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Managerial Incentives and Stackelberg Equilibria in Oligopoly

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Abstract

The paper investigates both quantity and price oligopoly games in markets with a variable number of managerial and entrepreneurial firms which defines market structure. Following Vickers (Economic Journal, 1985) which establishes an equivalence between the equilibrium under unilateral delegation and the Stackelberg quantity equilibrium, the outcomes of these games are compared with the ones in sequential multi-leaders and multi-followers games. The profitability of a managerial/entrepreneurial attitude vs leadership/followership is shown to critically depend upon the kind of strategy, price or quantity, and upon the assumed market structure. Indeed, the latter turns out to be crucial in determining the equivalence result that is shown to be contingent on the assumption that just one leader or one managerial firm operate in the market. A welfare analysis finally highlights the differences between the delegation and the sequential games, focusing on the impact of market structure and imperfect substitutability on the equilibria of the two games.

Jel Classification: C72, L13, L22
Keywords: Strategic delegation, sequential games, quantity and price competition, welfare analysis

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1 Introduction

The economic analysis of managerial incentive contracts starting from the contributions of Vickers (1985), Fershtman and Judd (1987) and Sklivas (1987) has given rise in recent years to an extensive literature on strategic delegation. A game theoretical approach characterizes this literature which basically aims at examining the strategic implications of delegating decisions to managers acting on the product market on behalf of the firms' owners. The general conclusion achieved in these models is that an owner concerned with profit maximization potentially benefits from letting a manager not maximize profits when this allows for a strategic advantage over competitors. By taking due account of rivals' reactions, the owners choose through a compensation contract the optimal extent of delegation to give their managers, thus motivating these agents according to their principals' objectives.

In the past few years the issue of strategic delegation has sparked an intense debate which contributes to explaining a wide range of competitive situations, most of which have been studied under a duopoly assumption. Moreover, the analysis of the strategic role played on the market by firms which manipulate incentive contracts has generally focused on the symmetric case in which both firms are manager-led. More recently, the analysis of strategic delegation has been extended to cases in which managerial firms compete against profit-maximizing (entrepreneurial) ones. In a similar context, and within a framework of quantity competition, Vickers (1985) examines an oligopoly with only one managerial firm and reinterprets unilateral delegation as a Stackelberg solution. Competition among heterogeneous firms has been also addressed within games which endogenize the choice of hiring a manager. Among these, Basu (1995) tackles quantity competition between two firms which have to take the costly decision of hiring a manager and demonstrates that the subgame perfect equilibrium of the underlying game coincides with the asymmetric solution in which only the most efficient firm hires a manager and act as the leader of a Stackelberg game.[1] The works of Barros and Grilo (2002) and White (2001) are also included in this literature.[2]

The present work is inspired by the Vickers' work, the analysis of which is extended in the following directions: we allow for oligopolistic competition among any given number of managerial and entrepreneurial firms[3] and address competition in prices besides quantities, under the assumption of product differentiation. The analysis of an oligopoly with a variable number of heterogeneous

[1] This equivalence result between the incentive equilibrium and the Stackelberg outcome basically highlights how a Stackelberg leadership can arise through the appropriate choice of an incentive contract. For this reason it can be included in the growing literature on endogenous Stackelberg leadership (Hamilton and Slutsky, 1990, van Damme and Hurkens, 1999, Huck and Rey-Biel, 2006).

[2] Barros and Grilo (2002) build a model of vertical differentiation in which two firms choose whether to be entrepreneurial or managerial is presentet; White (2001) addresses the endogenous decision to hire a manager taken by private and public firms in an oligopolistic market.

[3] The managerial hiring decisions are not modeled in this paper which assumes as exogenous the role played by each firm.

firms allows us to compare the managerial/entrepreneurial (ME) model with the correspondent Stackelberg model with multiple leaders and followers (LF), for any given market structure and any mode of competition, price or quantity. Given this background, the present paper has two related objectives. First, it aims at exploring competition in both managerial and sequential markets in order to examine the impact on firms' profits and welfare of changes in market structure, here defined as the number of delegating over profit-maximizing firms or as the number of leaders over followers. The profit and welfare comparisons across the different set-ups allow us to highlight the forces at work in shaping firms' incentives and creating welfare gains in different market configurations. As far as the second objective is concerned, the paper investigates the role of product substitutability, as a major determinant of firms' relative market position, within the above contexts. Comparative statics of the solutions with respect to the parameters describing market structure and product substitutability is performed in order to identify the potential benefits or damages associated with firms' heterogeneity induced by behavioral differences or a different order of moves.

By extending the analysis of the two models, mostly confined to quantity competition, to a price competition framework, this paper enriches both the existing literature on strategic delegation and that on multi-leader-follower games. As regards strategic delegation, Fershtman and Judd (1987) demonstrate that, in a duopoly with identical firms, price competition reduces firms' incentives for sales maximization and leads to a more collusive outcome. Price competition in a strategic delegation framework is also tackled in Lambertini (2000). Indeed, this paper examines a duopoly multi-stage game in which the endogenous assignment of roles is addressed as one of the different dimensions of competition,[4] and discusses the simultaneous price choices with just one delegating firm as one of the considered sub-games. The equilibrium of this sub-game is shown to be a Stackelberg-like equilibrium in prices, with a strategic advantage that is shown to be reverted in favour of entrepreneurial firm which assumes a role analogous to the second-mover in the correspondent sequential price game.[5] To the best of our knowledge, the analysis of price competition in this kind of framework has not been extended to the oligopoly case.

The focus of our analysis on sequential and delegating decisions closely relates the present paper to the literature on commitment, since both these strategies may imply the adoption of a commitment for strategic purposes.[6] In particular, firms which move first acting as Stackelberg leaders in a quan-

[4] In the model, firms have to choose to behave as price or quantity setters, to move early or later and, finally, to act as managerial or entrepreneurial firms.

[5] As well-known in the literature on sequential games and symmetric firms, when price competition is assumed, a second-mover advantage occurs due to the followers' ability to undercut the price set by the leaders. See Gal-Or (1985) and Amir and Stepanova (2006) for a discussion.

[6] The economic literature on commitment (Dixit and Nalebuff, 1991; Shelling, 1960) considers a number of ways through which a firm can bind itself to strategic credible actions. Within this body of literature it is shown how credibility can be established, among other ways, by engaging first mover strategies or by delegating decisions.

tity competition framework, gain a timing advantage by committing to actions that limits bargaining with late-movers.[7] In the same way, the owners in a delegation quantity model, through an irrevocable mandate to their managers, commit to an output prior to the profit-maximizer rivals, thus achieving market leadership.[8]

Using linear demand and cost functions, in this paper we find that the subgame perfect equilibrium of the ME quantity (price) model entails output (price) levels which coincide with those at the equilibrium of the correspondent LF model only when one manager and one leader are assumed to operate in the market, regardless of the number of followers and entrepreneurial firms and of the degree of product substitutability. However, the strategy space is shown to be determinant in defining the differences between the models' outcomes in the presence of a higher number of leaders or managerial firms. The analysis of inter-group and intra-group competition and the inspection of the welfare properties of the two games reveal that in a quantity competition framework social welfare is higher under strategic delegation rather than with sequential moves, while it is higher under sequential actions when price competition is assumed. The paper also compares the welfare properties of the Stackelberg and the incentive equilibria,[9] following changes in market structure and in the degree of product substitutability, and explicitly studies the interplay between these two key elements. We find that in both models, heterogeneity in the population of firms has a different impact on social welfare depending on whether firms compete in quantities or prices. Under quantity competition, firms' heterogeneity sustains a more aggressive behavior with respect to the rivals by the managerial or the leader firms which act as high producing firms and contribute positively to welfare. While the positive effect on welfare of a larger number of managerial firms always overcome the negative effect of a stronger competition which reduces their aggressiveness, an increasing number of leaders leads to welfare improvements provided that their number is low enough - i.e. the effect of competition among leaders is limited. Conversely, a market structure with a homogeneous population of profit-maximizers guarantees the highest welfare in both the models with price competition. Indeed, the existence of heterogeneous firms, i.e. managerial firms or firms moving sequentially, give scope to soften competition and generate welfare losses with respect to the symmetric cases. In each of the examined framework, product substitutability influences the overall welfare, by altering the strength of both inter-group and intra-group competition.

Our results are derived and discussed as follows. In Section 2 we solve the basic ME model in the two set-ups of quantity and price competition. The same procedure is applied in Section 3 for the LF model. Section 4 discusses the main results and presents a comparison across the models. The last section concludes.

[7] See Gal-Or (1985) as a seminal contribution to the literature on first/second-mover advantages.

[8] See Etro (2007) for a recent overview on market leadership theories in oligopoly.

[9] We refer to the equilibrium under strategic delegation as the incentive equilibrium.

2 The Managerial-Entrepreneurial (ME) model

We start by examining both quantity and price competition in a market with n firms ($n \geq 2$), m of which are delegating firms (with $0 \leq m \leq n$) allowing for revenues' maximization, and the other $n-m$ firms are entrepreneurial.[10] A two-stage game catches the two dimensions of competition for the managerial firms - competition in the delegation schemes and product market competition - and runs as follows: in the first stage the delegating owners choose the extent of control to delegate to their managers, namely the optimal compensation scheme, while in the second stage both the non-managerial owners and the delegated managers are in charge of quantity or price decisions.

Product differentiation is introduced in the model by using the linear demand function of Shubik and Levitan (1980), as in Vives (1985), defined in the form of an inverse demand function by:

$$p_z = v - \frac{1}{1+\mu}\left(nq_z + \mu\left(\sum_{z=1}^{n} q_z\right)\right)$$

which allows for imperfect substitute goods.[11] The parameter $\mu \in (0, +\infty)$ represents the degree of product substitutability: when $\mu = 0$ products are completely independent, while they are perfectly substitutable when $\mu \to +\infty$. We also assume that firms are symmetric with respect to costs; marginal costs are assumed to be constant and equal to c (with $c \neq 0$) and fixed costs are null. The parameter v is the intercept of the inverse demand function.

2.1 The quantity game between managerial and entrepreneurial firms

Let us consider the game with quantity-setting firms and solve it by backwards. At the second stage, the m managerial firms and the $n-m$ entrepreneurial firms compete on the final market choosing quantities simultaneously. Denote with q_i ($i = 1, .., m$) the individual quantity produced by the generic managerial firm i and with q_k ($k = m+1, .., n$) the quantity set by the generic entrepreneurial firm k.

Each manager maximizes the linear combination of profits and revenues $M_i = \theta_i \pi_i + (1-\theta_i) S_i$,[12] where $(1-\theta_i)$, ($i = 1, ..., m$), defines the weight assigned by the owner to revenues S_i, namely the incentive parameter. By

[10] According to this parameterization, we are able to describe competition in a wide range of market structures by simply assuming that m varies, keeping constant the total number of firms n. The number m of managerial firms can be also interpreted as a measure of market asymmetry, the highest being $m = n/2$.

[11] This demand function comes from the quadratic utility function of a representative consumer with quasi-linear preferences and is such that market size does not vary either with the degree of substitutability or the number of varieties.

[12] Fershtman and Judd (1987) use the same function as managerial objective, while in Vickers (1985) the managerial objective function is defined as a linear combination of firms' profits and sales. As shown in Lambertini and Trombetta (2002), the two formulations are equivalent.

summing the First Order Conditions (FOCs) calculated with respect to q_i over the m managerial firms, we get:

$$q_i m\,(2n+\mu) = (1+\mu)\left(mv - \sum_{i=1}^{m}\theta_i c\right) - \mu m Q \tag{1}$$

where Q is the total quantity produced in the market. Pure profit maximization with respect to q_k leads to the following expression which nests the FOCs of the $n-m$ entrepreneurial firms:

$$q_k\,(n-m)\,(2n+\mu) = (1+\mu)\,(n-m)\,(v-c) - \mu\,(n-m)\,Q \tag{2}$$

Since $Q = q_i m + q_k\,(n-m)$, from (1) and (2) we calculate the individual quantities of the two kinds of firms as functions of θ_i and θ_j $(j \neq i)$:

$$\begin{aligned} q_i &= \frac{v(1+\mu)\big((2n+\mu)+\mu c(n-m)-nc\theta_i(2+\mu)+\mu c\sum_{j\neq i}\theta_j\big)}{(2n+\mu(1+n))(2n+\mu)} \\ q_k &= \frac{(v-c)(1+\mu)(2n+\mu)-\mu cm+\mu c\sum_{i=1}^{m}\theta_i}{(2n+\mu(1+n))(2n+\mu)} \end{aligned}$$

At the first stage the owners of the managerial firms, by anticipating quantity competition at the last stage, compete with respect to the delegation parameters, deciding simultaneously upon the incentive contract for their manager. Profit maximization by the owner of firm i leads to the optimal delegation parameter for any given θ_j $(j \neq i)$ which gives, under the symmetry condition, the level of θ_i^{MEq} chosen at equilibrium by each managerial firm:

$$\theta_i^{MEq} = \frac{2cn^3(2+\mu)^2+2\mu n^2 c(3\mu+4)+2\mu^2 nc(1+\mu)+\mu^3 cm(n-1)-\mu^2 v(n-1)(2n+\mu)}{c(2n^2(2+\mu)((n+2)\mu+2n)+\mu^2 n(4+\mu)+\mu^3 m(n-1)+\mu^3)}$$

where the subscript denotes the ME model in quantities.[13] Note that $0 < \theta_i^{MEq} \leq 1$ for any $\mu \geq 0$ and increases in this interval with m and n. When in the limit the number of both the types of firms approaches infinity, the optimal parameter approaches the upper bound 1, i.e. pure profit maximization arises at equilibrium. Moreover, the negative sign of the derivative $\partial\theta_i^{MEq}/\partial\mu < 0$ (see Appendix A1 for a demonstration) shows that when product differentiation decreases (μ increases) the optimal delegation parameter monotonically decreases, starting from $\theta_i^{MEq} = 1$ when $\mu = 0$ and tending to a positive value which entails the highest degree of delegation when $\mu \to +\infty$, provided that the cost parameter c is sufficiently high as compared to the consumers' reservation price v.[14] The previous discussion introduces the following proposition.

Proposition 1 *The incentive to deviate from profit maximization is a decreasing function of both the number of managerial firms and the number of entrepreneurial firms. For any market structure, increasing product differentiation*

[13] As shown in Appendix 2A, the Second Order Conditions (SOCs) for the existence of a maximum are always satisfied.

[14] The marginal cost parameter c must be higher than $\overline{c} = (v\,(n-1))\,/\,(2n + m\,(n-1))$ for the lower bound of θ^{MEq} to be positive. The latter tends to zero under perfect substitutability when $c = \overline{c}$.

reduces the incentive to delegate and induces firms to move closer to profit-maximization.

The intuition for the above results is straightforward. Delegation is used in the quantity competition case as a device to gain an advantage over rivals by allowing managers to expand the output and induce a reduction of the competitors' quantities. The effectiveness of this strategy in generating this beneficial reply is dampened, due to strategic substitutability at both stages of the game, by an intense rivalry on the compensation schemes caused by an increase in the number of managerial firms or by an increase in the number of entrepreneurial firms which softens the aggressiveness of the delegating rivals. The effectiveness of delegation as a strategic device is also affected by increasing product differentiation which reduces the advantages of managerial firms over rivals and limits their need of being aggressive through delegation.

The values of the relevant market variables evaluated at the Subgame Perfect Nash Equilibrium (SPNE) are obtained by substitution and listed in Table 1a. The key results are summarized in the following prospect where π_i^{MEq} and π_k^{MEq} denote the profits gained by the managerial and the entrepreneurial firms, while CS^{MEq} and W^{MEq} denote respectively consumers' surplus and social welfare:

- $q_i^{MEq} > q_k^{MEq}$ and $\pi_i^{MEq} > \pi_k^{MEq}$, for any $m \in (0, n)$ and any given $\mu \in (0, +\infty)$;
- $\frac{\partial q_i^{MEq}}{\partial m} < 0$, $\frac{\partial q_k^{MEq}}{\partial m} < 0$; $\frac{\partial \left(q_i^{MEq} - q_k^{MEq}\right)}{\partial m} < 0$; $\frac{\partial \pi_i^{MEq}}{\partial m} < 0$, $\frac{\partial \pi_i^{MEq}}{\partial m} < 0$; $\frac{\partial \left(\pi_i^{MEq} - \pi_k^{MEq}\right)}{\partial m} < 0$; $\frac{\partial CS^{MEq}}{\partial m} > 0$; $\frac{\partial W^{MEq}}{\partial m} > 0$;
- $\frac{\partial \left(q_i^{MEq} - q_k^{MEq}\right)}{\partial \mu} > 0$, $\frac{\partial \left(\pi_i^{MEq} - \pi_i^{MEq}\right)}{\partial \mu} > 0$; $\frac{\partial CS^{MEq}}{\partial \mu} > 0$; $\frac{\partial W^{MEq}}{\partial \mu} > 0$.[15]

Basically, our results show that the managerial firms have a competitive advantage over the profit-maximizers, for any market structure and any degree of product substitutability. The advantages from delegation, as the incentive to delegate, decrease with the number of managerial firms (both quantity and profit differentials decrease in m). The non-delegating firms also suffer from a larger presence of delegating firms, whose interactions enhance overall competitive pressure in the market. In any case, the presence of an increasing number of managerial firms is welfare improving for any degree of product substitutability, since the positive contribution to welfare of a higher number of such aggressive firms always dominates the welfare losses due to their reduced aggressiveness. In this situation $m^* = n$ turns out to be the optimal market structure.

As far as product differentiation is concerned, we find that the quantities q_i^{MEq} and q_k^{MEq} coincide when $\mu = 0$, since in this case firms act as local monopolists in their respective markets and produce the quantity $(v - c)/2n$,

[15] The derivation of all these comparative results are available from the author upon request. Consumer welfare has been obtained using the following expression calculated at the optimal quantities: $CS = \frac{\left(n\left(m(q_i)^2 + (n-m)(q_k)^2\right) + \mu(m(q_i) + (n-m)(q_k))^2\right)}{2(1+\mu)}$.

while they diverge progressively when μ increases, the difference being maximum when $\mu \to +\infty$. Our results indeed show that product differentiation on the one hand weakens the overall market competition raising firms' mark-up; on the other hand, by pushing managerial firms towards profit maximization and shifting sales from the delegating to the non-delegating firms, it reduces the extent of the managerial firms' advantage and thus the effects of firms' disparities (both quantity and profit differentials increase in μ). The net result is an overall detrimental effect on welfare which, however, leaves unchanged the optimal market structure. The pattern of social welfare is illustrated in Figure 1.

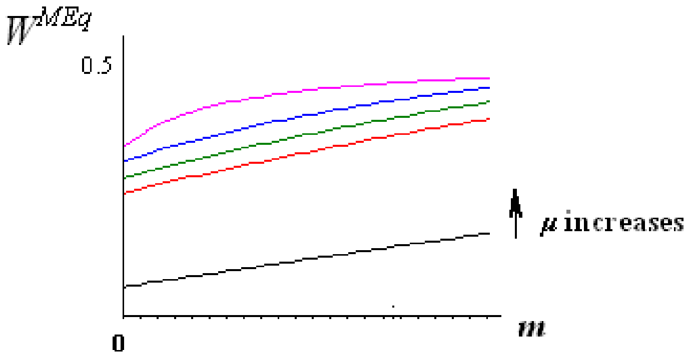

Figure 1: Social welfare in the ME quantity model ($n = 10$).

2.2 The price game between managerial and entrepreneurial firms

We turn now to solve the game when delegating and non-delegating firms engage price competition. Let p_i $(i = 1, .., m)$ be the price set by each managerial firm i and p_k $(k = m + 1, .., n)$ the price set by the entrepreneurial firm k.

By using the direct demand function *à la* Shubik and Levitan, we write the managers' objective function as a function of prices:

$$M_i = \theta_i \pi_i + (1 - \theta_i) S_i = (p_i - \theta_i c) \left(\frac{v - p_i (1 + \mu) + \frac{\mu}{n} (p_i + P_{-i})}{n} \right) \quad (3)$$

where P_{-i} is the sum of the prices set by all firms with the exception of firm i's.

At the second stage we solve the maximization problem of the function in (3) with respect to p_i. Summing the FOCs of the m managerial firms, we obtain:

$$p_i m (2n (1 + \mu) - \mu) = c \sum_{i=1}^{m} \theta_i (n (1 + \mu) - \mu) + m (nv + \mu P) \quad (4)$$

where P is the sum of all prices.

By summing the FOCs of all the $n-m$ profit-maximizing firms, we get:

$$p_k\,(n-m)\,(2n\,(1+\mu)-\mu)=c\,(n-m)\,(n\,(1+\mu)-\mu)+(n-m)\,(nv+\mu P) \tag{5}$$

Since $P=mp_i+(n-m)\,p_k$, we are able to determine P from (4) and (5) and get the prices of the managerial and the entrepreneurial firms as functions of θ_i and θ_j $(j\neq i)$:

$$\begin{aligned}
p_i &= \frac{n^2(1+\mu)(2v+\mu c)-n\mu(v+cm)-\mu^2c(n+m(n-1))-c(\mu-n(1+\mu))\left(\theta_i n(2+\mu)+\sum_{j\neq i}\theta_j\mu\right)}{(2n+\mu(n-1))(2n+\mu(2n-1))}\\
p_k &= \frac{2n^2(1+\mu)(v+c(1+\mu))-n\mu(v+c(m+3))-\mu^2c(n(m+3)-(m+1))-\mu c(\mu-n(1+\mu))\left(\theta_i+\sum_{j\neq i}\theta_j\right)}{(2n+\mu(n-1))(2n+\mu(2n-1))}
\end{aligned}$$

Profit maximization at the incentive-setting stage yields the optimal delegation parameter θ_i^{MEp} (see Appendix A3 for its analytical expression), where the subscript denotes the ME model in prices.[16] It can be checked that when $\mu\geq 0$ the optimal contract entails $\theta_i^{MEp}\geq 1$ which implies that at equilibrium managers are discouraged from putting some weight on sales and behave less aggressively than in the presence of all profit-maximizing firms.[17] Indeed, in contrast to the quantity competition case, owners competing in prices motivate their managers to keep prices beyond the profit maximization level, so that if all firms delegated control to managers, the incentive-equilibrium would be more favorable than the profit-maximizing equilibrium. The benefits from delegation in this heterogenous environment also accrue to the non-delegating firms which gain from operating in a less competitive market.

Our analysis also reveals that the parameter θ_i^{MEp} increases in m and decreases in n, showing that when there is an increasing number of managerial firms in the market, deviation from profit maximization is more likely and a collusion-like outcome can be sustained. Conversely, managers are more motivated towards profit maximization when the number of entrepreneurial firms increases since, for any given m, the presence of a high number of non-delegating firms reduces the ability of managerial firms to sustain a collusive outcome through delegation. In the limit, when the number of both the types of firms approaches infinity, θ_i^{MEp} equals the lower bound 1, namely firms find it optimal to maximize profits. Finally, we find that the impact of product differentiation on the optimal delegation parameter is interestingly non-monotone.[18] Indeed, at the extremes of the interval $(0,+\infty)$ of the differentiation parameter θ_i^{MEp} tends to 1, implying pure profit-maximization under both the hypotheses of independent markets and perfect substitutability.[19] Within this interval we

[16] The second order conditions for a maximum are always satisfied (see Appendix A4 for a formal proof).

[17] This compensation mechanism resembles the effects of a tax imposed by the owners on managers' expenditure. See Fershtman and Judd (1987, p.13) for a discussion on this point.

[18] See Appendix A3 for a formal proof.

[19] This is in contrast to the quantity competition case in which perfect substitutability entails the highest deviation from profit maximization and is consistent with the idea that for $\mu=0$

find $\partial\theta_i^{MEp}/\partial\mu < 0$ for sufficiently high values of μ: product differentiation widens the distortion from pure profit maximization when products are sufficiently substitutable; however, a further decrease of it induces delegating firms to move back towards profit maximization, entailing $\partial\theta_i^{MEp}/\partial\mu < 0$ when μ is low enough.

The above discussion is synthesized in the following proposition.

Proposition 2 *Deviation from profit maximization increases in the number of managerial firms and decreases in the number of entrepreneurial firms. For any given market structure, the managerial distortion from profit maximization is non-monotone in μ, reaching its maximum for a positive, finite value of μ.*

The solution for the relevant variables at the SPNE are reported in Table 1b. Again, we can summarize the main results as follows:

- $p_i^{MEp} > p_k^{MEp}$ and $\pi_i^{MEp} < \pi_k^{MEp}$ for any $m \in (0, n)$ and any given $\mu \in (0, +\infty)$;
- $\frac{\partial p_i^{MEp}}{\partial m} > 0$; $\frac{\partial p_k^{MEp}}{\partial m} > 0$; $\frac{\partial\left(p_i^{MEp} - p_k^{MEp}\right)}{\partial m} > 0$; $\frac{\partial \pi_i^{MEp}}{\partial m} > 0$, $\frac{\partial \pi_i^{MEp}}{\partial m} > 0$; $\frac{\partial\left(\pi_k^{MEp} - \pi_i^{MEp}\right)}{\partial m} > 0$; $\frac{\partial CS^{MEp}}{\partial m} < 0$; $\frac{\partial W^{MEp}}{\partial m}$ is negative over $(0, n)$ provided that μ is low enough, and exhibits a non-monotone pattern for higher values of μ;
- $\frac{\partial\left(p_i^{MEp} - p_k^{MEp}\right)}{\partial \mu}$ and $\frac{\partial\left(\pi_k^{MEp} - \pi_i^{MEp}\right)}{\partial \mu}$ are non-monotone in μ, being positive for low values of μ, and negative thereafter; $\frac{\partial CS^{MEp}}{\partial \mu} > 0$; $\frac{\partial W^{MEp}}{\partial \mu} > 0$.[20]

Intuitively, by committing to a less aggressive conduct, managerial firms keep high prices, beyond the profit maximization level, and relax the overall competition, exerting a positive externality on those firms that maximize profits in the same market. The higher prices and the output restriction mimic a collusive outcome which leads the profits of all firms to increase. For any degree of product substitutability and any market structure, the price managerial game gives a competitive advantage to the entrepreneurial firms. Furthermore, both the price and profit differentials increase in m, as well as the aggregate profits, showing an increasing advantage in favour of the entrepreneurial firms and an overall higher profitability for all firms when the number of managerial firms increases. The model's solutions also reveal that the presence of an increasing number of managerial firms is always welfare detrimental, provided that product substitutability is sufficiently low. For higher values of μ, social welfare is shown to rise when the managerial firms represent the largest share of firms in the market, since in such circumstances the positive impact due to the higher profits gained by all firms overcomes the negative effect of a larger market quantity contraction. Indeed, Figure 2 shows that the social welfare rises

the market outcomes of a price setting and a quantity setting model coincide, for any given market structure.

[20] The derivation of these comparative results are available from the author upon request.

when m and μ are high enough. Moreover, we find that the minimum number of managerial firms needed to generate such a boost is decreasing in μ, since increasing substitutability reduces the welfare detrimental effects of delegation and requires a progressively lower number of managerial firms for the positive effect of higher profits to prevail. The socially optimal market structure will entail the presence of only entrepreneurial firms in the market ($m^* = 0$).

With respect to changes in μ, it should be also remarked that the price and profit differentials reflect the non-monotone pattern of the optimal delegation parameter over the interval $(0, +\infty)$.[21] When μ is high, its reduction allows for a wider price difference which is achieved through higher delegation by the managerial firms; this widens the extent of the entrepreneurial firms' advantage. When μ is low, all firms are isolated from competition and this induces the owners of managerial firms to move towards pure profit maximization, as they would behave in a monopoly context; this reduces price and profit differentials. Analogously to the quantity competition case, increasing product differentiation has a twofold impact on the market outcomes: besides softening the overall competition, it may amplify or reduce, according to its initial value, the firms' disparities due to the differences of roles, so enhancing or shrinking the advantage enjoyed by the profit maximizers. The second effect is shown to be negligible as compared to the first one, so that the overall welfare monotonically decreases in μ, without altering the market structure required for social optimality.

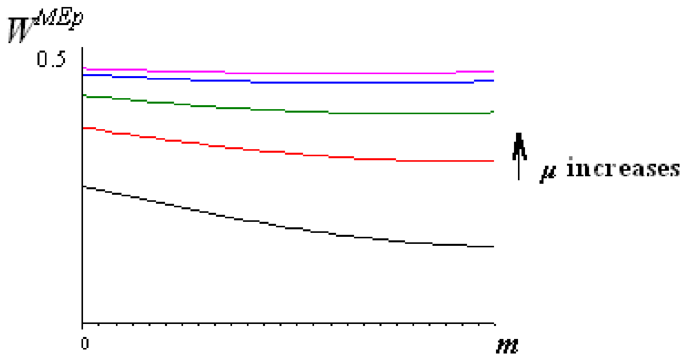

Figure 2: Social welfare in the ME price model ($n = 10$).

[21] The two prices coincide at the extremes of this interval, approaching the monopoly price $(v + c)/2$ when $\mu = 0$ and the marginal cost c when $\mu \to +\infty$.

3 The Stackelberg model (LF model)

In this section we discuss the Stackelberg equilibria of a n-firm model with differentiated product, in which m leaders and $n-m$ followers compete in quantities or in prices.[22] Stackelberg models in which m leaders and $n-m$ followers compete in a homogeneous good market have been considered, among others, by Daughety (1990), Huck et al (2001) and Ino-Matsumura (2009).[23] Within this literature, the paper by Daughety (1990) is closely related to ours. He models quantity competition in a homogenous product market and shows the existence of an inverse U-shaped relationship between social welfare, measured by the aggregate output, and the number of leaders. The welfare-maximizing market structure is associated in the model with the highest asymmetry among firms.

In this section the analysis underlying the Daughety's work is extended to embody product substitutability and price competition. We model sequential interactions in a standard two-stage game in which multiple leaders move simultaneously and independently in the earlier stage, and multiple followers choose their variables simultaneously and independently in the latest stage, given the leaders' choices.[24] We retain the above assumptions on demand and costs, allowing in this case for $c \geq 0$.

3.1 The quantity game in sequential moves

Let q_i (with $i = 1, .., m$) and q_k (with $k = m+1, .., n$) denote respectively the generic leader's and the generic follower's output. For any quantity chosen by the leaders, each follower maximizes his profits with respect to q_k. The solution of the maximization problem for each follower at the second stage gives his optimal quantity as a function of the aggregate leaders' output Q_L:

$$q_k\left(Q_L\right) = \frac{(v-c)(\mu+1)-\mu Q_L}{2n+\mu(1+n-m)} \tag{6}$$

At the first stage each leader i, by incorporating in his objective function the best reply function in (6) for the $n-m$ followers, maximizes his profits with respect to q_i. The following solution is obtained:

$$q_i^{LFq} = \frac{(v-c)(\mu+1)(\mu+2n)}{2n^2(2+\mu)+\mu(\mu(1+m)+4n)}$$

so that at equilibrium:

$$q_k^{LFq} = \frac{(v-c)(1+\mu)\left((\mu+2n)^2+2n\mu(n-m)\right)}{(2n^2(\mu+2)+\mu(\mu m+4n)+\mu^2)(\mu(n-m)+\mu+2n)}$$

[22] In the LF model the number m of leaders is taken as a measure of market structure asymmetry: $m = 0$ and $m = n$ represent the symmetric cases which imply simultaneous moves, while any $m \in]0,1[$ implies sequential moves.

[23] The Daughety's analysis focuses on the welfare-enhancing character of horizontal mergers in Stackelberg markets and is revisited by Ino-Matsumura (2009) under more general demand and cost functions, while Huck et al (2001) deal with the profitability of mergers for any pair of firms (leaders or followers).

[24] Sequential interactions can be also modeled as a hierarchical Stackelberg game, namely a multi-stage game in which firms choose their market variable sequentially at each stage. See Okuguchi and Yamazaki (1994) and Pal and Sarkar (2002) as possible examples.

These quantities coincide when $\mu = 0$, collapsing to the quantities $(v - c)/2n$ produced in independent markets, and diverge progressively when μ increases. Quantity differentials are maximum under perfect substitutability, case in which the optimal quantities collapse to the solutions of the Stackelberg asymmetric game found in Daughety (1990) and Huck et al (2001), $q_i^{LFq} = \frac{v-c}{1+m}$ and $q_k^{LFq} = \frac{v-c}{(1+m)(n-m+1)}$.[25]

An inspection of the SPNE outcomes (see Table 1c for the expression of profits) reveals that $q_i^{LFq} > q_k^{LFq}$ and $\pi_i^{LFq} > \pi_k^{LFq}$ for any m in the open interval $(0, n)$ and any $\mu \in (0, +\infty)$. Indeed, individual quantities produced by the leaders are always higher than those of the followers, for any degree of product substitutability and any market structure; higher profits accrue to leaders as a result of their higher sales.

Let us now investigate the way in which the equilibrium is affected by changes in the parameters m and μ. The analysis of the role of market structure on firms' output and profits is stated in the following proposition.

Proposition 3 *For any given μ, as the number of leaders m increases over the interval $(0, n)$, there is a first interval $(0, \widetilde{m}_1)$ in which the total production of both leaders and followers increases in m, leading to $\partial Q^{LFq}/\partial m > 0$, and profits are such that $\partial \pi_i^{LFq}/\partial m < 0$ and $\partial \pi_k^{LFq}/\partial m < 0$. In the second interval $(\widetilde{m}_1, m_1)$, $\partial \pi_i^{LFq}/\partial m < 0$ and $\partial \pi_k^{LFq}/\partial m > 0$, while in the third interval (m_1, n), $\partial \pi_i^{LFq}/\partial m > 0$ and $\partial \pi_k^{LFq}/\partial m > 0$: in these two subsets $\partial Q^{LFq}/\partial m < 0$.*

A proof is provided in Appendix A5.

For any given μ, our results show an inverse U-shaped behavior of Q over the interval $(0, n)$ that can be explained as follows. The higher production by leaders pushes towards a lower production by followers which are induced to be accommodating due to strategic substitutability of quantities. The effectiveness of such an aggressive strategy in increasing the leader's profits, namely their commitment power, depends on the competitive pressure among leaders - the tougher the competition among leaders, the weaker their ability to implement aggressive and profitable strategies. Overall, two effects of an increase of m on total quantity and welfare can be identified: a positive effect due to the presence of more leaders which contributes to a greater output expansion and to lower prices; a negative effect associated with the increased competition among leaders which reduces overall production and raises prices. The first effect prevails when $m \in (0, \widetilde{m}_1)$. Indeed, when m is low, competition among leaders is still soft, so that an increasing number of leaders contributes significantly to raising market quantity, notwithstanding the negative effect of a decreasing followers' output. Profits enjoyed by both leaders and followers will decrease as a consequence of the decreasing individual quantities. In contrast, the second effect prevails in the two intervals $(\widetilde{m}_1, m_1)$ and (m_1, n) where the followers' output reduction is

[25] This quantity game with sequential moves collapses to a standard Cournot model with differentiated products when $m = 0$ (all firms are 'followers') or $m = n$ (all firms are 'leaders').

not offset anymore by the progressively slighter output expansion by the leaders. The consequence is a reduction of the aggregate output in these intervals. Notice that only in the third interval (m_1, n) the profits of both leaders and followers increase, since a more significant market quantity reduction enables all firms to benefit from higher prices.

Moreover, we show that the way in which market structure affects social welfare is more articulated in the presence of product differentiation as compared to a homogeneous product market. By considering for analytical tractability the aggregate quantity behavior as a proxy of the correspondent welfare behavior, we find that with respect to the Daughety's result imperfect substitutability reduces the degree of market asymmetry required for social optimality. Indeed, we find $\widetilde{m}_1(\mu) \geq n/2$ for $\mu \in]0, +\infty]$,[26] i.e. welfare is maximized in the presence of a higher number of leaders. Decreasing values of μ, for any given m, reduce market quantity and social welfare, raising the profits of both leaders and followers. More interestingly, we find $\partial \widetilde{m}_1 / \partial \mu < 0$ when $\mu > 2n$, which shows that increasing product differentiation makes the critical value $\widetilde{m}_1$ to increase, provided that μ is high enough. When $\mu < 2n$, differentiation progressively lowers $\widetilde{m}_1$. As shown in Figure 3 in which the social welfare function is plotted, this non-monotone pattern of $\widetilde{m}_1$ is mirrored in a non-monotone behavior of welfare with respect to changes of μ.

This analysis is synthesized in the following proposition.

Proposition 4 *As long as substitutability among goods is high enough ($\mu > 2n$), increasing product differentiation widens the interval $(0, \widetilde{m}_1)$ in which overall quantity and welfare increase in the number of leaders. When products become less alike ($\mu < 2n$), increasing differentiation shrinks the interval $(0, \widetilde{m}_1)$. Product differentiation alters the socially optimal market structure accordingly to these changes.*

The non-monotone relationship between the parameter μ and $\widetilde{m}_1$ is crucial in explaining the role of product differentiation on the socially optimal market structure. In this regard, we argue that increased product differentiation on the one hand softens competition among leaders, limiting the loss of commitment power associated with an increase in m; on the other hand, it creates market niches and reduces the incentives for leaders to behave aggressively, thus reducing the effects of differences between leaders and followers.[27] Indeed, when firms' products are close substitutes (μ high) the differences of roles are still relevant, so that the main channel through which μ affects the optimal market structure is the competition-among-leaders effect - as μ decreases the optimal number of leaders progressively increases. Conversely, when substitutability is very low, the main effect of a further decrease in μ is that related to the reduction of the differences of roles - the optimal number of leaders decreases. The

[26] The optimal market structure $\widetilde{m}_1$ collapses to $n/2$ when $\mu \to +\infty$, as found by Daughety.

[27] As in the quantity LF model, the sign of the derivative $\partial \left(q_i^{MEq} - q_k^{MEq} \right) / \partial \mu$ is always positive, showing that product differentiation reduces monotonically the effects of firms' differences in a quantity model.

differences of roles become irrelevant when $\mu = 0$: in this case all the agents act as local monopolists and the critical value $\widetilde{m}_1$ is undetermined, clearly showing that when markets are completely independent, welfare does not depend on the relative number of leaders and followers.

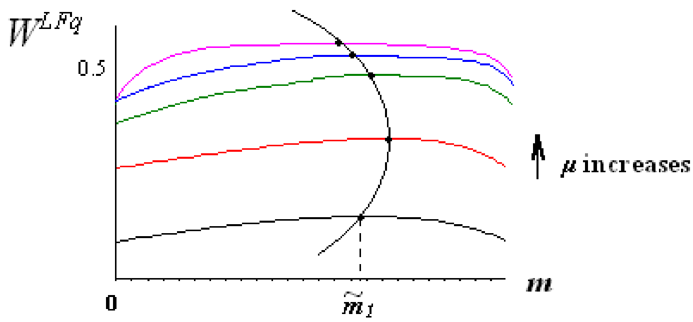

Figure 3: Social welfare in the LF quantity model ($n = 10$).

3.2 The price game in sequential moves

The above leader-follower game is now examined under the hypothesis of price competition. By solving the profit maximization problem of a generic follower k , we obtain the following optimal choice as a function of any price set by the first-movers:

$$p_k\left(P_L\right) = \frac{n(v+c)+c\mu(n-1)+\mu P_L}{n(\mu+2)+\mu(m-1)} \tag{7}$$

where P_L is the sum of prices set by leaders at the first stage.

At the first stage each leader, by taking into account the best reply function in (7) for the $n-m$ followers, sets the following profit-maximizing price:

$$p_i^{LFp} = \frac{2n^2(v+c)+\mu n v(2n-1)+\mu n c(4n-3)+\mu^2 c(2n(n-2)+m+1)}{2\mu n(3n-2)+\mu^2(2n(n-2)+m+1)+4n^2}$$

By substituting the latter in (7), we get the optimal price of each follower:

$$p_k^{LFp} = \frac{v\left(\left(2n^3+(2m-4)n^2+n\right)\mu^2+\left(6n^3+(2m-4)n^2\right)\mu+4n^3\right)}{4n^2\mu(4n-3+m)+2\mu^2 n(n(5n-9)+m(3n-1)+3)+\mu^3(2n(n-2)+m+1)(m+n-1)+8n^3} + \frac{c\left(\left(n(2n(n-3)+(2n-3)m+5)+m^2-1\right)\mu^3+\left(2n^2(4n-7)+2nm(2n-1)+5n\right)\mu^2+\left(10n^3+(2m-8)n^2\right)\mu+4n^3\right)}{4n^2\mu(4n-3+m)+2\mu^2 n(n(5n-9)+m(3n-1)+3)+\mu^3(2n(n-2)+m+1)(m+n-1)+8n^3}$$

It is easy to check that these prices coincide at the extremes of the interval $(0, +\infty)$ in which μ lies, and collapse to the marginal cost c when $\mu \to +\infty$ and to the monopoly price $(v+c)/2$ when $\mu = 0$.

It can be also checked that $p_i^{LFp} > p_k^{LFp}$ and $\pi_i^{LFp} < \pi_k^{LFp}$, for any m in the open interval $(0, n)$ and any $\mu \in (0, +\infty)$ (the equilibrium profits are reported in Table 1d). Indeed, followers are able to undercut the leaders' price and gain larger market shares, thus achieving a competitive advantage over the first-movers.[28]

The impact on profits and welfare of changes in market structure is outlined in the following proposition.

Proposition 5 *For any given μ, as the number of leaders m increases over the interval $(0, n)$, there is a first interval $(0, \widetilde{m}_2)$ in which $\partial Q^{LFp}/\partial m < 0$, and profits are such that $\partial \pi_i^{LFp}/\partial m > 0$ and $\partial \pi_k^{LFp}/\partial m > 0$. In a second interval $(\widetilde{m}_2, m_2)$, $\partial \pi_i^{LFp}/\partial m > 0$ and $\partial \pi_k^{LFp}/\partial m < 0$, while in the third interval (m_2, n), $\partial \pi_i^{LFp}/\partial m < 0$ and $\partial \pi_k^{LFp}/\partial m < 0$. In the latter two subsets $\partial Q^{LFp}/\partial m > 0$.*

A proof is provided in Appendix A6.

Our results show a U-shaped relationship between m and welfare, when the latter is approximated by total market quantity. In order to explain this pattern, it can be noticed that the price game with sequential moves collapses to a Bertrand differentiated model when $m = 0$ or $m = n$. Contrary to the quantity game, these symmetric solutions represent the socially optimal market structures of the model. In a price game the presence of one or more first-movers reduces the aggressiveness of the other firms and hence the overall market competitiveness. Starting from $m = 0$, the effect on welfare of a progressive increase in m is indeed twofold. On the one hand, the number of high-pricing firms increases - with an obvious negative effect on welfare; on the other hand a tougher competition among leaders emerges, which lowers their prices and induces, due to strategic complementarity, a followers' reaction in the same direction. The net effect on welfare is negative in the interval $(0, \widetilde{m}_2)$ - in which the number of leaders is not high enough to ensure that competition among them enhances welfare significantly - and is positive otherwise. Indeed, welfare is minimized in $\widetilde{m}_2 < n/2$ for $\mu \in]0, +\infty[$: a marginal increase in the number of leaders turns to generate overall welfare improvements before their number has reached that of the followers.

The overall effect of product substitutability in this game is stated in the following proposition.

Proposition 6 *The introduction of imperfect substitutability does not affect the socially optimal market structures $m^* = 0$ and $m^* = n$. Increasing product differentiation widens the interval $(0, \widetilde{m}_2)$ in which the overall quantity and welfare decrease in m, altering accordingly the welfare-minimizing market structure.*

Indeed, increasing product differentiation progressively causes the critical value $\widetilde{m}_2$ to increase, since $\partial \widetilde{m}_2/\partial \mu < 0$ for $\mu \in]0, +\infty[$. The intuition is that, as products become less substitutable, price competition becomes less

[28] The issue of price leadership in duopolistic markets has been extensively investigated in the literature. See Amir and Stepanova (2006) for a generalization of the main results in Bertrand markets.

hard and the price-competition-among-leaders effect dominates the leader-high-price effect for a higher number of leaders. This negative relationship between μ and $\widetilde{m}_2$ is shown in Figure 4 where the social welfare function is plotted.

As remarked in the quantity competition case, imperfect product substitutability affects welfare both directly - through an increase in the equilibrium mark-ups - and by influencing the relative behavior of the two types of firms. As for this latter channel, imperfect product substitutability widens the scope for leaders to set higher prices, but at the same time makes firms more independent from each other. While the first effect amplifies the differences associated with the roles played by firms on the market, the second dampens these differences. The first effect prevails when the products are still close substitutes, the second when μ is sufficiently low.[29] However, in the analysis of the welfare properties of market structure, the nature of price competition is such that the effect of changes of μ on the differences of roles is negligible as compared to its impact on the competition-among-leaders effect, so that an increasingly higher number of leaders causes welfare improvements when μ decreases. It can be noticed that ; no differences between leaders or followers are observable in the two limit cases $\mu = 0$ or $\mu = +\infty$, since all the agents act respectively as local monopolists or as firms pricing at marginal cost. In these latter cases, the critical value $\widetilde{m}_2$ is undetermined, clearly showing that when markets are completely independent, or when they are perfectly competitive, welfare does not depend on the number of leaders over followers.

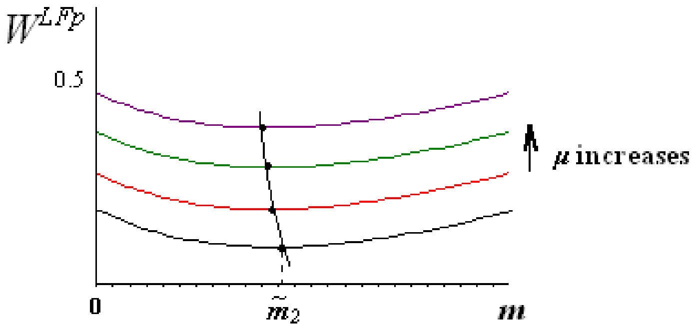

Figure 4: Social welfare in the LF price model ($n = 10$).

[29]As in the ME price model, the impact of product substitutability on the effects of role differences is captured by the derivative $\partial\left(p_i^{MEp} - p_k^{MEp}\right)/\partial\mu$ which is non-monotone in μ, being positive for low values of μ and negative thereafter.

4 Discussion

4.1 The main results

In this section we summarize the main results of the previous sections and highlight the different impact of market structure on the market outcomes of the ME and the LF models. The analysis developed in the quantity ME model has revealed that an increasing number of managerial firms causes the aggregate output and welfare to increase for any degree of product substitutability, so that the socially optimal market structure requires all firms to be managerial. This result is consistent with the fact that a quantity ME game collapses to a simultaneous Cournot model when all firms are entrepreneurial and converges monotonically to a welfare-enhancing market structure when all firms are managerial. Conversely, the inspection of a ME price game has shown that the presence of an increasing number of managerial firms favours collusive behavior and, for any market structure, has an adverse effect on welfare provided that product substitutability is low enough. For a higher degree of the latter, the presence of a sufficiently large number of managerial firms is responsible of limiting the negative effects of the collusive attitude, contributing positively to social welfare through the profit channel. In any case, the socially optimal market structure requires all firms to be entrepreneurial and to play a standard simultaneous Bertrand game.

Let us now discuss the results of the LF model. In the quantity competition case, we find that heterogeneity among firms, which implies sequential moves in this model, always yields higher welfare than in a model with simultaneous moves. As a result, social optimality requires a significant degree of asymmetry among firms. As far as price competition in sequential games is concerned, we have shown that the socially optimal market structures entail perfect symmetry among firms, so that any heterogeneity in roles causes a detrimental effect on welfare. Clearly, in both these settings, welfare varies non-monotonically with the number of leaders, since it depends on the degree of market asymmetry which determines the intensity of competition between and within the groups of leaders and followers. A fiercer competition among leaders induced by their increasing number, is shown to enhance welfare provided that: a) their number is not too high in the quantity model, that is when the toughness of competition among leaders does not reduce significantly their beneficial aggressiveness; b) their number is sufficiently high in a price model, so that the beneficial competition among leaders can compensate the welfare losses due to the sequentiality of moves.

4.2 The ME model and the LF model: a comparison

A comparison between the market outcomes of the ME and the LF models reveals that, for any mode of competition and any given total number of firms, the two games produce the same outcome when there is only one delegating firm on the market. This outcome entails identical market variables set by the managerial firm and the first-mover on the one hand, and by followers and entrepreneurial firms on the other hand. Indeed, the only managerial firm assumes, in the presence of quantity competition, the role of a Stackelberg leader for any $\mu \in]0, +\infty]$ and, in the presence of price competition, the less favorable status of a first-mover in a sequential game, for any $\mu \in]0, +\infty[$. Observationally equivalent market outcomes derive from an equivalent strategic behavior of each leading firm which faces competition by firms identical with respect to motives and timing. In the same intervals of the substitutability parameter, differences between the two models, resulting in different profitability and welfare conditions, arise when the number of delegating/leader firms increases. The presence of a higher number of firms playing at the first stage of each game is decisive in determining the different results in the two games, since it is the introduction of competition among these firms that modifies rivalry between and within the groups of firms in the two game structures, and consequently the strategic behavior of the leading firms.

A comparison between the two models for any given mode of competition, reveals that quantity competition between the managerial and the entrepreneurial firms leads to more competitive and efficient market outcomes than competition between leaders and followers. Conversely, under price competition, more desirable welfare properties are associated with interactions in a sequential model rather than in a delegation model. Moreover, in a quantity competition framework, the commitment advantage of leaders is always higher than the advantage gained in an equally structured market by firms which commit to delegation. However, a comparison of profits gained by the most successful firms under price competition shows that, for any market structure, the entrepreneurial firms gain more than the late-movers, thus suggesting that a managerial model gives scope for higher advantages to the leading firms. By comparing the models with respect to the mode of competition, we finally find that quantity setting behavior turns out to be always more profitable than price setting, whatever the role played by firms and for any degree of product substitutability. The opposite conclusion applies as far as welfare is concerned: social welfare is indeed found to be always higher in any price setting as compared to the corresponding quantity setting.

The overall discussion allows us to rank welfare according to the following relationship:

$$W^{LFp} \geq W^{MEp} > W^{MEq} \geq W^{LFq}$$

The implications associated with the LF models nest those obtained by Boyer and Moreaux (1987) in a duopoly framework with product differentiation. By examining the equilibria in simultaneous or sequential actions under both quantity and price competition, and ranking correspondingly social welfare, they basically find that the Bertrand simultaneous equilibrium dominates in terms of higher welfare the Stackelberg equilibrium in prices which, in turn, dominates the Stackelberg quantity equilibrium which finally dominates the Cournot equilibrium. Our analysis, indeed, extends these results, showing that, for any considered oligopolistic market structure, sequential actions are welfare enhancing with respect to simultaneous moves under quantity competition and are welfare detrimental under price competition.

By comparing the ME and the LF models, we finally want to focus on the impact of product substitutability on the outcomes of the two games, underlying the analogous properties they exhibit when firms compete in the same strategy space. Indeed, and as already pointed out throughout the paper, increasing product differentiation reduces welfare in each set-up by also affecting the firms' relative positions on the market. On the one hand, the analysis of the quantity competition models reveals that increasing product differentiation progressively reduces the effects of behavioral heterogeneity between leaders and followers (or between managerial and entrepreneurial firms) and contributes to a lower degree of competition which is welfare-reducing. On the other hand, in each price competition model the possibility for heterogeneous firms to set through product differentiation different prices with respect to the perfect substitutability case, makes relevant their behavioral differences which contribute to softening competition and impacting negatively welfare.

5 Conclusions

In this paper oligopolistic competition among heterogeneous producers of differentiated products has been modeled in the two frameworks of strategic delegation and sequential competition. Besides providing the conditions for an equivalence result to hold under both quantity and price competition, the paper has clarified the circumstances under which delegation and timing strategies contribute to firms' profitability by inducing pro-competitive or anti-competitive rivals' reactions. For any mode of competition, our analysis has shed light on the market forces enhancing or reducing competition within and between two groups of different firms in each model, providing a welfare ranking which captures the impact of firms' heterogeneity. By exploring the effects of changes in market structure and the role of product substitutability in altering the differences among any given number of mixed firms, this paper has drawn attention to the role of behavioral heterogeneity among technologically identical firms in pursuing individually and mutually beneficial objectives. Useful implications for market policies, such as merger control and measures against dominant position, can be derived from the presented welfare analysis.

Finally the paper, by discussing delegation and first mover strategies as possible sources of commitment power, sheds light on the determinants of the degree of market power gained through different commitments and thus provides a measure of firms' comparative advantages on the market. Along this research line, the paper addresses a point raised by Schelling (1960), according to which "a commitment is beneficial for a player who is the only one able to make a commitment". This conclusion applies to the analysis of duopolistic competition centered around the advantages gained from the unilateral adoption of a commitment, and on the coordination problem that may prevent firms from benefiting from it. The analysis of oligopolistic interactions presented throughout this paper allows us to assess the profitability of delegation or timing strategies under more general assumptions on market structure, when the latter is viewed as the result of the exogenous adoption of those commitments. We leave the analysis of the complex interactions under the hypothesis of endogenous commitment choice as a topic for future research.

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Table 1

a) SPNE outcomes in the ME quantity model

The optimal quantities of respectively managerial and entrepreneurial firms:

$$q_i^{MEq} = \frac{n(v-c)(1+\mu)(2+\mu)(2n+\mu)}{8n^2(n(\mu+1)+\mu)+2n\mu^2(n(n+2)+2)+\mu^3(n+m(n-1)+1)}$$
$$q_k^{MEq} = \frac{(v-c)(1+\mu)\left(2n^2(2+\mu)+\mu(2n+\mu)\right)}{8n^2(n(\mu+1)+\mu)+2n\mu^2(n(n+2)+2)+\mu^3(n+m(n-1)+1)}$$

The optimal firms' profits:

$$\pi_i^{MEq} = \frac{n(v-c)^2(\mu+2n)^2(1+\mu)(2+\mu)\left(n^2(\mu+2)+\mu(2n+\mu)\right)}{(8n^2\mu(n+1)+2\mu^2n(n^2+2n+2)+\mu^3(n+m(n-1)+1)+8n^3)^2}$$
$$\pi_k^{MEq} = \frac{(v-c)^2(\mu+n)(1+\mu)\left(2n^2(\mu+2)+\mu(2n+\mu)\right)^2}{(8n^2\mu(n+1)+2\mu^2n(n^2+2n+2)+\mu^3(n+m(n-1)+1)+8n^3)^2}$$

b) SPNE outcomes in the ME price model

The optimal prices of managerial and entrepreneurial firms: :

$$p_i^{MEp} = \frac{4n^3(v+c)+2\mu n^2v(3n-1)+\mu^2n^2v(2n-1)+\mu c\left(2n^2(5n-3)+n^2\mu(8n-11)+4\mu n+\mu^2(n-1)(2n(n-1)-m+1)\right)}{8n^2\mu(2n-1)+2n\mu^2(n(5n-6)+2)+\mu^3(n-1)(2n(n-1)-m+1)+8n^3}$$
$$p_k^{MEp} = \frac{4n^3(v+c)+2\mu n^2v(3n-1)+\mu^2nv(2n(n-1)+1)+\mu c\left(2n^2(5n-3)+2n^2\mu(4n-5)+3\mu n+\mu^2(n-1)(2n(n-1)-m+1)\right)}{8n^2\mu(2n-1)+2n\mu^2(n(5n-6)+2)+\mu^3(n-1)(2n(n-1)-m+1)+8n^3}$$

The optimal firms' profits:

$$\pi_i^{MEp} = \frac{n(v-c)^2(\mu+2)(2n(\mu+1)-\mu)^2\left(n\mu^2(n-2)+n\mu(3n-2)+\mu^2+2n^2\right)}{((\mu+1)(\mu+2)(2n^3(\mu+2)-4\mu n^2)+\mu^2(4n+\mu(3n-1))-\mu^3m(n-1))^2}$$
$$\pi_k^{MEp} = \frac{(v-c)^2(\mu(n-1)+n)\left(2n^2(\mu+2)(\mu+1)-\mu(2n(\mu+1)-\mu)\right)^2}{((\mu+1)(\mu+2)(2n^3(\mu+2)-4\mu n^2)+\mu^2(4n+\mu(3n-1))-\mu^3m(n-1))^2}$$

c) Profits of leaders and followers in the LF quantity model

$$\pi_i^{LFq} = \frac{(v-c)^2(1+\mu)(\mu+2n)^2\left(\mu n(n-m+3)+\mu^2+2n^2\right)}{(n(\mu+2)-\mu(m-1))(2n(n(\mu+2)+2\mu)+\mu^2(1+m))^2}$$
$$\pi_k^{LFq} = \frac{(v-c)^2(1+\mu)(\mu+n)\left(2\mu n(n-m+2)+\mu^2+4n^2\right)^2}{(n(\mu+2)-\mu(m-1))^2(2n(n(\mu+2)+2\mu)+\mu^2(1+m))^2}$$

d) Profits of leaders and followers in the LF price model

$$\pi_i^{LFp} = \frac{(v-c)^2(2n(1+\mu)-\mu)^2\left(n^2(\mu+2)(1+\mu)-n\mu^2(3-m)+\mu(\mu+nm)-3\mu n\right)}{(n(\mu+2)-\mu(1-m))(2n(\mu+1)(\mu n-2\mu+2n)+\mu^2(1+m))^2}$$
$$\pi_k^{LFp} = \frac{(v-c)^2(n(\mu+1)-\mu)(2n(\mu+1)(n(\mu+2)+\mu m)-\mu(4n(\mu+1)-\mu))^2}{(n(\mu+2)-\mu(1-m))^2(2n(\mu+1)(\mu n-2\mu+2n)+\mu^2(1+m))^2}$$

Appendix

A1 PROOF OF THE NEGATIVE SIGN OF the derivative $\partial\theta_i^{MEq}/\partial\mu$.

By calculating $\dfrac{\partial\theta^{MEq}}{\partial\mu}$ we obtain that $\dfrac{\partial\theta^{MEq}}{\partial\mu} < 0$ when:
$m < \frac{8n^3(\mu+2)+n^2\mu\left(8\mu+\mu^2+20\right)+\mu^2(\mu n+8n+\mu)}{\mu^3(n-1)} = \overline{m}$

where $\overline{m} > n$. This condition is always met in our model in which $m \leq n$.

A2 THE SOCS AT THE DELEGATION STAGE OF A ME QUANTITY GAME

The second order conditions require the following second derivative with respect to θ_i to be always negative:

$\frac{\partial^2\pi_i^{MEq}}{\partial\theta_i^2} = -\frac{(1+\mu)2nc^2\left(n^2(\mu+2)^2+2\mu^2(n+1)+\mu\left(\mu^2+4n\right)\right)}{((2n+\mu(1+n)))^2(2n+\mu)^2}$
Clearly, the condition is always satisfied in our model.

A3 PROOF OF THE NON-MONOTONE PATTERN OF THE DERIVATIVE $\partial\theta_i^{MEp}/\partial\mu$.

The optimal delegation parameter of the delegation game in prices is:

$$\begin{aligned}\theta_i^{MEp} &= \frac{2n^4c\mu(\mu+3)(\mu(\mu+3)+4)-\mu^2(n-1)(+\mu cm(\mu(n-1)+n)-nv(\mu(2n-1)+2n))}{c\left(2n^3(\mu+1)(\mu+2)^2-\mu^3(n(4n-3)+1)-4\mu n(\mu(3n-1)+2n)-\mu^3m(n-1)\right)(n+\mu(n-1))}+\\ &+\frac{2\mu^2nc(\mu(11n-3)+7n)+\mu^4c(n(7n-4)+1)+8n^4c-2\mu n^3c(3\mu+8)(\mu+1)^2}{c\left(2n^3(\mu+1)(\mu+2)^2-\mu^3(n(4n-3)+1)-4\mu n(\mu(3n-1)+2n)-\mu^3m(n-1)\right)(n+\mu(n-1))}\end{aligned}$$

The derivative $\frac{\partial\theta_i^{MEp}}{\partial\mu}$ is shown to be negative when $m < \overline{m}$, where:

$$\begin{aligned}\overline{m} &= \frac{2n^4(\mu+1)\left(2n(\mu+1)(\mu+2)(\mu(\mu-2)-4)-\mu\left(\mu^2(7\mu+5)-28(1+\mu)\right)\right)}{\mu^3(n-1)\left(2n^2(\mu+1)^2-n\mu(3\mu+4)+\mu^2\right)}+\\ &+\frac{4n^3\mu^4(5\mu+7)-2n^3\mu^2(11\mu+16)-\mu^3n^2(\mu(15\mu+16)-2)+2\mu^4n(2+3\mu)-\mu^5}{\mu^3(n-1)\left(2n^2(\mu+1)^2-n\mu(3\mu+4)+\mu^2\right)}\end{aligned}$$

As shown in the figure below where $\overline{m}$ is described with $n = 2$, $n = 3$, $n = 4$ and $n = 10$ (and where it is also assumed $v = 1$ and $c = \frac{1}{2}$), this threshold level of m increases in μ and n, converging asymptotically to the positive value $2n^2 - 2n + 1$ when $\mu \to +\infty$.

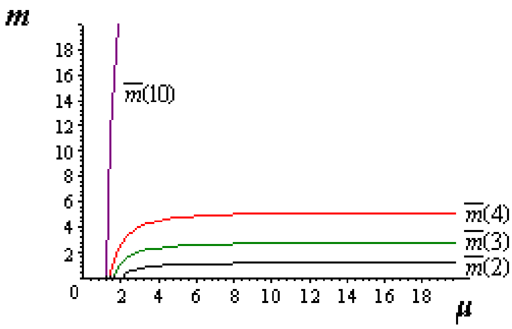

Figure 5: The threshold values $\overline{m}\,(\mu)$ as functions of different values of n.

The functions $\overline{m}\,(2)$ and $\overline{m}\,(3)$ and the lines for $n = 2$ and $n = 3$ are drawn in the following figure:

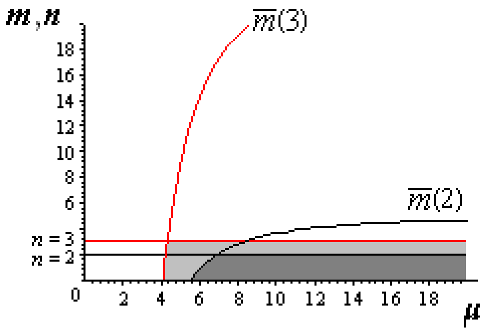

Figure 6: The set of values of m consistent with $\partial\theta_i^{MEp}/\partial\mu < 0$ when $n = 2$ and $n = 3$.

It can be noticed that the condition for the negativity of $\frac{\partial\theta_i^{MEp}}{\partial\mu}$ is met in our model when $m < \overline{m} < n$, that is for all the values m in the areas below each curve and the correspondent line n (the shadow areas in Figure 6). These values are clearly associated with the highest degree of product substitutability.

A4 The SOCs at the delegation stage of a ME price game

The second order conditions require the following second derivative with respect to θ_i to be always negative:

$$\frac{\partial^2 \pi_i^{MEp}}{\partial \theta_i^2} = -\frac{2c^2(2+\mu)\left(\mu^2+2n^2+n\mu(3n-2)+n\mu^2(n-2)\right)(n+\mu(n-1))^2}{n(2n+\mu(n-1))^2(2n(1+\mu)-\mu)^2}$$

which is always met in our model.

A5 Proof of proposition 3

By considering the leaders' profits, we find that $\frac{\partial \pi_i^{LFq}}{\partial m} > 0$ when $m_1 < m < m_2$,
where $m_1 = \frac{A-\sqrt{B}}{4\mu n}$ and $m_2 = \frac{A+\sqrt{B}}{4\mu n}$
with $A = 4n^2(\mu+2) + \mu(3\mu+10n)$
and $B = \left(\mu(9\mu+16n) + 8n^2(\mu+2)\right)(2n+\mu)^2$.

Since $m_2 > n$, we conclude that the profits of each leader increase in m over the interval (m_1, n).

As regards the followers' profits, we find that $\frac{\partial \pi_k^{LFq}}{\partial m} > 0$ when $\widetilde{m_1} < m < \frac{2n+\mu(n+1)}{\mu}$
and $\frac{4n^2+\mu\left(2n^2+4n+\mu\right)}{2\mu n} < m < \widetilde{m_2}$,
where $\widetilde{m}_1 = \frac{4n^2+\mu\left(2n^2+4n+\mu\right)}{2\mu n} - \frac{\sqrt{D}}{2\mu n}$ and $\widetilde{m}_2 = \frac{4n^2+\mu\left(2n^2+4n+\mu\right)}{2\mu n} + \frac{\sqrt{D}}{2\mu n}$
with $D = \left(2n^2(\mu+2) + \mu(4n+\mu)\right)(2n+\mu)^2$.

We can conclude that the followers' profits increase in m over the interval $(\widetilde{m_1}, n)$, since $\frac{2n+\mu(n+1)}{\mu}$, $\frac{4n^2+\mu\left(2n^2+4n+\mu\right)}{2\mu n}$ and $\widetilde{m_2}$ are greater than n.

We find $\partial Q^{LFq}/\partial m < 0$ in the same interval $(\widetilde{m_1}, n)$. We finally find that $m_1 > \widetilde{m}_1$ is always verified under our hypotheses.

A6 Proof of proposition 5

We find that $\dfrac{\partial \pi_i^{LFp}}{\partial m} > 0$ when $m_1 < m < m_2$,
where $m_1 = \frac{A-\sqrt{B}}{4\mu n(1+\mu)}$ and $m_2 = \frac{A+\sqrt{B}}{4\mu n(1+\mu)}$
with $A = -2n(1+\mu)(\mu(2n-5)+4n) - 3\mu^2$
and $B = \left(8n(1+\mu)(\mu(n-2)+2n) + 9\mu^2\right)(\mu(2n-1)+2n)^2$.

Since $m_1 < 0$, the profits of each leader increase in m over the interval $(0, m_2)$.

As regards the profits of followers, we find that $\frac{\partial \pi_k^{LFp}}{\partial m} > 0$ when
$\widetilde{m}_1 < m < -\frac{\mu(n-1)+2n}{\mu}$
and $-\frac{2\mu^2 n(n-2)+2\mu n(3n-2)+4n^2+\mu^2}{2\mu n(1+\mu)} < m < \widetilde{m}_2$
where $\widetilde{m}_1 = \frac{C-\sqrt{D}}{2\mu n(1+\mu)}$ and $\widetilde{m}_2 = \frac{C+\sqrt{D}}{2\mu n(1+\mu)}$
with $C = -2n\,(1+\mu)\,(\mu\,(n-2)+2n) - \mu^2$
and $D = \left(2n\,(1+\mu)\,(\mu\,(n-2)+2n)+\mu^2\right)(\mu\,(2n-1)+2n)^2$.

Since $\widetilde{m}_1$, $-\frac{\mu(n-1)+2n}{\mu}$ and $-\frac{2\mu^2 n(n-2)+2\mu n(3n-2)+4n^2+\mu^2}{2\mu n(1+\mu)}$ are always negative, the profits of each follower increase in m over the interval $(0, \widetilde{m}_2)$. In the same interval we find $\partial Q^{LFp}/\partial m < 0$. We finally find that $\widetilde{m}_2 < m_2$ is always verified under our hypotheses.