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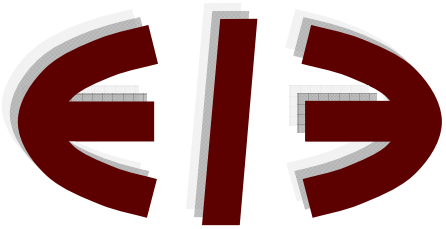

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News Shocks or Correlated Sunspots?
An Observational Equivalence Result in Linear Rational Expectations Models*

Marco M. Sorge †

Abstract

This paper studies identification of linear rational expectations models under news shocks. Exploiting the general martingale difference solution approach, we show that news shocks models are observationally equivalent to a class of indeterminate equilibrium frameworks which are subject only, though arbitrarily, to i.i.d. fundamental shocks. The equivalent models are characterized by a lagged expectations structure, which arises typically when choice variables are predetermined or rather based on past information with respect to current observables. This particular feature creates room for serially correlated sunspot variables to arise in equilibrium reduced forms, whose dynamics can be equivalently induced by news shocks processes. This finding, which is inherent to the rational expectations theoretical construct, calls for carefully designing empirical investigations of news shocks in estimated DSGE models.

Keywords: Rational expectations; News shocks; Indeterminacy; Observational equivalence

JEL Classification: C1; E3

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1 Introduction

In recent years, economists have been paying great attention to the idea that advance information on shifts in fundamentals, as opposed to current changes in opportunities or preferences, may configure an important driver of aggregate fluctuations (e.g. Beaudry and Portier, 2006; Schmitt-Grohé and Uribe, 2008; Jaimovich and Rebelo, 2009). While the classic sunspot literature (e.g. Azariadis, 1981; Benhabib and Farmer, 1994, 1999) emphasizes the importance of forecast revisions driven by extrinsic (non-fundamental) uncertainty in rational expectations (RE) frameworks, these studies rather investigate the possibility of expectations-driven fluctuations in regular economies when accounting for an information structure under which forthcoming developments in the economy are (possibly imperfectly) anticipated.

While sensibly improving the time series properties of dynamic stochastic models, news shocks have been shown to introduce non-invertible MA components into equilibrium reduced forms (e.g. Leeper and Walker, 2011). As a consequence, the success of the SVAR approach to proper uncovering this type of shock is controversial (Leeper et al., 2008; Fève et al., 2009).

The purpose of this paper is to show, in the same spirit of Beyer and Farmer (2008), that an issue of observational equivalence also exists, which stems from the impossibility to distinguish between news shocks models and indeterminate equilibrium frameworks in which news shocks are not present. The equivalent models are generally characterized by a lagged expectations structure, which can be associated with different microfoundations, like the presence of staggered-price setting under past information (e.g. Woodford, 2003), information stickiness (e.g. Mankiw and Reis, 2002) or imperfect information in monetary policy making (e.g. McCallum and Nelson, 2000). More generally, lagged expectations terms arise in DSGE models whenever (some of the) control variables are predetermined, or based on past information with respect to current observables. By exploiting the general martingale difference solution approach (e.g. Broze and Szafarz, 1991), we show that, for any exactly identified news shocks model with VARMA equilibrium reduced form, an observationally equivalent class of lagged expectations linear RE (LRE) systems exists which is subject solely, though arbitrarily (via indeterminacy), to i.i.d. fundamental shocks[1].

[1]The papers goal is to point to this identification problem which is likely to arise in models subject to different sources of expectations revisions, without claiming that news and sunspot models are inherently equivalent.

A key assumption of the literature on DSGE models with news shocks, the one which crucially departs from standard business cycle analysis, is that forward-looking agents are endowed with a richer information set than the one containing current and past realizations of exogenous variables. Indeed, future shocks to the latter are typically assumed to be - possibly only in part - anticipated (e.g. Schmitt-Grohé and Uribe, 2008). Interestingly, the observational equivalence result established in this paper involves stochastic difference models featuring some degree of information stickiness, according to which past state variables partly determine the current behavior of endogenous variables, and the largest σ-field upon which conditional forecasts are built - i.e. the one containing current and past observables - is only a subset of the one available to economic agents acting in news shocks frameworks.

The main implication of introducing lagged expectations terms in LRE systems is that serially correlated sunspot variables arise in equilibrium reduced forms. Wang and Wei (2006) were the first to point out this fact. However, Wang and Wei (2006)' solution algorithm for LRE systems with lagged expectations configures a particular case of the general martingale difference approach presented in Broze and Szafarz (1991), as the former restricts attention to linear stationary solutions and thus implicitly excludes arbitrariness in the equilibrium MA coefficients, which can be pinned down from structural parameters knowledge. We rather follow Broze and Szafarz (1991) to exploit the possibility of equilibrium forecast errors that feature an arbitrary correlation structure with respect to fundamental shocks[2].

The analysis presented here is more closely related to the econometric literature on the identification and estimation of indeterminate equilibrium RE systems. Observational equivalence is a general issue in RE models, as noticed in the the early work of Sims (1980) and Pesaran (1987) and recently emphasized by Beyer and Farmer (2007, 2008). It lies indeed on the largely untestable nature of identifying restrictions on the random processes which generate exogenous variables or the dynamic structure of RE frameworks. We contribute to this literature by extending the equivalence result to news shocks versus indeterminate equilibrium LRE models. Given the prominent position gained by theories regarding news shocks and anticipation effects in the business cycle debate, this finding points to the opportunity of supplementing likelihood-based empirical investigations of news shocks in estimated DSGE models with testing strate-

[2]That is, the analysis presented here does not rely on the existence of non-fundamental sunspot noise, as in Beyer and Farmer (2007) and Sorge (2010).

gies for the indeterminacy hypothesis.

The paper is organized as follows. Section 2 introduces the problem by looking at a highly stylized model economy under a news shock. In Section 3 a general equivalence result for multivariate news shocks LRE models is then established. Section 4 offers concluding remarks.

2 News shocks or correlated sunspots? A simple example

We first consider a simple single-equation model under a news shock, studied in Fève et al. (2009). Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathcal{P})$ be a filtered probability space, and y_t be a scalar endogenous variable adapted to $\mathcal{F}_t$ which evolves according to[3]:

$$y_t = \frac{\phi}{1+\phi\gamma} y_{t-1} + \frac{\gamma}{1+\phi\gamma} E_t(y_{t+1}) + \frac{1}{1+\phi\gamma} z_t \tag{1}$$

$$\phi \in (-1, 0) \cup (0, 1), \quad |\gamma| < 1$$

where $E_t(\cdot) := E(\cdot|\mathcal{F}_t)$ denotes the conditional expectation operator[4]. The realization of the exogenous variable z_t is assumed to be (fully) anticipated one period in advance (news shock):

$$z_t = v_{t-1}, \quad v_t \sim i.i.d.\, N(0, \sigma_v)$$

If the sequence for $E(y_t)$ is bounded[5], the following determinate stationary solution obtains, that only involves fundamentals, as expressed in ARMA (1,1) form:

$$(1 - \phi L) y_t = \gamma(1 + \gamma^{-1} L) v_t \tag{2}$$

where L denotes the lag operator. The non-invertibility of the MA component suggests that statistical inference based on simple autoregressions is generally invalid (e.g. Leeper et al., 2008).

Model identification also involves difficulties for empirical analysis. Consider the

[3]Structural parameters are chosen so as to ensure the existence of a (locally) unique RE equilibrium.

[4]Typically, $\mathcal{F}_t := \sigma(y_k, k \leq t)$

[5]This is usually enforced by the existence of a transversality condition in the underlying optimization framework.

following RE model featuring a lagged expectation but no news shocks[6]:

$$y_t = a_{01}E_t(y_{t+1}) + a_{11}E_{t-1}(y_t) + bv_t \tag{3}$$

$$a_{01} \neq 0, \quad a_{11} \neq 1$$

The reduced form of (3) is given by:

$$\sum_{i=0}^{1} a_i^* y_{t+i-1} = \lambda_t - bv_{t-1} \tag{4}$$

$$a_0^* = a_{11} - 1, \qquad a_1^* = a_{01}$$

where $\lambda_t = \sum_{k=0}^{1} a_{k1}\xi_{t-k}$, with $\xi_t := y_t - E_{t-1}(y_t)$ being an arbitrary martingale difference sequence with respect to $\mathcal{F}_t$. Thus, model (3)'s equilibrium is subject to a correlated sunspot shock λ_t.

Let us assume that $\xi_t = \pi v_t$ for some arbitrary parameter $\pi \in \Re$. The forecast errors can indeed be expressed as $\xi_t = \pi v_t + \delta s_t$, i.e. as a linear combination of the fundamental shock and an extraneous (non-fundamental) sunspot variable s_t satisfying $E_{t-1}(s_t) = 0$, which is assumed to be observed by the agents[7]. The impact parameters π and δ are to be determined endogenously; as demonstrated in Lubik and Schorfheide (2003), not all the reduced form parameters are uniquely determined under equilibrium indeterminacy.

Here, we focus on the case where $s_t := 0$ almost surely, $\forall t$ (parametric indeterminacy)[8]. Then from (4) we obtain the ARMA (1,1) representation:

$$\left(1 + \frac{a_0^*}{a_1^*}L\right) y_t = \pi \left(1 + \frac{(a_{11}\pi - b)}{a_1^*\pi}L\right) v_t \tag{5}$$

Let $\pi = \gamma$. Then for any element in the set:

$$S := \left\{(a_{01}, a_{11}, b) : a_{01} = \frac{(\gamma - b)}{(1 + \phi\gamma)};\ a_{11} = 1 - \phi a_{01};\ b \neq \gamma\right\}$$

[6]We will be more specific about the role of restrictions imposed to the alternative model when presenting the general result in Section 3.

[7]That is, $\mathcal{S}_t := \sigma(s_k, k \leq t) \subset \mathcal{F}_t$.

[8]This is the case when the equilibrium reduced form for y_t is to have a moving average structure with respect to the innovation v_t. Indeed, for such a type of solutions the following property holds: $\xi_t^j := E_t(y_{t+j}) - E_{t-1}(y_{t+j}) = \pi_j v_t$, $\forall j \in \mathcal{N}$. However, not all the real parameters π_j are necessarily arbitrary (see Broze and Szafarz, 1991).

which is non-empty[9], model (1) and (3) generate the same reduced form forecasts, and thus are observationally equivalent.

It is important to emphasize the role of the lagged expectation term $E_{t-1}(y_t)$ in this simple model economy. Equilibrium indeterminacy allows for self-fulfilling expectations revisions even in the absence of any fundamental shock in (3), i.e. when $b = 0$ (e.g. Benhabib and Farmer, 1994). Then observational equivalence would obtain for any (non-fundamental) sunspot variable $\xi_t := s_t \sim i.i.d.\, N(0, \gamma\sigma_v)$[10]. However, in this case the presence of a lagged expectation is crucial for the result, as it creates room for correlated sunspots to arise in equilibrium.

3 A general equivalence result for multivariate news shocks models

Several studies on the implications of news shocks for DSGE modeling have shown that this type of shock typically involves a non-fundamental VARMA equilibrium reduced form; as a consequence, the structural MA representation cannot be uncovered from SVAR models on the observables (e.g. Leeper et al, 2008; Fève et al., 2009; Leeper and Walker, 2011). For the purpose of the paper, we assume that the news shocks model is *exactly identified*, i.e. we assume that unique values for the structural parameters can be recovered from the estimates of the reduced form and from independent linear restrictions suggested by the theory (e.g. Rothenber, 1971; Beyer and Farmer, 2008). This Section shows that a family of lagged expectations LRE models under i.i.d. fundamental shocks always exists, which generate the same likelihood function as the news shocks model.

More specifically, let $Y_t = [y_{1t}, \ldots, y_{nt}]'$ be a vector-valued endogenous variable, and let the following VARMA process define the equilibrium reduced form of a general multivariate RE model featuring (potential) news shocks $Z_t = [z_{1t}, \ldots, z_{nt}]'$, whose j-th element is $z_{jt} = v_{j,t-s_j}$, $\{s_j \in \{0, 1, \ldots, q\}\, ; V_t = [v_{1t}, \ldots, v_{nt}]', V_t \sim i.i.d.\, N(0, \Sigma_v)\}$,

[9] In particular, the restrictions imposed to (3) are preserved.

[10] While it would be harder to justify coordination of agents' expectations revisions on fundamental shocks if the latter were not present, this result is still consistent with the RE framework.

where q represents the maximum anticipation horizon:

$$(I - \Phi L)Y_t = \Gamma(L)V_t, \qquad \Gamma_L = \Gamma_0 + \sum_{i=1}^{q} \Gamma_i L^i \tag{6}$$

with Γ_0 non-singular. We prove the following[11]:

Theorem 1. *Let $G_0(E_{\mathcal{F}}(Y_-, Y, Y_+), Z; \Theta_0) = 0$ be an exactly identified news shocks RE model with structure $\{G_0, \Theta_0\}$, whose equilibrium reduced form is the VARMA process (6). Then a class of observationally equivalent RE models $G_1(E_{\mathcal{F}}(Y_-, Y, Y_+), V; \Theta_1) = 0$ with structure $\{G_1, \Theta_1\}$ exists, which is only subject to the i.i.d. shocks V_t.*

Proof. The proof works as follows. We first introduce a general multivariate LRE system without news shocks whose reduced form admits the existence of forecast errors in VMA representation, which can be assumed to be arbitrarily related with the fundamental shocks V_t. A set of conditions, in the form of linear restrictions, is then found on this correlation structure and the parameters of the alternative multivariate RE system in order to construct an observationally equivalent model. The assertion is proven by showing that this system of constraints always admits solutions.

To begin with, let us consider the following LRE system:

$$G_1(E_{\mathcal{F}}(Y_-, Y, Y_+), V; \cdot) = 0 :$$

$$Y_t = \sum \sum\nolimits_{(k,i)\in\mathcal{I}} A_{k,i+k} E_{t-k}(Y_{t+i}) + BV_t \tag{7}$$

where $A_{k,i+k}$ are conformable matrices and:

$$\mathcal{I} := \{(k, i = h - k) : \, k \in \{0, 1, \ldots, K\}, \, h \in \{0, 1, \ldots, H\}\}$$

that is, h and k refer to the lead of the expectations and to the conditioning σ-field $\mathcal{F}_{t-k}$.

Any solution to (7) satisfies the recursive system:

$$\sum_{i=J_0}^{J_1} A_i^* Y_{t+i} = \sum_{k=0}^{K} \sum_{h=1}^{H} \sum_{j=0}^{h-1} A_{kh} \Xi_{t+h-k-j}^{j} - BV_t \tag{8}$$

[11]The t-dated information set is $\mathcal{F}_t := \sigma(Y_k, k \leq t)$. The time the forecast is formed cannot be posterior to the dating period of the expected variables.

where:

(i) $A_0^* = -I + \sum_{k:(k,0)\in\mathcal{W}]} A_{kk}, \quad A_i^* = \sum_{k:(k,i)\mathcal{W}]} A_{k,i+k},\ i \neq 0;$

(ii) $J_0 = \min\{i \in \mathcal{Z} :\ A_i^* \neq 0\}, \quad J_1 = \max\{i \in \mathcal{Z} :\ A_i^* \neq 0\};$

(iii) $-K \leq J_0 \leq 0 \leq J_1 \leq H;$

(iv) $\Xi_t^j := E_t(Y_{t+j}) - E_{t-1}(Y_{t+j})$ are H n-dimensional (forecasts) revision processes.

However, (8) is an equilibrium reduced form of (7) if and only if a set of constraints is imposed to the revision processes Ξ_t^j, i.e. the components of the latter may not generally be chosen as arbitrary martingale difference sequences. Let us set $J_1 = H = 1$, $J_0 = 0$ and $K = q$. Equation (8) then rewrites:

$$A_1^* Y_t + A_0^* Y_{t-1} = \Lambda_t - BV_{t-1}, \qquad A_i^* \neq 0 \quad i = 0, 1 \tag{9}$$

where $\Lambda_t = \sum_{k=0}^{q} A_{k1}\Xi_{t-k}$.This reduced form only involves $(n - n_1)$ arbitrary martingale differences as components of the revision process Ξ_t, where n_1 is the number of zero roots of the following characteristic equation:

$$det\left(\sum_{i=J_0}^{J_1} A_i^* \mu^{J_1 - i}\right) = 0 \tag{10}$$

The matrix A_0^* is generically invertible. By requiring A_1^* to be non-singular[12], no zero root exists, and Ξ_t is fully arbitrary. Serially correlated sunspot variables Λ_t then enter model (7)'s equilibrium reduced form. Given arbitrariness of forecasts revision processes, we can set $\Xi_t = \Pi V_t$ for some matrix $\Pi \in \Re^{n\times n}$. Then, for any non-singular A_1^* matrix and $\Pi = \Gamma_0$, we can obtain a system of $n^2(2q + 1)$ linear restrictions[13] of the form:

$$A_{11} = I - A_{01}\Phi \tag{11}$$

$$B = A_{11}\Gamma_0 - A_{01}\Gamma_1 \tag{12}$$

$$A_{i1} = A_{01}^{-1}\Gamma_i\Gamma_0^{-1}, \qquad i \in \{2, \ldots, q\} \tag{13}$$

[12]This step involves no loss of generality for the elements of A_0^* are really free parameters.

[13]Of which, $n^2(K - J_0) = n^2 q$ are zero-restrictions imposed on some of the A_i^* matrices by the assumption $H = J_1 = 1$ and $J_0 = 0$.

$$A_{i-1,0} = -A_{i1}, \qquad i \in \{2, \ldots, q\} \tag{14}$$

$$A_{q0} = 0 \tag{15}$$

in exactly $n^2(2q+1)$ 'unknowns' (i.e. the structural parameters of the alternative RE model)[14]. Clearly, once A_1^* is chosen, system (11)-(15) always admits a solution, which we label Θ_1. Along with (7), this yields the class of observational equivalent models $G_1(E_{\mathcal{F}}(Y_-, Y, Y_+), V; \Theta_1) = 0$. □

The established identification issue is inherent to the RE hypothesis (e.g. Sims, 1980)[15]. It allows to construct alternative economies driven by different types of expectations shocks which cannot be disentangled empirically. This observational equivalence problem is analogous to that discussed in Beyer and Farmer (2007), in which an econometrician with no independent information on the true variances of the (unobservable) fundamental shocks and the (unobservable) sunspot variables may well be unable to uncover the latter from the real world data.

One way of coping with this problem may consist in testing the hypothesis that the data are generated by an indeterminate equilibrium model. In this respect, testing strategies which are able to control for dynamic misspecification - i.e. the omission of lags, expectational leads or variables with respect to the actual data generating process (Fanelli, 2010) - should be preferred over system-based ones which exploit information on autocovariance patterns of observed time series to deliver evidence on determinacy versus indeterminacy (Lubik and Schorfheide, 2004)[16].

4 Concluding remarks

In this paper we study identification of linear dynamic RE models under news shocks. The main question addressed is whether these models are empirically distinguishable from lagged expectations RE systems which allow for equilibrium correlated sunspots. By means of the general martingale solution approach, it is shown that, for any exactly

[14]Equivalently, we could have imposed $A_{i-1,0} = A_{i1} = 0$ in (14).

[15]A simple corollary of this result is that not only may alternative specifications of news shocks in RE models - i.e. i.i.d news and correlated news processes - have identical information content to rational agents (e.g. Leeper andWalker, 2011), but even lead to observationally equivalent equilibrium dynamics.

[16]This latter approach to testing for indeterminacy is in fact affected by dynamic misspecification problems. A different direction is followed by studies on how to address the non-invertibility property of news shocks models (e.g. Kriwoluzky, 2009; Dupor and Han, 2011).

identified news shocks model, there exists an observationally equivalent class of indeterminate RE systems, which are only subject to i.i.d. fundamental shocks. Since the alternative models possess different determinacy properties, different implications for policy making are also likely to arise. We believe that this issue should be carefully addressed in likelihood-based estimation exercises intended to evaluate the quantitative importance of news shocks and anticipation effects as drivers of business cycles.

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