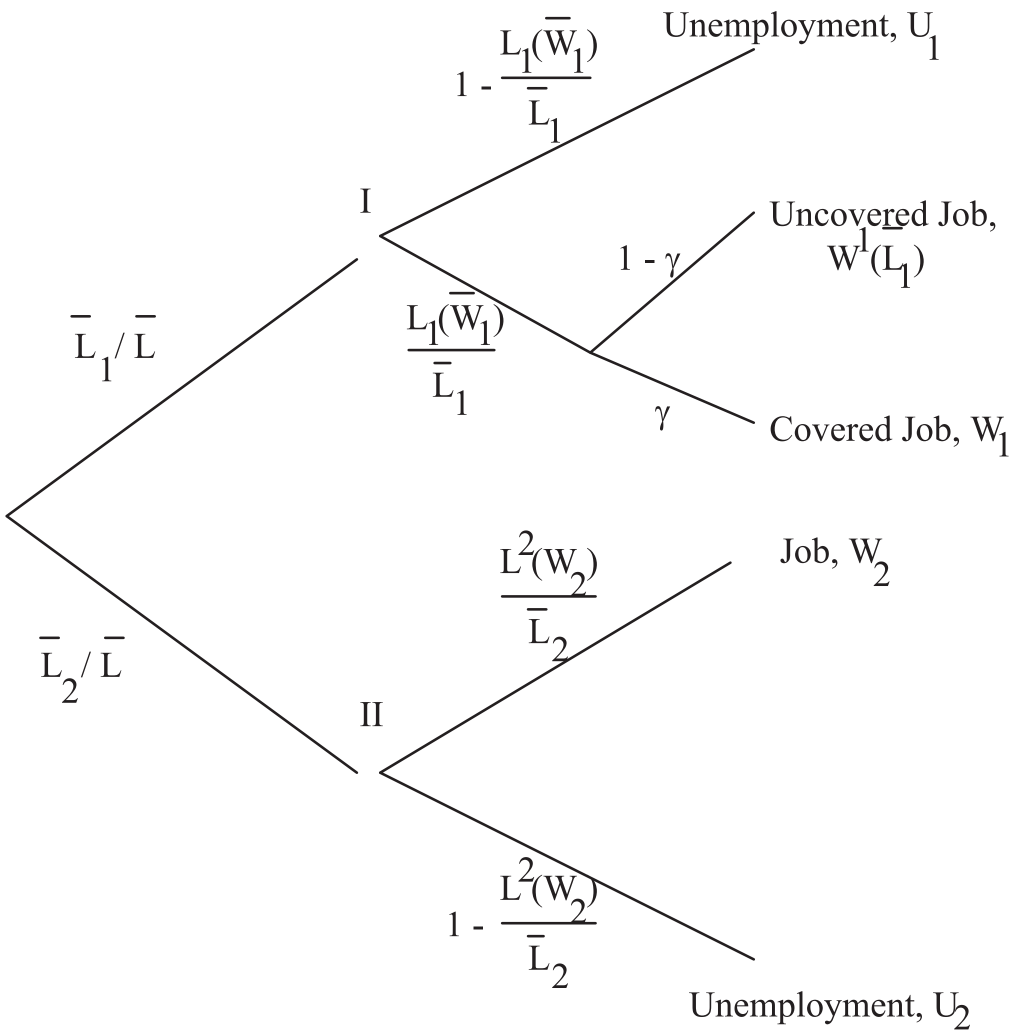
1. individuals are risk neutral and maximize expected income.
2. job rotation is accomplished locally or within the industry.
3. wage in sector 1 is partially determined by market conditions.
4.a. wage in sector 2 is market determined (H.-T. type); alternatively:
4.b. wage in sector 2 is institutionally determined and coverage is complete (B.-H type).
5. local/industry labor demand depends negatively on its argument. There are no cross effects, i.e., $dL_i/dW_j = 0$ for $i \neq j$.

When 4.a. is assumed, a generalization of the Harris-Todaro model is considered. When 4.b. is assumed, we consider a generalization of the Bhagwati-Hamada model.

4. We can model partial coverage in several ways. Let us consider two solutions or versions of equilibrium dynamics which will be denoted Type - A and Type - B respectively:

ii. Partial Coverage Equilibrium: Type - A.

1. Assume that the dynamics of the model are described by the tree below (I and II define the two main branches of the decision node):



Consider that we have assumption 4.a. Individuals can go to sector 2 and earn wage W_2, which will go down till $U_2 = 0$. Expected wage in sector 2 will be:

(7) $$W_2^e = W_2 = W^2(L; \bar{\ }_2)$$

If an individual chooses to go to sector 1, he may find employment, but of the available jobs, only a proportion γ pay the institutional wage, W_1, with the other $(1 - \gamma)$ paying the marginal product of the total population in sector 1. Average wage in sector 1 will be:

$$\bar{W}_1 = [\gamma W_1 + (1 - \gamma) W^1(\bar{L}_1)] \tag{8}$$

Technically, condition (8) implies that wages in uncovered jobs of sector 1 only partially obey market conditions, not being driven down till no unemployment is generated in the economy. It can be justified by the existence of real world frictions such as ex-ante uncertainty about the accepted job type and implied time loss from switching jobs, more costly in sector 1 where there are more alternatives.

And employment in sector 1 will obey demand:

$$L_1^e = L_1 = L^1[\gamma W_1 + (1 - \gamma) W^1(\bar{L}_1)] = L^1(\bar{W}_1) \tag{9}$$

Notice that being $\bar{W}_1$ a weighted average of W_1 and $W^1(\bar{L}_1)$ - and for the institutional wage to be binding $W^1(\bar{L}_1) < W_1$ -, we must have:

$$W^1(\bar{L}_1) < \bar{W}_1 < W_1 \tag{10}$$

and also

$$L^1(W_1) < L_1^e = L^1(\bar{W}_1) < \bar{L}_1 \tag{11}$$

Expected wage in sector 1 will be:

$$(12) \qquad W_1^e = [\gamma W_1 + (1 - \gamma) W^1(\bar{L}_1)]\ L^1[\gamma W_1 + (1 - \gamma) W^1(\bar{L}_1)] / \bar{L}_1$$

and the wage bill is $\bar{W}_1 L^1[\gamma W_1 + (1 - \gamma) W^1(\bar{L}_1)] = \bar{W}_1 L^1(\bar{W}_1)$.

Internal solutions will require equalization of expected wages across sectors and, thus:

$$(13) \quad [\gamma W_1 + (1 - \gamma) W^1(\bar{L}_1)] \quad L^1[\gamma W_1 + (1 - \gamma) W^1(\bar{L}_1)] / \bar{L}_1 = W^2(\bar{L}_2) =$$

$$= W^2(\bar{L} - \bar{L}_1)$$

which solves for $\bar{L}_1$. In equilibrium,

$$(14) \qquad \bar{W}_1 (1 - u_1) = W_2$$

2. If $\gamma = 0$, the model will yield the free market solution, i.e., the one for which W* obeys:

$$(15) \qquad L^1(W^*) + L^2(W^*) = \bar{L}$$

If $\gamma = 1$, the model will yield the Harris-Todaro solution, i.e., the one for which W_2 is such that:

$$(16) \qquad W_1 L^1(W_1) / [\bar{L} - L^2(W_2)] = W_2$$

or

(17) $$W_1 \, L^1(W_1) / \bar{L}_1 = W^2(\bar{L}_2) = W^2(\bar{L} - \bar{L}_1)$$

3. In the internal solution of the general model, it must be the case that:

(18) $$W_1 > W^*$$

or the wage restriction is not binding. And

(19) $$\bar{W}_1 = \gamma \, W_1 + (1 - \gamma) \, W^1(\bar{L}_1) > W_2$$

However, $\bar{W}_1$ may be larger (if $\partial \bar{W}_1 / \partial\gamma > 0$) or smaller (if $\partial \bar{W}_1 / \partial\gamma < 0$) than W*. Also, W_2 may be larger or smaller than W*; and it may be larger or smaller than $W^1(\bar{L}_1)$, i.e., we can have either:

(20) $$W_1 > \bar{W}_1 > W^1(\bar{L}_1) > W_2$$

or

(21) $$W_1 > \bar{W}_1 > W_2 > W^1(\bar{L}_1)$$

Proposition 1: With incomplete coverage in one sector and a completely uncovered sector and the internal equilibrium solutions described by:

$$[\gamma \, W_1 + (1 - \gamma) \, W^1(\bar{L}_1)] \; L^1[\gamma \, W_1 + (1 - \gamma) \, W^1(\bar{L}_1)] \, / \, \bar{L}_1 = W^2(\bar{L} - \bar{L}_1)$$

it must be the case that:

- $W_1 > \bar{W}_1 > W^1(\bar{L}_1) > W_2$

Or

- $W_1 > \bar{W}_1 > W_2 > W^1(\bar{L}_1)$

Of course, the expected value of the wage is equalized across sectors 1 and 2, but to have a chance at the high wage W_1, an individual must also risk ending up with – or rotatively get also - $W^1(\bar{L}_1)$. With the described equilibrium, this informal wage of the covered sector may fall below the wage of the uncovered sector itself.

4. If we have institutional wage in sector 2 but with complete law enforcement in this sector, expected wage in sector 2 will be:

$$(22) \qquad W_2^e = W_2\, L^2(W_2) / \bar{L}_2$$

Then, the equilibrium condition becomes:

$$(23) \qquad [\gamma\, W_1 + (1 - \gamma)\, W^1(\bar{L}_1)] \quad L^1[\gamma\, W_1 + (1 - \gamma)\, W^1(\bar{L}_1)] / \bar{L}_1 = = W_2\, L^2(W_2) / \bar{L}_2 = W_2\, L^2(W_2) / (\bar{L} - \bar{L}_1)$$

which solves for $\bar{L}_1$. In equilibrium,

$$(24) \qquad \bar{W}_1\, (1 - u_1) = W_2\, (1 - u_2)$$

If $\gamma = 0$, the model will yield the Harris-Todaro solution, i.e., the one for which W_1 is such that:

$$(25) \qquad W_2\, L^2(W_2) / \bar{L}_2 = W^1(\bar{L}_1) = W^1(\bar{L} - \bar{L}_2)$$

If $\gamma = 1$, the model will yield the Bhagwati-Hamada solution, i.e.,

(26) $$W_1 L^1(W_1) / L;\bar{\ }_1 = W_2 L^2(W_2) / L;\bar{\ }_2$$

We can look at the equilibrium solution as if the left hand-side was a weighted average of $W_1 L^1(\bar{\ };W_1) / L;\bar{\ }_1$ and $W^1(L;\bar{\ }_1) L^1(\bar{\ };W_1) / L;\bar{\ }_1$. As $W_1 > W^1(L;\bar{\ }_1)$, then it must be the case that:

(27) $$W^1(L;\bar{\ }_1) L^1(\bar{\ };W_1) / L;\bar{\ }_1 < W_2 L^2(W_2) / L;\bar{\ }_2 < W_1 L^1(\bar{\ };W_1) / L;\bar{\ }_1$$

If $\bar{\ };W_1 > W_2$, then, by the equilibrium condition, $L^1(\bar{\ };W_1) / L;\bar{\ }_1 < L^2(W_2) / L;\bar{\ }_2$. Dividing the inequality (27) by $L^1(\bar{\ };W_1) / L;\bar{\ }_1$:

(28) $$W^1(L;\bar{\ }_1) < W_2 [L^2(W_2) / L;\bar{\ }_2] / [L^1(\bar{\ };W_1) / L;\bar{\ }_1] < W_1$$

Well, $[L^2(W_2) / L;\bar{\ }_2] / [L^1(\bar{\ };W_1) / L;\bar{\ }_1] > 1$. Then

(29) $$W_2 < W_2 [L^2(W_2) / L;\bar{\ }_2] / [L^1(\bar{\ };W_1) / L;\bar{\ }_1] < W_1$$

Consider now that $\bar{\ };W_1 < W_2$; then, by the equilibrium condition, $L^1(\bar{\ };W_1) / L;\bar{\ }_1 > L^2(W_2) / L;\bar{\ }_2$. Dividing the inequality by $L^2(W_2) / L;\bar{\ }_2$:

(30) $$W^1(L;\bar{\ }_1) [L^1(\bar{\ };W_1)/L;\bar{\ }_1] / [L^2(W_2)/L;\bar{\ }_2] < W_2 < \\ < W_1[L^1(\bar{\ };W_1)/L;\bar{\ }_1] / [L^2(W_2)/L;\bar{\ }_2]$$

Well, $[L^{1}(\bar{\ };W_{1}) / L;\bar{\ }_{1}] / [L^{2}(W_{2}) / L;\bar{\ }_{2}] > 1$. Then

(31) $\quad W^{1}(L;\bar{\ }_{1}) < W^{1}(L;\bar{\ }_{1})\, [L^{1}(\bar{\ };W_{1}) / L;\bar{\ }_{1}] / [L^{2}(W_{2}) / L;\bar{\ }_{2}] < W_{2}$

Therefore, we can now have two possibilities for the internal solution:
If $W_2 > W_1$:

(32) $\quad W_{2} > W_{1} > \bar{\ };W_{1} > W^{1}(L;\bar{\ }_{1})$, $\quad L^{2}(W_{2}) / L;\bar{\ }_{2} < L^{1}(\bar{\ };W_{1}) / L;\bar{\ }_{1}$ and $u_2 > u_1$

If, $W_2 < W_1$, then, either:

(33) $\quad W_{1} > W_{2} > \bar{\ };W_{1} > W^{1}(L;\bar{\ }_{1})$, $\quad L^{2}(W_{2})/ L;\bar{\ }_{2} < L^{1}(\bar{\ };W_{1}) / L;\bar{\ }_{1}$ and $u_2 > u_1$

or

(34) $\quad W_{1} > \bar{\ };W_{1} > W_{2} > W^{1}(L;\bar{\ }_{1})$, $\quad L^{2}(W_{2})/ L;\bar{\ }_{2} > L^{1}(\bar{\ };W_{1}) / L;\bar{\ }_{1}$ and $u_2 < u_1$

or

(35) $\quad W_{1} > \bar{\ };W_{1} > W^{1}(L;\bar{\ }_{1}) > W_{2}$, $\quad L^{2}(W_{2})/ L;\bar{\ }_{2} < L^{1}(\bar{\ };W_{1}) / L;\bar{\ }_{1}$ and $u_2 < u_1$

With global coverage, i.e., $W_1 = W_2 = W$, it must be the case that

(36) $\quad W > \bar{\ };W_{1} > W^{1}(L;\bar{\ }_{1})$, $\quad L^{2}(W)/ L;\bar{\ }_{2} < L^{1}(\bar{\ };W_{1}) / L;\bar{\ }_{1}$ and $u_2 > u_1$

Proposition 2: With incomplete coverage in one sector and a completely covered sector and the internal equilibrium solutions described by:

$$[\gamma\, W_1 + (1 - \gamma)\, W^1(L\bar{;}_1)] \quad L^1[\gamma\, W_1 + (1 - \gamma)\, W^1(L\bar{;}_1)] / L\bar{;}_1 =$$

$$= W_2\, L^2(W_2) / (L\bar{;} - L\bar{;}_1)$$

2.1. In general:

. If $W_2 > W_1$, then:

$W_2 > W_1 > \bar{;}W_1 > W^1(L\bar{;}_1)$, $L^2(W_2) / L\bar{;}_2 < L^1(\bar{;}W_1) / L\bar{;}_1$ and $u_2 > u_1$

. If $W_2 < W_1$, then, either:

$W_1 > W_2 > \bar{;}W_1 > W^1(L\bar{;}_1)$, $L^2(W_2)/ L\bar{;}_2 < L^1(\bar{;}W_1) / L\bar{;}_1$ and $u_2 > u_1$

or

$W_1 > \bar{;}W_1 > W_2 > W^1(L\bar{;}_1)$, $L^2(W_2)/ L\bar{;}_2 > L^1(\bar{;}W_1) / L\bar{;}_1$ and $u_2 < u_1$

or

$W_1 > \bar{;}W_1 > W^1(L\bar{;}_1) > W_2$, $L^2(W_2)/ L\bar{;}_2 < L^1(\bar{;}W_1) / L\bar{;}_1$ and $u_2 < u_1$

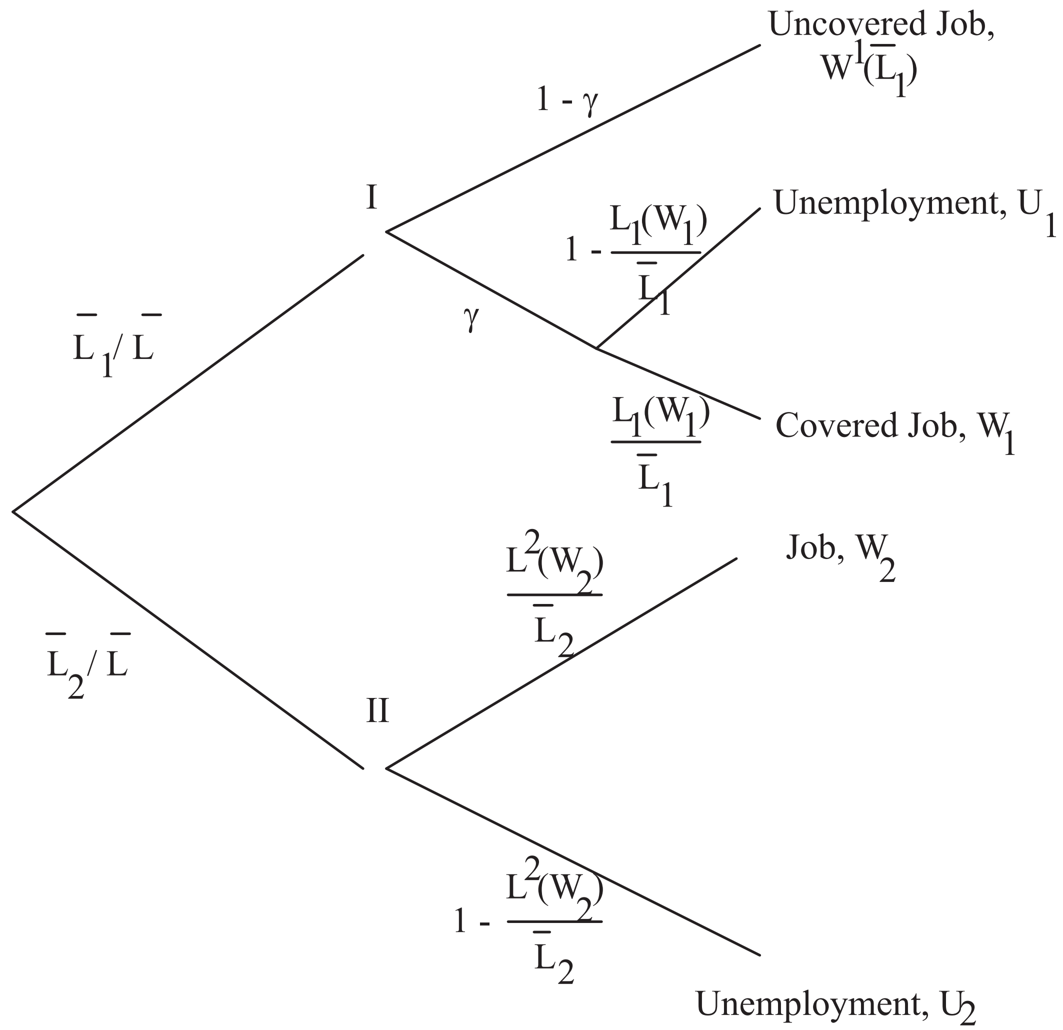
2.2. If $W_2 = W_1 = W$:

$W > \bar{;}W_1 > W^1(L\bar{;}_1)$, $L^2(W)/ L\bar{;}_2 < L^1(\bar{;}W_1) / L\bar{;}_1$ and $u_2 > u_1$

With both sectors being covered, if the institutional wage in the partially informal sector is not higher than in the other, the informal jobs' wage will fall below both. But if the opposite occurs, dependent on demands and on the relative distance between the institutional wages, a variety of relative magnitudes – as well as of the local unemployment rates – are compatible with (a unique) equilibrium.

iii. Partial Coverage Equilibrium: Type - B.

1. We can consider, instead of (8) and (9), that a proportion γ of first sector firms pays the institutional wage, employing people in accordance, and a proportion 1 - γ of the firms pay the marginal product of applicants, in such a way that dynamics may be represented by the tree below (I and II define the two main branches of the decision node):



Then, it is as if γ represented the proportion of unionized firms in sector 1. Employment in sector 1 will be:

$$L_1^e = \gamma \; L^1(W_1) + (1 - \gamma) \; \bar{L}_1 \tag{37}$$

Of course, the institutional wage coverage in the first sector will only be relevant if:

$$W^1(\bar{L}_1) < W_1 \tag{38}$$

Expected wage in sector 1 will be:

$$W_1^e = \gamma \, W_1 \, L^1(W_1) / \bar{L}_1 + (1 - \gamma) \, W^1(\bar{L}_1) \tag{39}$$

The wage bill in sector 1 is:

$$\gamma \, W_1 \, L^1(W_1) + (1 - \gamma) \, W^1(\bar{L}_1) \, \bar{L}_1 \tag{40}$$

From (37), we will observe in an interior solution that:

$$L^1(W_1) < L_1^e < \bar{L}_1 \tag{41}$$

The average wage paid to people employed in sector 1 will be:

$$\bar{W}_1 = [\gamma \, W_1 \, L^1(W_1) + (1 - \gamma) \, W^1(\bar{L}_1) \, \bar{L}_1] / L_1^e = \tag{42}$$

$$= [\gamma \, W_1 \, L^1(W_1) + (1 - \gamma) \, W^1(\bar{L}_1) \, \bar{L}_1] / [\gamma \, L^1(W_1) + (1 - \gamma) \; \bar{L}_1]$$

Then, we can look at $\bar{W}_1$ as a weighted average of W_1 and $W^1(\bar{L}_1)$ with weights $\gamma \, L^1(W_1) / L_1^e$ and $(1 - \gamma) \, \bar{L}_1 / L_1^e$ (that sum 1). Then:

(43) $$W^1(\bar{L}_1) < \bar{W}_1 < W_1$$

2. Consider that in sector 2 the wage is market determined. Equalization of expected wage in both sectors will yield the equilibrium condition:

(44) $$\gamma W_1 L^1(W_1) / \bar{L}_1 + (1 - \gamma) W^1(\bar{L}_1) = W^2(\bar{L}_2) = W^2(\bar{L} - \bar{L}_1)$$

When $\gamma = 0$, the model reproduces the free market (with no unemployment) equilibrium condition. When $\gamma = 1$, we have the Harris-Todaro model.

Looking at (44), in the interior solution $W^2(\bar{L}_2)$ is a weighted average of $W_1 L^1(W_1) / \bar{L}_1$ and $W^1(\bar{L}_1)$. Then, either:

(45) $$W^1(\bar{L}_1) < W^2(\bar{L}_2) < W_1 L^1(W_1)/ \bar{L}_1 < W_1$$

Or,

(46) $$W_1 > W^1(\bar{L}_1) > W^2(\bar{L}_2) > W_1 L^1(W_1)/ \bar{L}_1$$

This second case occurs when the institutional wage in the first sector is very high and $L^1(W_1) / \bar{L}_1$ is low.

For infinitesimal increases of W_1 around the unconstrained equilibrium, $W^1(\bar{L}_1)$ >(<) $W_1 L^1(W_1)/ \bar{L}_1$ iff $W^1(\bar{L}_1) \bar{L}_1$ >(<) $W_1 L^1(W_1)$; as W_1 is set at a higher level than $W^1(\bar{L}_1)$, that is expected to occur for a wage elasticity of demand in sector 1 - of $L^1(W_1)$ - larger (smaller) than 1 in absolute value. That is, $|\varepsilon^1| = -W^1(\bar{L}_1) / [W^1(\bar{L}_1)' \bar{L}_1] > (<) 1$.

Proposition 3: With incomplete coverage in one sector and a completely uncovered sector and the internal equilibrium solutions described by:

$$\gamma W_1 L^1(W_1) / \bar{L}_1 + (1 - \gamma) W^1(\bar{L}_1) = W^2(\bar{L}_2) = W^2(\bar{L} - \bar{L}_1)$$

we may have either that:

. $W^1(\bar{L}_1) < W^2(\bar{L}_2) < W_1 L^1(W_1)/ \bar{L}_1 < W_1$, expected if $|\varepsilon^1| < 1$

Or

. $W_1 > W^1(\bar{L}_1) > W^2(\bar{L}_2) > W_1 L^1(W_1) / \bar{L}_1$, expected if $|\varepsilon^1| > 1$

Interestingly, the possibilities differ from those of the equilibrium of Type – A. Now, the informal wage may be below that of the totally uncovered sector, but, in the opposite extreme, the expected value of the covered job wage may fall below all of the other employment situations.

3. Take now the case where sector 2 is subject to institutional rules. Then, the equilibrium condition becomes:

$$(47) \qquad \gamma W_1 L^1(W_1) / \bar{L}_1 + (1 - \gamma) W^1(\bar{L}_1) = W_2 L^2(W_2)/ \bar{L}_2$$

When $\gamma = 0$, the model reproduces the Harris-Todaro equilibrium condition. When $\gamma = 1$, we have the Bhagwati-Hamada model. γ may be interpreted as the degree of coverage in sector 1. Equilibrium expected wage in the economy is the expected wage in sector 2.

In the optimal solution, it must be the case that:

$$(48) \qquad W^1(\bar{L}_1) < W_1$$

for an interior solution to be observed - otherwise the institutional rule is not binding in the first sector. Given (47), in the optimal interior solution, the expected wage in the completely covered sector, 2 - and in the economy -, will lie between $W_1 L^1(W_1) / \bar{L}_1$ and $W^1(\bar{L}_1)$ - because it is a weighted average of the two -, but $W_1 L^1(W_1) / \bar{L}_1$ may be larger or smaller than $W^1(\bar{L}_1)$. That is, we may have that in the optimal solution either:

$$(49) \qquad W^1(\bar{L}_1) < W_2 L^2(W_2) / \bar{L}_2 < W_1 L^1(W_1) / \bar{L}_1 < W_1$$

and it must be the case that

$$(50) \qquad W^1(\bar{L}_1) < W_2.$$

Or,

$$(51) \qquad W_1 > W^1(\bar{L}_1) > W_2 L^2(W_2) / \bar{L}_2 > W_1 L^1(W_1) / \bar{L}_1$$

and then:

$$(52) \qquad W_2 > W_1 L^1(W_1) / \bar{L}_1$$

This second case occurs when the wage in the first sector is very high and $L^1(W_1)/ \bar{L}_1$ is low.

One can see that both situations are consistent with (47) and (48), and with W_2 larger or smaller than W_1.

Homogeneous coverage will guarantee $W_1 = W_2 = W$. The equilibrium condition becomes:

$$(53) \qquad \gamma W L^1(W) / \bar{L}_1 + (1 - \gamma) W^1(\bar{L}_1) = W L^2(W) / \bar{L}_2$$

In this case, due to (48), it will always be the case that

$$W > \bar{W}_1 \tag{54}$$

and therefore, from (53)

$$L^2(W) / \bar{L}_2 < L_1^e / \bar{L}_1 \quad \text{or} \quad u_2 > u_1 \tag{55}$$

Again, we may have, either of the two cases (49) and (51): either (49) holds

$$W^1(\bar{L}_1) < W L^2(W) / \bar{L}_2 < W L^1(W) / \bar{L}_1 < W \tag{56}$$

and, then, necessarily, using (55):

$$L^2(W) / \bar{L}_2 < L^1(W) / \bar{L}_1 < L_1^e / \bar{L}_1 \tag{57}$$

Or (51) holds:

$$W > W^1(\bar{L}_1) > W L^2(W) / \bar{L}_2 > W L^1(W) / \bar{L}_1 \tag{58}$$

which implies, with (55) that:

$$L_1^e / \bar{L}_1 > L^2(W) / \bar{L}_2 > L^1(W) / \bar{L}_1 \tag{59}$$

Proposition 4: With incomplete coverage in one sector and a completely covered sector and the internal equilibrium solutions described by:

$$\gamma W_1 L^1(W_1) / \bar{L}_1 + (1 - \gamma) W^1(\bar{L}_1) = W_2 L^2(W_2) / \bar{L}_2$$

4.1. In the general case, we may have either that:

. $W^1(\bar{L}_1) < W_2 \, L^2(W_2)/ \bar{L}_2 < W_1 \, L^1(W_1)/ \bar{L}_1 < W_1$ and $W^1(\bar{L}_1) < W_2$

Or

. $W_1 > W^1(\bar{L}_1) > W_2 L^2(W_2)/\bar{L}_2 > W_1 L^1(W_1)/\bar{L}_1$ and $W_2 > W_1 L^1(W_1)/\bar{L}_1$

4.2. If $W_2 = W_1 = W$, either:

. $W^1(\bar{L}_1) < W \, L^2(W) / \bar{L}_2 < W \, L^1(W) / \bar{L}_1 < W$ and, then

$$L^2(W) / \bar{L}_2 < L^1(W) / \bar{L}_1 < L_1^e / \bar{L}_1$$

Or

. $W > W^1(\bar{L}_1) > W \, L^2(W) / \bar{L}_2 > W \, L^1(W) / \bar{L}_1$ and, then

$$L_1^e / \bar{L}_1 > L^2(W) / \bar{L}_2 > L^1(W) / \bar{L}_1$$

With both sectors subject to some fixed wage regulation, in the new equilibrium, $W^1(\bar{L}_1)$ may be below or above the expected value of earnings in the completely covered sector – and, but not simultaneously, of the expected wage of a covered job in its own vicinity, i.e., of $W_1 \, L^1(W_1)/ \bar{L}_1$.

iv. Some Comments to the Two Structures, Type - A and Type - B.

. The two structures are such that corner values for γ yield, respectively, the free market wage (when $\gamma = 0$) and the expected wage in a completely covered sector (when $\gamma = 1$) for the expected wage in sector 1. However, they originate different features of the equilibrium outcome. Looking at the equilibrium conditions and expected wage formation according to each decision tree, we can interpret the applicability of either scenario as follows:

In Type-A dynamics, the same firm would hire individuals both at the institutional wage (in a proportion γ) and at the implied free market wage that would accommodate all population in sector 1 (in a proportion 1 - γ); therefore demand in the partially covered sector is a function of average wage.

Type-B would be more consistent with a higher degree of segregation between formal and informal firms. The formal firms, in a proportion γ of the economy, would respond according to the labor demand argument that they face, the institutional wage; the informal firms would pay the market wage that would employ all population in sector 1.

Implicitly, and if γ is related to probability of detection (and subsequent law enforcement) of non-compliance, we could link type-A solutions to cases where workers are sampled by the legal system, whereas type-B would correspond to firms being the subjects of the inspection sampling pool (the employment decision being made after the firm having been - or not - the target of inspection) [3]. Even if Type-B would seem, in that sense, more adequate, that does not have to be the case: if detection comes from workers' complaint, or previous employment decisions by the firm are binding after detection, Type-A could be more appropriate.

If one interprets γ as degree of unionisation in sector 1, the two scenarios confront unionisation by, say, profession – Type A - with firm-level unionisation – Type B.

. We always model a partially covered sector with some unemployment generating ability. That did (does) not have to be the case – and we might have the informal wage decreasing till all the labor force in the sector is employed.

Technically, that would advise the departure from two different labor demands of the formal and informal sectors and the homogeneity – and randomness, on which the stochastic equilibrium relies - of employment chances in the partially covered sector would be lost. Otherwise, other correspondences had to be devised between the inverse demand and the appropriate argument to define the informal wage which would eventually complicate the analysis without gains in further insights.

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3 Type-B can be seen as in line with the assumptions about firm behavior of Ashenfelter and Smith (1979) and Grenier (1982), if the market wage $W^{1}(\bar{\ };L_{1})$ in our equilibrium corresponds to the free market wage the firm faces in their structures - with full payment of the minimum wage if detection occurs. Chang and Ehrlich (1985) define a profit function on mean wage; hence, his assumptions would be in line with our type-A.

III. Degree of Coverage and Equilibrium in H.-T. Dualistic Models.

This section assesses the impact of a change in degree of coverage in a primary sector when a secondary sector is competitive – in sub-section i for Type-A equilibria, in ii for Type-B. Additionally, a contrast with the consequences of altering the formal wage is provided.

i. Type - A Model.

1. Consider the equilibrium described in Proposition 1. Take a change in the degree of coverage in sector/region 1. We have that:

(60) $\partial L;\bar{}_1/\partial\gamma = -\,\partial L;\bar{}_2/\partial\gamma = [W_1 - W^1(L;\bar{}_1)]\,[L^1(\bar{};W_1) + \bar{};W_1\, L^1(\bar{};W_1)']\,/$

$/\ \{W^2(L;\bar{}_2) - W^2(L;\bar{}_2)`\ L;\bar{}_1 - (1-\gamma)\, W^1(L;\bar{}_1)`\ [L^1(\bar{};W_1) + \bar{};W_1\, L^1(\bar{};W_1)']\} =$

$= [W_1 - W^1(L;\bar{}_1)]\ /$

$/\ \{[W^2(L;\bar{}_2) - W^2(L;\bar{}_2)`\ L;\bar{}_1] \,/\, [L^1(\bar{};W_1) + \bar{};W_1\, L^1(\bar{};W_1)'] - (1-\gamma)\, W^1(L;\bar{}_1)`\ \}$

It is a sufficient condition for this to be positive that:

(61) $[L^1(\bar{};W_1) + \bar{};W_1\, L^1(\bar{};W_1)'] > 0$, or $|\,\varepsilon^1\,| = -\,\bar{};W_1\, L^1(\bar{};W_1)' \,/\, L^1(\bar{};W_1) < 1$

However, this is not a necessary condition.

Being $W^1(L;\bar{}_1)`\ L^1(\bar{};W_1)'$ close to 1, and if

(62) $W^2(\bar{L}_2) = \bar{W}_1 (1 - u_1) > (1 - \gamma) \bar{W}_1$ i.e., $u_1 < \gamma$ or $U < \gamma \bar{L}_1$

then, $\partial \bar{L}_1/\partial\gamma < 0$ if $| \varepsilon^1 | > 1$.

Expected wage in the economy - equal to $W^2(\bar{L}_2)$ -, total wage bill - which will equal $W^2(\bar{L}_2)\, \bar{L}$ - and wage bill in sector 1 - equal to $W^2(\bar{L}_2)\, \bar{L}_1$ - will move in the same direction as $\bar{L}_1$. The wage bill in sector 2 will move in the same direction as $W^2(\bar{L}_2)$ - and of $\bar{L}_1$ - iff

$$(63) \qquad | \varepsilon^2 | = - W^2(\bar{L}_2) / [W^2(\bar{L}_2)'\, \bar{L}_2] < 1$$

If $| \varepsilon^2 | > 1$, it will move in the same direction as $\bar{L}_2$.

As for mean wage in first sector:

$$(64) \qquad \partial \bar{W}_1/\partial\gamma = [W_1 - W^1(\bar{L}_1)] + (1 - \gamma)\, W^1(\bar{L}_1)'\, \partial \bar{L}_1/\partial\gamma =$$

$$= [W_1 - W^1(\bar{L}_1)]\, [W^2(\bar{L}_2) - W^2(\bar{L}_2)'\, \bar{L}_1] \,/$$

$$/ \{[W^2(\bar{L}_2) - W^2(\bar{L}_2)'\, \bar{L}_1] - (1 - \gamma)\, W^1(\bar{L}_1)'\, [L^1(\bar{W}_1) + \bar{W}_1\, L^1(\bar{W}_1)']\}$$

The numerator is always positive. A sufficient condition for the denominator to be also positive is (61). Or, being $W^1(\bar{L}_1)'\, L^1(\bar{W}_1)'$ close to 1, that (62) holds, i.e., if $u_1 < \gamma$, mean wage in sector 1 increases with coverage. Neither are necessary conditions. Local employment will move in the opposite direction. As for unemployment in region 1 - and therefore, total unemployment:

(65) $\partial U_1/\partial\gamma = \partial L;\bar{}_1/\partial\gamma - L^1(\bar{};W_1)' \, \partial W;\bar{}_1/\partial\gamma =$

$= [W_1 - W^1(L;\bar{}_1)] \; \{L^1(\bar{};W_1) + \bar{};W_1 \; L^1(\bar{};W_1)' - L^1(\bar{};W_1)'[W^2(L;\bar{}_2) -$

$W^2(L;\bar{}_2)` \, L;\bar{}_1]\}/$

$/ \; \{[W^2(L;\bar{}_2) - W^2(L;\bar{}_2)` \, L;\bar{}_1] - (1 - \gamma) \, W^1(L;\bar{}_1)` \, [L^1(\bar{};W_1) + \bar{};W_1$

$L^1(\bar{};W_1)']\} =$

$= [W_1 - W^1(L;\bar{}_1)] \; \{L^1(\bar{};W_1) - L^1(\bar{};W_1)' \, [W^2(L;\bar{}_2) - W^2(L;\bar{}_2)` \, L;\bar{}$

$_1 - \bar{};W_1]\} /$

$/ \; \{[W^2(L;\bar{}_2) - W^2(L;\bar{}_2)` \, L;\bar{}_1] - (1 - \gamma) \, W^1(L;\bar{}_1)` \, [L^1(\bar{};W_1) + \bar{};W_1$

$L^1(\bar{};W_1)']\}$

This will be positive if $| \varepsilon^1 | < 1$. Being $L^1(\bar{};W_1)' \, W^1(L;\bar{}_1)`$ close to 1, we can write:

(66) $\partial U_1/\partial\gamma = - \, L^1(\bar{};W_1)' \; [W_1 - W^1(L;\bar{}_1)]$

$\{[W^2(L;\bar{}_2) - W^2(L;\bar{}_2)` \; L;\bar{}_1] - W^1(L;\bar{}_1)` \; [L^1(\bar{};W_1) + \bar{};W_1$

$L^1(\bar{};W_1)']\} /$

$/ \; \{[W^2(L;\bar{}_2) - W^2(L;\bar{}_2)` \, L;\bar{}_1] - (1 - \gamma) \, W^1(L;\bar{}_1)` \, [L^1(\bar{};W_1) + \bar{};W_1$

$L^1(\bar{};W_1)']\}$

When $| \varepsilon^1 | > 1$, the denominator is always larger than the numerator, because only a proportion $(1 - \gamma)$ of what is subtracted in the numerator is there a negative term. Then, if the denominator is negative, the denominator will be negative and $\partial U_1/\partial\gamma > 0$.

Also, when the numerator is positive, the denominator will be positive and, again, $\partial U_1/\partial\gamma > 0$.

Then, (65) will only be negative when $|\,\varepsilon^1\,| > 1$ and if de numerator is negative and the denominator is positive: for instance, if (62) holds and the numerator is negative:

(67) $$|\,\varepsilon^1\,| \ \{1 - [L^1(\bar{\ };W_1) / \bar{L}_1 + (1 / |\,\varepsilon^2\,|)\, L^1(\bar{\ };W_1) / \bar{L}_2]\} > 1$$

Total employment will move in the opposite direction of U_1.

Finally, for local unemployment rate:

(68) $$\partial u_1/\partial\gamma = \{L^1(\bar{\ };W_1)\ \partial\bar{L}_1/\partial\gamma \ - \ \bar{L}_1\ L^1(\bar{\ };W_1)'\ \partial\bar{W}_1/\partial\gamma\ ] / \bar{L}_1^{\,2} =$$

$$= (1/ \bar{L}_1^{\,2})\ [W_1 - W^1(\bar{L}_1)]\ \{L^1(\bar{\ };W_1)^2 + L^1(\bar{\ };W_1)'\ W^2(\bar{L}_2)'\ \bar{L}_1^{\,2}] /$$

$$/ \ \{[W^2(\bar{L}_2) - W^2(\bar{L}_2)'\ \bar{L}_1] - (1 - \gamma)\ W^1(\bar{L}_1)'\ [L^1(\bar{\ };W_1) + \bar{\ };W_1\ L^1(\bar{\ };W_1)']\}$$

Then $\partial u_1/\partial\gamma$ has the same sign as $\partial\bar{W}_1/\partial\gamma$.

2. Consider an alternative policy measure: increase the wage in the partially covered sector, W_1.

(69) $$\partial\bar{L}_1/\partial W_1 = - \partial\bar{L}_2/\partial W_1 = \gamma\ [L^1(\bar{\ };W_1) + \bar{\ };W_1\ L^1(\bar{\ };W_1)'] /$$

$$/ \ \{W^2(\bar{L}_2) - W^2(\bar{L}_2)'\ \bar{L}_1 - (1 - \gamma)\ W^1(\bar{L}_1)'\ [L^1(\bar{\ };W_1) + \bar{\ };W_1\ L^1(\bar{\ };W_1)']\} =$$

$$= \gamma / \{[W^2(L;\bar{\ }_2) - W^2(L;\bar{\ }_2)‘\ L;\bar{\ }_1] / [L^1(\bar{\ };W_1) + \bar{\ };W_1\ L^1(\bar{\ };W_1)'] - (1 - \gamma)\ W^1(L;\bar{\ }_1)‘\}$$

One can see that an increase in the wage has the same effect as increasing coverage.

Proposition 5: With incomplete coverage in one sector and a completely uncovered sector and the internal equilibrium solutions described by:

$$[\gamma\ W_1 + (1 - \gamma)\ W^1(L;\bar{\ }_1)] \quad L^1[\gamma\ W_1 + (1 - \gamma)\ W^1(L;\bar{\ }_1)] / L;\bar{\ }_1 = W^2(L;\bar{\ } - L;\bar{\ }_1)$$

5.1. If $| \varepsilon^1 | < 1$, population will flow to the partially covered sector when coverage increases. If $| \varepsilon^1 | > 1$ and $u_1 < \gamma$, population will flow away from the partially covered sector when coverage increases.

5.2. If $| \varepsilon^1 | < 1$ or $u_1 < \gamma$, mean wage in the partially covered sector will increase with coverage. Local unemployment rate will move in the same direction.

5.3. Wage in the uncovered sector, expected wage in the economy and total wage bill will move in the same direction as population in the partially covered sector. Wage bill in the uncovered sector will move in the same direction as population in the partially covered sector iff $| \varepsilon^2 | < 1$.

5.4. An increase in the partially covered sector institutional wage has the same type of effect on the population flows as an increase in the degree of coverage.

ii. Type - B Model.

1. Consider the equilibrium described in Proposition 3. Take a change in the degree of coverage in sector/region 1. We have that:

$$(70) \qquad \partial \bar{L}_1/\partial\gamma = - \partial \bar{L}_2/\partial\gamma = [W_1 L_1(W_1) - \bar{L}_1 W^1(\bar{L}_1)] / \{[W^2(\bar{L}_2) - W^2(\bar{L}_2)' \bar{L}_1] - (1 - \gamma) [W^1(\bar{L}_1) + \bar{L}_1 W^1(\bar{L}_1)']\}$$

The denominator of the expression is always positive, once $W^2(\bar{L}_2) = \gamma W_1 L_1(W_1) / \bar{L}_1 + (1 - \gamma) W^1(\bar{L}_1) > (1 - \gamma) W^1(\bar{L}_1)$ for any interior solution.

We may have that in the optimal solution either (45) occurs and in this case the numerator of (70) is positive - i.e., an increase of the degree of coverage in the first sector attracts supply to this sector.

Or (46) holds, and then the numerator is negative. Intuitively, this means that chances of being employed become so low in the first sector when coverage increases, that people prefer to go to the second sector.

Expected wage in the economy and total wage bill - equal to $W^2(\bar{L}_2) \bar{L}$ -, and wage bill in sector 1 - equal to $W^2(\bar{L}_2) \bar{L}_1$ - in the economy(ies) will move in the same direction as $\bar{L}_1$. The wage bill in sector 2 will move in the same direction as $W^2(\bar{L}_2)$ - and of $\bar{L}_1$ - iff

$$(71) \qquad | \varepsilon^2 | = - W^2(\bar{L}_2) / [W^2(\bar{L}_2)' \bar{L}_2] < 1$$

If $| \varepsilon^2 | > 1$, it will move in the same direction as $\bar{L}_2$.

Employment in sector 1 will move according to:

$$(72)\qquad \partial L_1^e/\partial\gamma = -[\bar{L}_1 - L^1(W_1)] + (1-\gamma)\ \partial\bar{L}_1/\partial\gamma =$$

$$= \{[\bar{L}_1 - L^1(W_1)]\,\bar{L}_1\,[W^2(\bar{L}_2)' + (1-\gamma)\,W^1(\bar{L}_1)'] -$$

$$- W^2(\bar{L}_2)\,[\bar{L}_1 - L^1(W_1)] + (1-\gamma)\,[W_1 - W^1(\bar{L}_1)]\,L^1(W_1)\} \;/$$

$$/\ \{[W^2(\bar{L}_2) - W^2(\bar{L}_2)'\,\bar{L}_1] - (1-\gamma)\,[W^1(\bar{L}_1) + \bar{L}_1\,W^1(\bar{L}_1)']\}$$

(46) is a sufficient condition for this to be negative. Or that $\gamma = 1$. But neither are necessary.

As for unemployment, $U = U_1 = \bar{L}_1 - L_1^e = \gamma\,[\bar{L}_1 - L^1(W_1)]$:

$$(73)\qquad \partial U/\partial\gamma = [\bar{L}_1 - L^1(W_1)] + \gamma\ \partial\bar{L}_1/\partial\gamma =$$

$$= \{-[\bar{L}_1 - L^1(W_1)]\,\bar{L}_1\,[W^2(\bar{L}_2)' + (1-\gamma)\,W^1(\bar{L}_1)'] +$$

$$+ [\bar{L}_1 - L^1(W_1)]\,[W^2(\bar{L}_2) - (1-\gamma)\,W^1(\bar{L}_1)] + \gamma\,[W_1\,L_1(W_1) - \bar{L}_1\,W^1(\bar{L}_1)]\} \;/$$

$$/\ \{[W^2(\bar{L}_2) - W^2(\bar{L}_2)'\,\bar{L}_1] - (1-\gamma)\,[W^1(\bar{L}_1) + \bar{L}_1\,W^1(\bar{L}_1)']\}$$

This will likely be positive. A sufficient condition for it to be positive is that $\gamma = 0$, or that (45) holds. As for local unemployment rate in the partially covered sector, $u_1 = \gamma\,[\bar{L}_1 - L^1(W_1)]/\bar{L}_1$:

(74) $$\partial u_1/\partial\gamma = [\bar{L}_1\{[\bar{L}_1 - L^1(W_1)] + \gamma\, \partial\bar{L}_1/\partial\gamma\} - \partial\bar{L}_1/\partial\gamma\ \gamma\, [\bar{L}_1 - L^1(W_1)]]/\bar{L}_1^2 =$$

$$= \{\bar{L}_1\, [\bar{L}_1 - L^1(W_1)] + \gamma\, L^1(W_1)\, \partial\bar{L}_1/\partial\gamma\} / \bar{L}_1^2 =$$

$$= \{-[\bar{L}_1 - L^1(W_1)]\, [W^2(\bar{L}_2)' + (1-\gamma)\, W^1(\bar{L}_1)'] +$$

$$+ \gamma\, [W_1 - W^1(\bar{L}_1)]\, L^1(W_1) / \bar{L}_1\} /$$

$$/ \{[W^2(\bar{L}_2) - W^2(\bar{L}_2)'\, \bar{L}_1] - (1-\gamma)\, [W^1(\bar{L}_1) + \bar{L}_1\, W^1(\bar{L}_1)']\} > 0$$

Given the definitions and the equilibrium condition, $\bar{W}_1\, L_1^e = \gamma\, W_1\, L^1(W_1) + (1-\gamma)\, W^1(\bar{L}_1)\, \bar{L}_1 = W^2(\bar{L}_2)\, \bar{L}_1$. Hence, mean wage paid to first sector employees, $\bar{W}_1 = W^2(\bar{L}_2)\, \bar{L}_1 / [\gamma\, L^1(W_1) + (1-\gamma)\, \bar{L}_1] = W^2(\bar{L}_2)\, \bar{L}_1 / L_1^e$, and:

(75) $$\partial\bar{W}_1/\partial\gamma = \{L_1^e\, \partial[W^2(\bar{L}_2)\, \bar{L}_1]/\partial\gamma - W^2(\bar{L}_2)\, \bar{L}_1\, \partial L_1^e/\partial\gamma\} / L_1^{e\,2} =$$

$$= (1/L_1^{e\,2})\, \{ [\bar{L}_1 - L^1(W_1)]\, W^2(\bar{L}_2)\, \bar{L}_1 +$$

$$+ \{[W^2(\bar{L}_2) - W^2(\bar{L}_2)'\, \bar{L}_1]\, L_1^e - (1-\gamma)\, W^2(\bar{L}_2)\, \bar{L}_1\}\, \partial\bar{L}_1/\partial\gamma\} =$$

$$= (1/L_1^{e\,2})\, \{W^2(\bar{L}_2)\, \bar{L}_1\, \gamma\, L^1(W_1)\, [W_1 - W^1(\bar{L}_1)]\} -$$

$$- \bar{L}_1^2\, [W^2(\bar{L}_2)'\, \{\gamma\, L^1(W_1)\, [W_1 - W^1(\bar{L}_1)] + (1-\gamma)\, W^1(\bar{L}_1)\, [\bar{L}_1 - L^1(W_1)]\} +$$

$$+ (1 - \gamma)\, W^2(\bar{L}_2)\, W^1(\bar{L}_1)' \,[\bar{L}_1 - L^1(W_1)] \,] \,/$$

$$/ \,\{[W^2(\bar{L}_2) - W^2(\bar{L}_2)'\, \bar{L}_1] - (1 - \gamma)\, [W^1(\bar{L}_1) + \bar{L}_1\, W^1(\bar{L}_1)'] \} > 0$$

2. Consider an alternative measure: increase the wage in the partially covered sector, W_1.

$$\partial \bar{L}_1 / \partial W_1 = - \partial \bar{L}_2 / \partial W_1 = \gamma\, [L^1(W_1) + W_1\, L^1(W_1)'] \,/ \tag{76}$$

$$/ \,\{[W^2(\bar{L}_2) - W^2(\bar{L}_2)'\, \bar{L}_1] - (1 - \gamma)\, [W^1(\bar{L}_1) + \bar{L}_1\, W^1(\bar{L}_1)'] \}$$

One can see that an increase in the wage has now a different effect of increasing coverage. Now, people will flow to the covered sector with the covered sector wage iff $| \varepsilon^1 | = - W_1\, L^1(W_1)' / L^1(W_1) < 1$.

Proposition 6: With incomplete coverage in one sector and a completely uncovered sector and the internal equilibrium solutions described by:

$$\gamma\, W_1\, L^1(W_1) / \bar{L}_1 + (1 - \gamma)\, W^1(\bar{L}_1) = W^2(\bar{L}_2) = W^2(\bar{L} - \bar{L}_1)$$

6.1. If $W^1(\bar{L}_1) < W^2(\bar{L}_2)$, population will flow to the partially covered sector when coverage increases. If $W^1(\bar{L}_1) > W^2(\bar{L}_2)$, population will flow away from the partially covered sector when coverage increases.

6.2. Local unemployment rate and mean wage paid to employees in the partially covered sector will always increase with the degree of coverage.

6.3. Wage in the uncovered sector, expected wage in the economy and total wage bill will move in the same direction as population in the partially covered sector. Wage bill in the uncovered sector will move in the same direction as population in the partially covered sector iff | ε^2 | < 1.

6.4. An increase in the partially covered sector institutional wage will lead to an increase of population in that sector iff | ε^1 | < 1.

IV. Degree of Coverage and Equilibrium in B.-H. Dualistic Models.

This section reproduces the exercises of section III for the Type-A and B generalizations of the B.-H. scenario – that is, we assume that the alien sector to the partially informal one is completely institutionalized.

i. Type - A Model.

1. Consider the equilibrium described in Proposition 2. Take a change in the degree of coverage in sector/region 1. We have that:

$$(77)\qquad \partial L;^{-}{}_{1}/\partial\gamma = - \partial L;^{-}{}_{2}/\partial\gamma = L;^{-}{}_{2}\,[W_1 - W^1(L;^{-}{}_1)]\,[L^1(^{-};W_1) + {}^{-};W_1\, L^1(^{-};W_1)']\,/$$

$$/\ \{^{-};W_1\, L^1(^{-};W_1) + W_2\, L_2 - (1-\gamma)\ L;^{-}{}_{2}\, W^1(L;^{-}{}_1)'\ [L^1(^{-};W_1) + {}^{-};W_1\, L^1(^{-};W_1)']\} =$$

$$= L;^{-}{}_{2}\,[W_1 - W^1(L;^{-}{}_1)]\,/$$

$$/\ \{[^{-};W_1\, L^1(^{-};W_1) + W_2\, L_2]\,/\,[L^1(^{-};W_1) + {}^{-};W_1\, L^1(^{-};W_1)'] - (1-\gamma)\ L;^{-}{}_{2}\, W^1(L;^{-}{}_1)'\ \}$$

It is a sufficient condition for this to be positive, and expected wage, total wage bill, and wage bill in sector 1 - once wage bill in sector 2 is fixed - in the economy(ies) to increase with coverage, that:

$$(78)\quad [L^1(^{-};W_1) + {}^{-};W_1\, L^1(^{-};W_1)'] > 0\ ,\quad \text{i.e.,}\quad |\,\varepsilon^1\,| = - {}^{-};W_1\, L^1(^{-};W_1)'/L^1(^{-};W_1) < 1$$

However, this is not a necessary condition. When $|\varepsilon^1| > 1$, and being $L^1(\bar{W}_1)'\, W^1(\bar{L}_1)'$ close to 1, $\partial \bar{L}_1/\partial\gamma < 0$ if:

$$(79) \qquad 1 - u_1 = L^1(\bar{W}_1) / \bar{L}_1 \; > \; (1-\gamma)\; \bar{L}_2 / \bar{L} \quad \text{i.e., } u_1 < (\bar{L}_1 + \gamma\, \bar{L}_2) / \bar{L}$$

As for mean wage in first sector:

$$(80) \qquad \partial \bar{W}_1/\partial\gamma \;=\; [W_1 - W^1(\bar{L}_1)] \;+ (1-\gamma)\; W^1(\bar{L}_1)'\, \partial \bar{L}_1/\partial\gamma \;=$$

$$= \; [W_1 - W^1(\bar{L}_1)]\; [\bar{W}_1\, L^1(\bar{W}_1) + W_2\, L_2] \; /$$

$$/ \; \{\bar{W}_1\, L^1(\bar{W}_1) + W_2\, L_2 - (1-\gamma)\; \bar{L}_2\, W^1(\bar{L}_1)'\, [L^1(\bar{W}_1) + \bar{W}_1\, L^1(\bar{W}_1)']\}$$

If either (78) holds, or, in the equilibrium, being $W^1(\bar{L}_1)'\, L^1(\bar{W}_1)'$ close to 1, (79) holds, then mean wage in sector 1 increases with coverage. Neither are necessary conditions. Employment (local in sector 1 and total) will move in the opposite direction. Local unemployment, in case $L^1(\bar{W}_1)'\, W^1(\bar{L}_1)'$ is close to 1:

$$(81) \qquad \partial U_1/\partial\gamma \;= \partial \bar{L}_1/\partial\gamma - L^1(\bar{W}_1)'\, \partial \bar{W}_1/\partial\gamma \;=\; [W_1 - W^1(\bar{L}_1)]$$

$$\{\bar{L}_2\, L^1(\bar{W}_1) \; - L^1(\bar{W}_1)'\, [\bar{W}_1\, L^1(\bar{W}_1) + W_2\, L_2 - \bar{W}_1\, \bar{L}_2]\} \; /$$

$$/ \; \{\bar{W}_1\, L^1(\bar{W}_1) + W_2\, L_2 - (1-\gamma)\; \bar{L}_2\, W^1(\bar{L}_1)'\, [L^1(\bar{W}_1) + \bar{W}_1\, L^1(\bar{W}_1)']\}$$

This will be positive if $|\varepsilon^1| < 1$; or if $|\varepsilon^1| > 1$, $W^1(L;^{-}_{1})$‘ $L^1(^{-};W_1)'$ is close to 1 and:

$$(82) \qquad L^1(^{-};W_1) / L;^{-}_{1} > L;^{-}_{2} / L;^{-} \quad \text{i.e.,} \quad u_1 < L;^{-}_{1} / L;^{-}$$

Notice that if (82) holds, (79) will hold necessarily.

Finally, for local unemployment rate:

$$(83) \qquad \partial u_1/\partial\gamma = \{L^1(^{-};W_1)\, \partial L;^{-}_{1}/\partial\gamma - L;^{-}_{1}\, L^1(^{-};W_1)'\, \partial^{-};W_1/\partial\gamma\,] / L;^{-2}_{1} =$$

$$= \{[W_1 - W^1(L;^{-}_{1})]\, L^1(^{-};W_1) / L;^{-2}_{1}\}\ \{L^1(^{-};W_1)\, L;^{-}_{2} - L^1(^{-};W_1)'\ ^{-};W_1\, L;^{-}_{1}] /$$

$$/ \{^{-};W_1\, L^1(^{-};W_1) + W_2 L_2 - (1-\gamma)\ L;^{-}_{2}\, W^1(L;^{-}_{1})‘\, [L^1(^{-};W_1) + ^{-};W_1\, L^1(^{-};W_1)']\}$$

This will increase with coverage if elasticity of demand in sector 1 is lower than 1 (in absolute value); or, being $W^1(L;^{-}_{1})$‘ $L^1(^{-};W_1)'$ close to one, if in the optimal solution (79) holds.

2. With homogeneous wages - i.e., $W_1 = W_2 = W$ - the conclusions will not be altered.

3. Consider a change in wages. Then:

$$(84) \qquad \partial L;^{-}_{1}/\partial W_1 = -\,\partial L;^{-}_{2}/\partial W_1 = \gamma\ L;^{-}_{2}\, [L^1(^{-};W_1) + ^{-};W_1\, L^1(^{-};W_1)'] /$$

$/ \{ ^{-};W_1 \, L^1(^{-};W_1) + W_2 \, L_2 - (1 - \gamma) \; L;^{-}_{2} \, W^1(L;^{-}_{1})` \, [L^1(^{-};W_1) + ^{-};W_1$

$L^1(^{-};W_1)']\}$

We can see that it will be positive or negative under the same conditions as $\partial L;^{-}_{1}/\partial\gamma$ is: If $| \varepsilon^1 | < 1$, $\partial L;^{-}_{1}/\partial W_1 > 0$. When $| \varepsilon^1 | > 1$, and being $L^1(^{-};W_1)' \, W^1(L;^{-}_{1})`$ close to 1, $\partial L;^{-}_{1}/\partial W_1 < 0$ if (79) holds. Considering a change in the other sector's wage:

$$(85) \quad \partial L;^{-}_{1}/\partial W_2 = - \partial L;^{-}_{2}/\partial W_2 = - L;^{-}_{1} \, [L^2(W_2) + W_2 \, L^2(W_2)'] \, /$$

$$/ \{ ^{-};W_1 \, L^1(^{-};W_1) + W_2 \, L_2 - (1 - \gamma) \; L;^{-}_{2} \, W^1(L;^{-}_{1})` \, [L^1(^{-};W_1) + ^{-};W_1$$

$$L^1(^{-};W_1)']\}$$

When $| \varepsilon^1 | < 1$; or $| \varepsilon^1 | > 1$ and (79) holds: $\partial L;^{-}_{1}/\partial W_2 > 0$ iff $| \varepsilon^2 | > 1$ and $\partial L;^{-}_{1}/\partial W_2 < 0$ iff $| \varepsilon^2 | < 1$.

As for a general wage increase in case of homogeneous coverage:

$$(86) \quad \partial L;^{-}_{1}/\partial W = - \partial L;^{-}_{2}/\partial W = \{\gamma \; L;^{-}_{2} \, [L^1(^{-};W_1) + ^{-};W_1 \, L^1(^{-};W_1)'] -$$

$$- \; L;^{-}_{1} \, [L^2(W_2) + W_2 \, L^2(W_2)'] \, \} \, /$$

$$/ \{ ^{-};W_1 \, L^1(^{-};W_1) + W_2 \, L_2 - (1 - \gamma) \; L;^{-}_{2} \, W^1(L;^{-}_{1})` \, [L^1(^{-};W_1) + ^{-};W_1$$

$$L^1(^{-};W_1)']\}$$

$\bar{L}_1$ will decrease with the institutional wage if elasticity of demand in sector 1 is lower than 1 (in absolute value); or, being $W^1(\bar{L}_1)' \, L^1(\bar{W}_1)'$ close to one, if in the optimal solution (79) holds; and:

$$(87) \qquad \gamma L^1(\bar{W}_1) / \bar{L}_1 (1 - |\varepsilon^1|) < L^2(W) / \bar{L}_2 (1 - |\varepsilon^2|)$$

or

$$(88) \qquad \gamma W \ (1 - |\varepsilon^1|) < \bar{W}_1 \ (1 - |\varepsilon^2|)$$

As $\gamma W < \bar{W}_1$, this means that if $|\varepsilon^2| < |\varepsilon^1| < 1$, then $\partial \bar{L}_1 / \partial W < 0$. If $W^1(\bar{L}_1)' \, L^1(\bar{W}_1)'$ is close to one and in the optimal solution (79) holds, and if $|\varepsilon^2| > |\varepsilon^1| > 1$, then $\partial \bar{L}_1 / \partial W > 0$.

Proposition 7: With incomplete coverage in one sector and a completely covered sector and the internal equilibrium solutions described by:

$$[\gamma W_1 + (1 - \gamma) W^1(\bar{L}_1)] \ \ L^1[\gamma W_1 + (1 - \gamma) W^1(\bar{L}_1)] / \bar{L}_1 =$$
$$= W_2 L^2(W_2) / (\bar{L} - \bar{L}_1)$$

7.1. If $|\varepsilon^1| < 1$, population will flow to the partially covered sector when coverage increases. If $|\varepsilon^1| > 1$ and $u_1 < (\bar{L}_1 + \gamma \bar{L}_2) / \bar{L}$, population will flow away from the partially covered sector when coverage increases.

7.2. If $|\varepsilon^1| < 1$ or $u_1 < (\bar{L}_1 + \gamma \bar{L}_2) / \bar{L}$, mean wage in the partially covered sector will increase with coverage.

7.3. Expected wage in the economy, wage bill in the partially covered sector and total wage bill will move in the same direction as population in the partially covered sector.

7.4. An increase in the partially covered sector institutional wage has the same type of effect on the population flows as an increase in the degree of coverage.

ii. Type - B Model.

1. Consider the equilibrium described in Proposition 4. Take a change in the degree of coverage in sector/region 1. We have that:

$$(89) \qquad \partial \bar{L}_1/\partial\gamma = -\partial \bar{L}_2/\partial\gamma = \bar{L}_2\ [W_1\ L^1(W_1)\quad \bar{L}_1\ W^1(\bar{L}_1)]\ /$$

$$/\{\gamma\ W_1\ L^1(W_1)\ \bar{L}\ /\ \bar{L}_1 + (1-\gamma)\ W^1(\bar{L}_1)\ \bar{L}_1 - (1-\gamma)\ W^1(\bar{L}_1)'\ \bar{L}_1\ \bar{L}_2\}$$

The denominator of the expression is always positive.

We may have that in the optimal solution either (49) occurs and in this case the numerator of (89) is positive - i.e., an increase of the degree of coverage in the first sector attracts supply to this sector. Expected wage in the economy - $W_2\ L^2(W_2)/\ \bar{L}_2$ -, as $W_2\ L^2(W_2)$ is fixed, increases, and the unemployment rate in sector 2 decreases.

Or (51) holds, and then the numerator is negative. Intuitively, this means that chances of being employed become so low in the first sector when coverage increases, that people prefer to go to the other sector. In this case, expected wage in the economy decreases with γ, and the unemployment rate in sector 2 increases, as local supply decreases in sector 1 and switches to sector 2.

Consider the definition of effective employment (37):

$$(90) \qquad \partial L_1^{e}/\partial\gamma = -[\bar{L}_1 - L^1(W_1)] + (1-\gamma)\ \partial \bar{L}_1/\partial\gamma =$$

$$= \{(1-\gamma)\ \bar{L}_2\ [W_1\ L^1(W_1) - \bar{L}_1\ W^1(\bar{L}_1)]\ -$$

$$- [\bar{L}_1 - L^1(W_1)]\ [\gamma\, W_1\, L^1(W_1)\, \bar{L} / \bar{L}_1 + (1 - \gamma)\, W^1(\bar{L}_1)\, \bar{L}_1] +$$

$$+ [\bar{L}_1 - L^1(W_1)]\ (1 - \gamma)\, W^1(\bar{L}_1)'\, \bar{L}_1\, \bar{L}_2 \} /$$

$$/\{\gamma\, W_1\, L^1(W_1)\, \bar{L} / \bar{L}_1 + (1 - \gamma)\, W^1(\bar{L}_1)\, \bar{L}_1 - (1 - \gamma)\, W^1(\bar{L}_1)'\, \bar{L}_1\, \bar{L}_2\}$$

A sufficient condition for this to be negative is that $\gamma = 1$ or that (51) holds. But neither are necessary conditions.

As for the wage bill in sector 1 - and total wage bill -, $\gamma\, W_1\, L^1(W_1) + (1 - \gamma)\, W^1(\bar{L}_1)\, \bar{L}_1 = W_2\, L^2(W_2)\, \bar{L}_1 / \bar{L}_2$ and therefore it will move in the same direction as $\bar{L}_1$.

As for average wage in sector 1, $\bar{W}_1 = [\gamma\, W_1 L^1(W_1) + (1- \gamma)\, W^1(\bar{L}_1)\, \bar{L}_1] / L_1^e = W_2\, L^2(W_2)\, \bar{L}_1 / (\bar{L}_2\, L_1^e)$:

(91) $$\partial \bar{W}_1 / \partial\gamma = [W_2\, L^2(W_2) / (\bar{L}_2\, L_1^e)^2]$$

$$\{[\gamma\, L^1(W_1)\, \bar{L} + (1 - \gamma)\, \bar{L}_1^2]\ \partial\bar{L}_1/\partial\gamma + \bar{L}_1\, \bar{L}_2\ [\bar{L}_1 - L^1(W_1)]\} =$$

$$= [W_2\, L^2(W_2) / (\bar{L}_2\, L_1^e)^2]\, \bar{L}_1\, \bar{L}_2$$

$$\{L^1(W_1)\, [\gamma\, \bar{L}_2 + \bar{L}_1]\, [W_1 - W^1(\bar{L}_1)] - (1 - \gamma)\, W^1(\bar{L}_1)'\, \bar{L}_1\, \bar{L}_2\, [\bar{L}_1 - L^1(W_1)]\}$$

$$/\{\gamma\, W_1\, L^1(W_1)\, \bar{L} / \bar{L}_1 + (1 - \gamma)\, W^1(\bar{L}_1)\, \bar{L}_1 - (1 - \gamma)\, W^1(\bar{L}_1)'\, \bar{L}_1\, \bar{L}_2\} > 0$$

As for local unemployment:

$$(92) \qquad \partial U_1/\partial\gamma = \partial \bar{L}_1/\partial\gamma - \partial L_1^e/\partial\gamma =$$

$$= [\bar{L}_1 - L^1(W_1)] + \gamma \, \partial \bar{L}_1/\partial\gamma =$$

$$= \{ [\bar{L}_1 - L^1(W_1)] [\gamma W_1 L^1(W_1) \bar{L} / \bar{L}_1 + (1 - \gamma) W^1(\bar{L}_1) \bar{L}_1] +$$

$$+ \gamma \bar{L}_2 [W_1 L^1(W_1) - \bar{L}_1 W^1(\bar{L}_1)] - [\bar{L}_1 - L^1(W_1)] (1 - \gamma) W^1(\bar{L}_1)' \bar{L}_1 \bar{L}_2 \} /$$

$$/\{\gamma W_1 L^1(W_1) \bar{L} / \bar{L}_1 + (1 - \gamma) W^1(\bar{L}_1) \bar{L}_1 - (1 - \gamma) W^1(\bar{L}_1)' \bar{L}_1 \bar{L}_2\}$$

If $\gamma = 0$ or (49) hods, this will be positive but it is not a necessary condition.

As for local unemployment rate in region 1, $u_1 = U_1/ \bar{L}_1$, it will always increase with coverage:

$$(93) \qquad \partial u_1 / \partial\gamma = \partial\{\gamma [\bar{L}_1 - L^1(W_1)] / \bar{L}_1\} /\partial\gamma =$$

$$= \{\bar{L}_1 [\bar{L}_1 - L^1(W_1)] + \gamma \, L^1(W_1) \partial \bar{L}_1/\partial\gamma \} / \bar{L}_1^2 =$$

$$= (1/ \bar{L}_1) \{[\bar{L}_1 - L^1(W_1)] [\gamma W_1 L^1(W_1) + (1 - \gamma) W^1(\bar{L}_1) \bar{L}_1] +$$

$$+ \gamma L^1(W_1) \bar{L}_2 [W_1 - W^1(\bar{L}_1)] - [\bar{L}_1 - L^1(W_1)] (1 - \gamma) W^1(\bar{L}_1)' \bar{L}_1 \bar{L}_2 \} /$$

$$/\{\gamma W_1 L^1(W_1) \bar{L} / \bar{L}_1 + (1-\gamma) W^1(\bar{L}_1) \bar{L}_1 - (1-\gamma) W^1(\bar{L}_1)' \bar{L}_1 \bar{L}_2\} > 0$$

2. Consider that $W_1 = W_2 = W$. Then we may have either (56) or (58) and the general results above apply.

3. Consider a change in the wage rates. Then, we have:

$$\partial \bar{L}_1/\partial W_1 = -\partial \bar{L}_2/\partial W_1 = \{\bar{L}_2 \gamma [L^1(W_1) + W_1 L^1(W_1)']\} / \quad (94)$$

$$/\{\gamma W_1 L^1(W_1) \bar{L} / \bar{L}_1 + (1-\gamma) W^1(\bar{L}_1) \bar{L}_1 - (1-\gamma) W^1(\bar{L}_1)' \bar{L}_1 \bar{L}_2\}$$

If the wage elasticity of demand in sector 1 is smaller than one, then supply in sector 1 increases and in sector 2 decreases when W_1 rises.

$$\partial \bar{L}_1/\partial W_2 = -\partial \bar{L}_2/\partial W_2 = -\{\bar{L}_1 [L^2(W_2) + W_2 L^2(W_2)']\} / \quad (95)$$

$$/\{W_2 L^2(W_2) + \gamma W_1 L_1(W_1) + (1-\gamma) W^1(\bar{L}_1) \bar{L}_1 - (1-\gamma) W^1(\bar{L}_1) \bar{L}_2 -$$

$$-(1-\gamma) W^1(\bar{L}_1)' \bar{L}_1 \bar{L}_2\} = -\{\bar{L}_1 [L^2(W_2) + W_2 L^2(W_2)']\} /$$

$$/\{\gamma W_1 L^1(W_1) \bar{L} / \bar{L}_1 + (1-\gamma) W^1(\bar{L}_1) \bar{L}_1 - (1-\gamma) W^1(\bar{L}_1)' \bar{L}_1 \bar{L}_2\}$$

If the wage elasticity of demand in sector 2 is smaller than one, then supply in sector 1 decreases and in sector 2 increases when W_2 rises.

With homogeneous wages, and considering a general rise in the wage:

$$(96) \qquad \partial \bar{L}_1/\partial W = - \partial \bar{L}_2/\partial W =$$

$$= \{ \bar{L}_2 \, \gamma \, [L^1(W) + W L^1(W)'] - \bar{L}_1 \, [L^2(W) + W L^2(W)']\} /$$

$$/\{\gamma \, W L^1(W) \, \bar{L} / \bar{L}_1 + (1 - \gamma) \, W^1(\bar{L}_1) \, \bar{L}_1 - (1 - \gamma) \, W^1(\bar{L}_1)' \, \bar{L}_1 \, \bar{L}_2\}$$

This will be negative, i.e., population will flow to region 2 when there is a general rise in the institutional wage, iff

$$(97) \qquad \gamma \; [L^1(W)/ \bar{L}_1] \, (1 - | \varepsilon^1 |) < [L^2(W)/ \bar{L}_2] \, (1 - | \varepsilon^2 |)$$

This expression is similar to (87). As $\gamma \, L^1(W) / \bar{L}_1 < L^2(W) / \bar{L}_2$, this means that if $| \varepsilon^2 | < | \varepsilon^1 | < 1$, then $\partial \bar{L}_1/\partial W < 0$. If $| \varepsilon^2 | > | \varepsilon^1 | > 1$, then $\partial \bar{L}_1/\partial W > 0$.

Proposition 8: With incomplete coverage in one sector and a completely covered sector and the internal equilibrium solutions described by:

$$\gamma \, W_1 \, L^1(W_1) / \bar{L}_1 + (1 - \gamma) \, W^1(\bar{L}_1) = W_2 \, L^2(W_2)/ \bar{L}_2$$

8.1. If $W^1(\bar{L}_1) < W_1 \; L^1(W_1)/ \bar{L}_1$, population will flow to the partially covered sector when coverage increases. If $W^1(\bar{L}_1) > W_1 \, L^1(W_1)/ \bar{L}_1$, population will flow away from the partially covered sector when coverage increases.

8.2. Local unemployment rate in the partially covered sector and mean wage in this sector will always increase with the degree of coverage.

8.3. Expected wage in the economy, wage bill in the partially covered sector and total wage bill will move in the same direction as population in the partially covered sector.

8.4. An increase in the partially covered sector institutional wage will lead to an increase of population in that sector iff $|\varepsilon^1| < 1$.

V. Possible Extensions.

. General findings should not change significantly if we assumed workers are expected utility rather than just expected income or wage maximizers – they do not seem to in many applications of the dualistic principle. That was the conclusion of Bhattacharya (1993) for his survey of rural-urban migration models.

Say – as in Bhatia (1979), for example -, each individual enjoys utility $u(W_i)$ when he earns wage W_i for certain; being p_i the probability of employment and $(1 - p_i)$ of unemployment if in sector or region i, an individual decides his location maximizing $p_i\ u(W_i) + (1 - p_i)\ u(0)$ – being u(0) the utility workers obtain when unemployed. In this case, the region's wage bill is replaced by the aggregate sum of utilities of the individuals located in it in the equilibrium conditions of both H.-T. and B.-H. models, on the one hand. On the other, the wage elasticity of demand of sector i, ε^i, is expected to be replaced in several of the relations by

$$(98) \qquad [L^i(W_i)' / L^i(W_i)]\ \ [u(W_i) - u(0)] / u(W_i)'$$

once that also happens when we simulate a change in the formal wage in those models. $[u(W_i) - u(0)] / u(W_i)'$ is known in the game-theoretical literature – see Svejnar (1986); also Martins (2004) – as "fear of disagreement", assessing individual's aversion to large risks. It measures how incremental utility is exchanged per marginal utility - that is, per incremental utility per unit of the argument, here, W_i. It is inversely connected to "boldness" – see Aumann and Kurz (1977) -, the semi-elasticity of the utility with respect to the argument. $[u(W_i) - u(0)] / u(W_i)'$ is the increase in wage required to generate a unitary proportional increase of the excess utility of employed workers relative to unemployed ones in the sector.

One can manipulate (98) to generate:

$$(99) \qquad \varepsilon^i\ \ [u(W_i) - u(0)] / [u(W_i)'\ W_i]$$

ε^i is divided by the elasticity of the spread in utilities available to individuals in sector i with respect to the wage – see Martins (1998) for an example of the role of the elasticity of individual's utility with respect to the argument in cooperative game theory.

Hence, ε^i should be replaced by the product of the wage-semi-elasticity of demand by fear of disagreement. Or by its ratio to the elasticity of the utility gain of employed relative to unemployed workers generated by an increase in the wage in sector i.

VI. Summary and Conclusions.

1. The research was designed to analyze the effect of a change in the degree of coverage in a dualistic scenario. Two kinds of distinctions were considered:

- one with respect to the equilibrium mechanism - Type A and Type B, the second more applicable to a higher degree of segregation between formal and informal work in the partially covered sector. For example, probability of detection (and law enforcement) may be seen as affecting workers in type A and affecting firms (once a firm is "detected" all its workers are paid the legal wage) in type B; or type A applies to a case where unionisation is structured by profession and type B to firm-level unionisation.

- the other with respect to the wage setting arrangements in the other sector (i.e., the one in which coverage is not partial).

The structures have in common the fact that expected wage in the partially covered sector is modelled in such a way that it corresponds, respectively, to the free market wage and the expected wage in a completely covered sector in corner solutions.

Internal solutions of the four models considered require different relative magnitudes of the several wages in presence. Yet, wages of informal job holders are not necessarily lower than expectations of formal job ones, nor even of the other independent sector's – whether the latter is totally covered or not at all; in some instances, conditions for informal wages to be relatively higher were related to a wage-elasticity of the sector's labor demand larger than 1.

2. The impact on main macroeconomic aggregates of an increase in the degree of coverage were investigated. The sign effects are summarized in Tables 1, 1.1 and 1.2.

The increase in the degree of coverage, or probability of detection, will not increase necessarily the population installed in the partially covered sector - whatever the scenario considered. In Type A, that will occur if wage elasticity is lower than one in absolute value in the partially covered sector [4]. In Type B, only if the wage paid to uncovered workers (e.g., illegal immigrants) of the partially covered sector is very low (at least, lower than the wage paid in the other sector).

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[4] A low labor demand elasticity is found to increase the firms' incentive to comply in the partial equilibrium frameworks of both Grenier (1982) and Chang and Ehrlich (1985).

However, in a large number of situations, the increase in the degree of coverage will lead to a rise of the mean wage paid in the partially covered sector and its local unemployment rate.

In equilibrium, the expected wage in the economy, the wage bill in the partially covered sector and total wage bill will move in the same direction as population installed in the partially covered sector. Therefore, it is not indifferent which is the type of equilibrium in presence. Nor will it always be convenient to increase law enforcement if aggregate well-being of the workers in the partially covered sector is the aim of minimum wage legislation.

3. The increase in the degree of coverage may have a different impact in terms of sign than the increase in the institutional wage of the partially covered sector - they are confronted for population installed in the partially covered sector in Table 2 (for the cases where $W_1 = W_2$, the rise in the wage considered is simultaneous in both sectors, 1 and 2). All depends in the way degree of coverage is enforced: the two policy measures will have the same sign effect in structures of Type A (an increase in either implying a rise in population in the partially covered sector if wage elasticity of demand in the sector is lower than unity in absolute value), when detection is made by worker sampling, but not in Type B, with firm sampling.

Table 1.
Change in γ

	Uncovered Sector		Multiple Coverage	
Variable	Type A	Type B	Type A	Type B
$\bar{L}_1$	+ if $\lvert \varepsilon^1 \rvert < 1$; - if $\lvert \varepsilon^1 \rvert > 1$ and $u_1 < \gamma$	+ iff $W^1(\bar{L}_1) < W^2(\bar{L}_2)$	+ if $\lvert \varepsilon^1 \rvert < 1$; - if $\lvert \varepsilon^1 \rvert > 1$ and $u_1 <$ $(\bar{L}_1 + \gamma \bar{L}_2)/$ $\bar{L}$	+ iff $W^1(\bar{L}_1) < W_1$ $x\ L^1(W_1)/\bar{L}_1$
$\bar{W}_1$	+ if $\lvert \varepsilon^1 \rvert < 1$; or $\lvert \varepsilon^1 \rvert > 1$ and $u_1 < \gamma$	+	+ if $\lvert \varepsilon^1 \rvert < 1$; or $\lvert \varepsilon^1 \rvert > 1$ and $u_1 <$ $(\bar{L}_1 + \gamma \bar{L}_2)/$ $\bar{L}$	+
L^e_1	- if $\lvert \varepsilon^1 \rvert < 1$; or $\lvert \varepsilon^1 \rvert > 1$ and $u_1 < \gamma$	- if $W^2(\bar{L}_2) < W^1(\bar{L}_1)$ or $\gamma = 1$	- if $\lvert \varepsilon^1 \rvert < 1$; or $\lvert \varepsilon^1 \rvert > 1$ and $u_1 <$ $(\bar{L}_1 + \gamma \bar{L}_2)/$ $\bar{L}$	- if $W^1(\bar{L}_1) > W_1$ $x\ L^1(W_1)/\bar{L}_1$ or $\gamma = 1$
U_1	+ if $\lvert \varepsilon^1 \rvert < 1$	+ if $W^1(\bar{L}_1) < W^2(\bar{L}_2)$ or $\gamma = 0$	+ if $\lvert \varepsilon^1 \rvert < 1$; or $\lvert \varepsilon^1 \rvert > 1$ and $u_1 < \bar{L}_1 / \bar{L}$	+ if $W^1(\bar{L}_1) < W_1$ $x\ L^1(W_1)/\bar{L}_1$ or $\gamma = 0$

Table 1.1
Change in γ

	Uncovered Sector		Multiple Coverage	
Variable	Type A	Type B	Type A	Type B
u_1	+ if $\lvert\varepsilon^1\rvert < 1$; or $\lvert\varepsilon^1\rvert > 1$ and $u_1 < \gamma$	+	+ if $\lvert\varepsilon^1\rvert < 1$; or $\lvert\varepsilon^1\rvert > 1$ and $u_1 < (\bar{L}_1+\gamma \bar{L}_2)/\bar{L}$	+
$\bar{W}_1 L^e_1$	+ if $\lvert\varepsilon^1\rvert < 1$; - if $\lvert\varepsilon^1\rvert > 1$ and $u_1 < \gamma$	+ iff $W^1(\bar{L}_1)<W^2(\bar{L}_2)$	+ if $\lvert\varepsilon^1\rvert < 1$; - if $\lvert\varepsilon^1\rvert > 1$ and $u_1 < (\bar{L}_1+\gamma \bar{L}_2)/\bar{L}$	+ iff $W^1(\bar{L}_1) < W_1 \times L^1(W_1)/\bar{L}_1$
$\bar{L}_2$	- if $\lvert\varepsilon^1\rvert < 1$; + if $\lvert\varepsilon^1\rvert > 1$ and $u_1 < \gamma$	- iff $W^1(\bar{L}_1)<W^2(\bar{L}_2)$	- if $\lvert\varepsilon^1\rvert < 1$; + if $\lvert\varepsilon^1\rvert > 1$ and $u_1 < (\bar{L}_1+\gamma \bar{L}_2)/\bar{L}$	- iff $W^1(\bar{L}_1) < W_1 \times L^1(W_1)/\bar{L}_1$
W_2	+ if $\lvert\varepsilon^1\rvert < 1$; - if $\lvert\varepsilon^1\rvert > 1$ and $u_1 < \gamma$	+ iff $W^1(\bar{L}_1)<W^2(\bar{L}_2)$	·	·

L_2	- if $\lvert \varepsilon^1 \rvert < 1$; + if $\lvert \varepsilon^1 \rvert > 1$ and $u_1 < \gamma$	- iff $W^1(L;\bar{\ }_1) < W^2(L;\bar{\ }_2)$	.	.

Table 1.2
Change in γ

	Uncovered Sector		Multiple Coverage	
Variable	Type A	Type B	Type A	Type B
U_2	.	.	- if $\lvert\varepsilon^1\rvert < 1$; + if $\lvert\varepsilon^1\rvert > 1$ and $u_1 < (\bar{L}_1 + \gamma \bar{L}_2)/\bar{L}$	- iff $W^1(\bar{L}_1) < W_1 \times L^1(W_1)/\bar{L}_1$
u_2	.	.	- if $\lvert\varepsilon^1\rvert < 1$; + if $\lvert\varepsilon^1\rvert > 1$ and $u_1 < (\bar{L}_1 + \gamma \bar{L}_2)/\bar{L}$	- iff $W^1(\bar{L}_1) < W_1 \times L^1(W_1)/\bar{L}_1$
$W_2 L_2$	+ iff $(\lvert\varepsilon^1\rvert-1)(\lvert\varepsilon^2\rvert-1)>0$ and $u_1 < \gamma$ when $\lvert\varepsilon^1\rvert > 1$	+ iff $[W^2(\bar{L}_2) - W^1(\bar{L}_1)] \times (1 - \lvert\varepsilon^2\rvert) > 0$	.	.
W^e	+ if $\lvert\varepsilon^1\rvert < 1$; - if $\lvert\varepsilon^1\rvert > 1$ and $u_1 < \gamma$	+ iff $W^1(\bar{L}_1) < W^2(\bar{L}_2)$	+ if $\lvert\varepsilon^1\rvert < 1$; - if $\lvert\varepsilon^1\rvert > 1$ and $u_1 < (\bar{L}_1 + \gamma \bar{L}_2)/\bar{L}$	+ iff $W^1(\bar{L}_1) < W_1 \times L^1(W_1)/\bar{L}_1$

U	+ if $\mid \varepsilon^1 \mid < 1$	+ if $W^1(L;^{-}_{1}) < W^2(L;^{-}_{2})$ or $\gamma = 0$	+ if $\mid \varepsilon^1 \mid < 1$; or $\mid \varepsilon^1 \mid > 1$ and $u_1 <$ $(L;^{-}_{1} + \gamma L;^{-}_{2})/$ $L;^{-}$	+ if $W^1(L;^{-}_{1}) > W_1$ $x\, L^1(W_1)/ L;^{-}_{1}$ or $\gamma = 1$

Table 2

Effect on $\bar{L}_1$

Change of:	Uncovered Sector: Type A	Uncovered Sector: Type B	Multiple Coverage: Type A	Multiple Coverage: Type A $W_1 = W_2$	Multiple Coverage: Type B	Multiple Coverage: Type B $W_1 = W_2$
γ	+ if $\lvert\varepsilon^1\rvert < 1$; - if $\lvert\varepsilon^1\rvert > 1$ and $u_1 < \gamma$	+ iff $W^1(\bar{L}_1) < W^2(\bar{L}_2)$	+ if $\lvert\varepsilon^1\rvert < 1$; - if $\lvert\varepsilon^1\rvert > 1$ and $u_1 < (\bar{L}_1 + \gamma \bar{L}_2)/\bar{L}_2$	+ if $\lvert\varepsilon^1\rvert < 1$; - if $\lvert\varepsilon^1\rvert > 1$ and $u_1 < (\bar{L}_1 + \gamma \bar{L}_2)/\bar{L}_2$	+ iff $W^1(\bar{L}_1) < W_1 \times L^1(W_1)/\bar{L}_1$	+ iff $W^1(\bar{L}_1) < W \times L^1(W)/\bar{L}_1$
W_1	+ if $\lvert\varepsilon^1\rvert < 1$; - if $\lvert\varepsilon^1\rvert > 1$ and $u_1 < \gamma$	+ iff $\lvert\varepsilon^1\rvert < 1$	+ if $\lvert\varepsilon^1\rvert < 1$; - if $\lvert\varepsilon^1\rvert > 1$ and $u_1 < (\bar{L}_1 + \gamma \bar{L}_2)/\bar{L}_2$	- if $\lvert\varepsilon^2\rvert < \lvert\varepsilon^1\rvert < 1$; + if $\lvert\varepsilon^2\rvert > \lvert\varepsilon^1\rvert > 1$ and $u_1 < (\bar{L}_1 + \gamma \bar{L}_2)/\bar{L}$	+ iff $\lvert\varepsilon^1\rvert < 1$	- if $\lvert\varepsilon^2\rvert < \lvert\varepsilon^1\rvert < 1$; + if $\lvert\varepsilon^2\rvert > \lvert\varepsilon^1\rvert > 1$
W_2	·	·	when $\lvert\varepsilon^1\rvert < 1$; or $\lvert\varepsilon^1\rvert > 1$ and $u_1 < (\bar{L}_1 + \gamma \bar{L}_2)/\bar{L}_2$: + iff $\lvert\varepsilon^2\rvert > 1$	- if $\lvert\varepsilon^2\rvert < \lvert\varepsilon^1\rvert < 1$; + if $\lvert\varepsilon^2\rvert > \lvert\varepsilon^1\rvert > 1$ and $u_1 < (\bar{L}_1 + \gamma \bar{L}_2)/\bar{L}$	+ iff $\lvert\varepsilon^2\rvert > 1$	- if $\lvert\varepsilon^2\rvert < \lvert\varepsilon^1\rvert < 1$; + if $\lvert\varepsilon^2\rvert > \lvert\varepsilon^1\rvert > 1$

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