Estimating Import-Demand Function in ARDL Framework: The Case of Pakistan

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Abstract

We develop a structural econometric model of import demand for Pakistan, with binding foreign exchange constraint. ARDL and DOLS techniques are used to estimate the log-run coefficients of price and income elasticities. The empirical results from ARDL bound testing approach and Johansen’s method for cointegration show strong evidence of the existence of a long-run stable relationship among the variables included in the import demand model. The price and income elasticity estimates have correct signs and are statistically significant. The coefficient of scarcity premium, as it appeared statistically significant with correct sign, confirms the presence of a binding foreign exchange constraint on aggregate import demand, particularly before the period of trade liberalization.

JEL Classification: F14; O16
Keywords: Import Demand, Foreign Exchange Constraint, ARDL, DOLS, Pakistan
1. Introduction

The econometric analysis of income and price elasticity of import demand has been a one of the most active research areas in international economics. It has been accomplished the empirical literature for more than a quarter century. Some of the important studies are Goldstein and Khan (1985), Reinhart (1995), Carporale and Chui (1999), Oskooee (2005). Although a large number of studies have been done, however, the main issue which is ignored almost in most of these studies is about the theoretical foundation or microeconomic foundation of the theoretical models. These foundations have been drawn from the optimality condition of an intertemporal maximization program, under the assumptions of rational expectation permanent income hypothesis (RE/PIH), where resources are consumed between present and future periods.

Another important issue has been neglected in the import demand models, is prevalence of foreign exchange constraint. The foreign exchange constraint binds each-period consumption. The models used this idea combines traditional model of import demand with the stock of real reserves (see, for instance, Arize (2004)), or combines with the sum of foreign exchange receipts and foreign exchange reserves (see, for details, Moran (1988)), or combines with contemporaneous export receipts (see, for details, Mazeri (1995)). In these cases foreign exchange variable only determines the volume of import demand, which creates the problem of near identity. Price and income elasticities also receive non-sensational results.

This study aims to modeling aggregate import demand function for Pakistan. A structural econometric equation of import demand is derived by applying a two good version of rational expectation permanent income hypothesis with binding foreign exchange constraints. The tow alternative techniques namely Autoregressive Distributed Lag (ARDL) and Dynamic Ordinary Least Square (DOLS) are used to estimate the price and income elasticities. Moreover, the trade liberalization effects are also analyzed on demand for imports.
2. Theoretical Model

Following Clarida (1994), the rational expectation permanent income model of representative agent is used to derive the import demand function. This model incorporates a binding foreign exchange constraint. The representative agent consumes two goods, a home good \((H_i)\) and an imported good \((M_i)\). The optimization problem is defined by two constraints, first one is budget constraint describing the assets accumulation and the second is an inequality constraint describing the foreign exchange availability constraint. The optimization problem of the representative agent is as follows:

\[
\max_{[H_i, M_i, A_i]} V = E \int_{t=0}^{\infty} e^{-\delta t} U (H_i, M_i) dt
\]

Subjected to

\[
\dot{A} = r A_i + \ddot{Y}_i - H_i - P_i H_i \quad (1)
\]

\[
P_i M_i \leq F_i \quad (2)
\]

where \(P_i\) = relative price of imports, \(A_i\) = assets, \(\ddot{Y}_i\) = labour income, \(F_i\) = total amount of foreign exchange available, \(r\) = constant real interest rate, \(\delta\) = the subjective rate of time preference which the representative agent used to discount the future value. Finally, \(\dot{A} = \frac{dA_i}{dt}\) is a time derivative. If constraint (2) is binding then the volume of imports is equal to foreign exchange availability and the standard price and income variables are irrelevant. The current value Hamiltonian function of the optimization problem of the representative agent can be written as:

\[
L = U(H_i, M_i) + \lambda_i [r A_i + \ddot{Y}_i - H_i - P_i M_i] + \mu_i [F_i - P_i M_i]
\]

where \(H_i\) and \(M_i\) are control variables because they are included in the objective function which is dependent upon control as well as a state variable which is \(A_i\) and \(\lambda_i\).
is the costate variable and is called marginal utility of wealth and \( \mu \) is the Lagrange multiplier associated with the foreign exchange constraint. The maximum principle of the optimization problem is defined as:

\[
U_H = \frac{\partial L}{\partial H} = \lambda_t, \tag{3}
\]

\[
U_M = \frac{\partial L}{\partial M} = P_t(\lambda_t + \mu_t), \tag{4}
\]

\[
\dot{\lambda} = \frac{\partial L}{\partial A} = \lambda_t(\delta - r). \tag{5}
\]

Following Clarida (1994), it is assumed that \( U(.) \) is an addilog utility function:

\[
U(H_t, M_t) = C_t \frac{H_t^{1-a}}{1-a} + B_t \frac{M_t^{1-\eta}}{1-\eta}
\]

where \( C_t \) and \( B_t \) are random, strictly stationary shocks to preference. By inserting the Clarida’s addilog utility function into the original current value, the Hamiltonian equation is rewritten as follows:

\[
L = C_t \frac{H_t^{1-a}}{1-a} + B_t \frac{M_t^{1-\eta}}{1-\eta} + \lambda_t [rA_t + \tilde{Y}_t - H_t - P_t M_t] + \mu_t [F_t - P_t M_t]
\]

The first order condition of the optimization problem is defined as follows:

\[
C_t H_t^{-a} = \lambda_t \tag{7}
\]

\[
B_t M_t^{-\eta} = P_t \lambda_t (1 + \mu_t^*) = \lambda_t P_t^* \tag{8}
\]

where \( \mu_t^* = \frac{H_t}{\lambda_t} = \frac{\mu}{U_H} \) is the scarcity premia, and \( P_t^* \) is the scarcity price at which transaction occur at the shop floor in the secondary market if the secondary market fails to clear (Shilpi (2001)). Equation (7) is used to eliminate \( \lambda_t \) from equation (8) and take logarithm to get the following equation:
\[ b_i - \eta m_i = c_i + p_i - ah_i + \ln(1 + \mu_i^*) \]  

(9)

where the lower case letter denote natural logarithm of the corresponding upper case letters. In order to derive the long run import demand model, the steady state conditions is applied in the model that is \(A = \dot{\lambda} = 0\). Also, the steady state is characterized by the equilibrium price relations implying \(P_i = P_i^*\). The total household income evaluated at equilibrium price is denoted by \(Y^*\) and it includes both labor and assets income. The steady state solution implies the following condition:

\[ Y^* = H + P^* M \]  

(10)

Using the steady state condition and taking logarithm, we get the following expression for \(h_i\):

\[ h_i = \ln(Y_i^* - P_i^* M_i) \]

\[ = \ln(Y_i - P_i M_i) \]  

(11)

By inserting equation (11) into equation (9) we eliminate the \(h_i\) and solve for \(m_i:\)

\[ m_i = \frac{a}{\eta} \ln(Y_i - P_i M_i) - \frac{1}{\eta} p_i - \frac{1}{\eta} (1 + \mu_i^*) + \xi_i \]  

(12)

where \(\xi_i = \frac{1}{\eta} (b_i - c_i)\) is the composite preference shock. If the foreign exchange constraint is not binding than \(\mu_i^*\) is zero and the remaining import demand equation from equation (12) is the same as used by many studies for developed and developing countries which used traditional model of import demand (see, for instance, Sinha (1997), Goldstein and Khan (1985), Houthakker (1984), Bahmani-Oskooe (2005), Faini et al. (1992) and many others).

In equation (12), \(Y_i\) denotes the total expenditures which include expenditures on domestic goods as well as on imported items. Thus, \(\ln(Y_i - P_i M_i)\) can be defined as GDP minus imports. Traditional import demand models include value of GDP or GNP.
But equation (12) includes expenditure on home goods, which is achieved by excluding imports from GDP variable. When the foreign exchange constraint is binding, the Kuhn-Tucker theorem requires that \( \mu_i > 0 \), and hence \( \mu_i^* > 0 \).

If we use foreign exchange variable in the regression equation it creates the problem of near identity. So real total expenditure ((GDP + import - export)/foreign exchange available) is used instead of \( \mu_i^* \) and that new variable is denoted by \( Z_i^* \). There is no direct effect of \( Z_i \) on import demand, but through the medium of \( \mu_i^* \) and they are positively related. Since \( \frac{\partial \mu_i^*}{\partial Z_i} > 0 \), import demand will very negatively with \( Z_i \). Therefore,

\[
\frac{\partial M_i}{\partial Z_i} = \frac{\partial M_i}{\partial \mu_i^*} \times \frac{\partial \mu_i^*}{\partial Z_i} < 0
\]

To check the effect of trade liberalization a dummy for trade liberalization is used. It is one for the period from 1975-1986 and zero for the period from 1987-2008, the post liberalization period and it is multiplied with \( Z_i \) variable and new variable is denoted by \( Z_i^2 \). Finally, the following equation is derived for estimating import demand function:

\[
m_i = \frac{a}{\eta} \ln(Y_i - P_iM_i) - \frac{1}{\eta} p_i - \frac{\theta_i}{\eta} Z_i^* + \xi_i
\]

\[
= \pi_1 \ln(Y_i - P_iM_i) - \pi_2 p_i - \pi_3 Z_i^* + \xi_i
\]

3. Literature Review

Clarida (1994) used rational expectation permanent income model to develop a structural econometric equation. Quarterly data is used beginning from 1967:1 to 1990:2 for non-durable consumer goods. The Engle and Granger casualty test is applied for estimation. The results show that all variables in the regression equation are cointegrated and they are highly significant with correct signs.
Another study by Reinhart (1995) estimated two separate equations for imports and exports demand. These empirical equations are applied on 12 developing countries from three regions Africa, Asia and Latin America. Annual data is used from the period 1970-92. The estimates on Engle and Granger causality test show that income and relative prices parameters are significant for all of these 12 countries.

Amano and Wirjanto (1996) examined intertemporal substitution in import consumption of US non durable goods by using permanent income model. He used addilog utility function and the concept of preference shock. Quarterly data for the period 1967:1 to 1993:2 is used. Two approaches are used for estimation one is Engle and Granger causality test and the second is GMM estimates.

The estimation results provide evidence that intertemporal substitution is an important feature of import consumption and the conventional import demand models that do not account for this feature are required to compare with this feature. Import and domestic consumption estimates are highly significant, with correct signs and well within the range of previous estimates.

Carporale and Chui (1999) estimated income and relative price elasticity of trade in a cointegration framework for 21 countries using annual data for the period from 1960 to 1992. The ARDL and DOLS estimates confirm the existence of cointegration relationship between growth rates and income elasticity estimates. It shows that faster growing economies have high income elasticity of their exports but lower import elasticities.

Abrishami and Mehrara (2000) conducted a study by estimating the demand equations for import of consumer, intermediate and capital goods based on ARDL methodology. The quarterly data (198-1999) is used for estimation. The model of long- and short-run demand for imports are estimated using proper selection of criteria for each variable in different groups. The results confirm that the variable parallel market exchange rate, best explains the behavior of the different categories of imported goods in Iran. The results also show proximity of parallel market exchange rate for opportunity cost of importers.
Emran and Shilp (2001) used structural econometric model of aggregate imports for India and Sri Lanka. To estimate the model time series data is used for the period 1952-99 for India and 1960-1995 for Sri Lanka. ARDL and DOLS method is used for estimation. The estimates of income and price elasticities derived from the model satisfy the theoretical sign restriction and are highly significant for both the countries. The mean of income elasticity is 1.07 which shows long run unitary income elasticity. The mean of price elasticity is -0.72 and foreign exchange availability variable is also highly significant with correct positive signs for both of the countries.

Ernkle-Rousse and Danial (2002) analyzed the difference of trade price elasticities. Bilateral annual trade data is used for estimation, for 14 countries, 16 trading partners, and 27 industries for the period 1960 to 1994. Transformed least square and instrumental variables are used for estimation. The results support the recent studies on substitution elasticity estimates using monopolistic competition.

Ooskooee (2005) estimated the trade elasticities for 28 countries. Import demand is dependant upon income, relative process and exchange rate variable. The estimated coefficients have unique results for each country. But the general conclusion is that, the sum of trade elasticities is greater than one. It shows that the Marshall-Lerner condition is met and currency depreciation could improve the trade balance in the long run.

Narayan and Narayan (2005) estimated long-run relationship between import volumes, domestic income and relative prices for Fiji in a cointegration framework. Their results confirm this finding that domestic income has a positive impact on import volumes, while an increase in relative prices reduce import volumes. Growth in income has a significant and elastic impact on import demand in the long run, which suggests that higher growth will induce higher demand for imports.

Frimpong and Fosu (2007), investigated the import demand behavior for Ghana by using disaggregated expenditure components to total national income, and that are total consumption expenditures, investment and expenditures on total exports. Autoregressive distributed lag (ARDL) and bound F test is used for estimation, under the sample period
1970 – 2002. And error correction model is used to separate the short and long run elements of import demand. The results show a positive relationship between the three expenditure components and aggregate import demand. Relative price is also inelastic, but have negative impact on import demand. It is required that Ghana will improve its price competitiveness in external trade to reduce its trade deficit.

Tang (2008) reexamined the cointegration relationship of Japan's aggregate import demand through Autoregressive Distributed lag approach to cointegration. He used rolling windows technique which is applied to the (ARDL) bounds testing procedure. The sample period of quarterly data covers the period of 1973Q1 to 2007Q2. The estimated results show the instability of Japan's import demand function over the examined period. This instability shows the presence of cointegration for certain periods and also its absence for other periods.

4. Empirical Framework

Unlike the residual based test such as Engle-Granger (1987) and the maximum likelihood based test such as Johansen (1991 and 1995) for testing the long-run association, the ARDL approach does not require that the underlying series included in system have same order of integration. Another advantage of this approach is that the model takes sufficient number of lags to reduce the intensity of serial correlation of residuals in a general to specific modeling framework. Furthermore, a dynamic error correction model (ECM) can be derived from ARDL through simple linear transformation. The ECM emerges the short-run dynamics with the long-run stable equilibrium without losing long-run information.

The ARDL regression yields a test statistic which can be compared to two asymptotic critical values (upper and lower critical values). If the test statistic is above an upper critical value at the given level of significance, the null hypothesis of no long-run relationship is rejected regardless whether the orders of integration of the variables are one or zero. Alternatively, if the calculated test statistic is below the lower critical value at given level of significant, the null hypothesis of no long-run relationship is accepted.
However, if the test statistic falls between upper and lower bounds, the result is inconclusive. Another advantage of this approach is that an appropriate specification of the ARDL equation helps to fix the problems of endogenous variables and residual serial correlation. Finally, it performs better than Engle-Granger (1987) and Johansen (1990 and 1995) cointegration tests in case of small samples. We begin with an unrestricted VAR in level with an intercept term:

$$X_t = \alpha + \sum_{i=1}^{p} \beta_i X_{t-i} + e_t$$

(14)

where $\gamma_i$ is a $k \times 1$ vector of variables, which can be either $I(0)$ or $I(1)$. $\alpha$ is a vector of constants and $\beta_i$ is a matrix of VAR parameters for lag $^i$. The vector of error terms $e_t$ has zero mean and positive definite variance.

Next, following Banerjee et al. (1993), a simple linear manipulation of equation (14) allows this VAR model to be written as a vector correction model (VECM). Specifically, it is defined as:

$$\Delta X_t = \alpha + \Psi X_{t-1} + \sum_{i=1}^{p-1} \gamma_i \Delta X_{t-i} + e_t$$

(15)

where $\Delta$ is the difference operator. Here $\Psi$ is the long-run multiplier matrix and is given by $\Psi = -(I - \sum_{i=1}^{p} \beta_i)$. The sum of the short-run coefficient is defined by:

$$\gamma = I - \sum_{i=1}^{p-1} \gamma_i = -\Psi + \sum_{k=r+1}^{p} \beta_k$$

where $I$ is a $k \times k$ identity matrix, here $^k$ denotes the number of variables included in the system. The diagonal elements of this matrix are left unrestricted. This implies that each of the variables can be integrated of order one or zero. This procedure allows for the

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$^1$ For details on this, see Laurenceson and Chai (2003).
testing of at most one long-run relationship and so requires a zero restriction on one of the off diagonals of the $\gamma$ matrix.

To analyze the long-run effects of the level of the variables on the level of demand for imports, we impose the restriction $\Psi_{ij} = 0$, where $i \neq j$. This condition implies that there is no long-run feedback from import demand, but there is feedback in the short-run. Under this condition, the empirical equation for the import demand function from the VECM of equation (15) can be obtained as:

$$
\Delta D_t = \alpha_0 + \alpha_t t + \Psi_{D_0} D_{t-1} + \Psi_{DG} G_{t-1} + \sum_{i=1}^{m} \gamma_i \Delta X_{t-i} + \phi \Delta G_t + \epsilon_t
$$

(16)

where $t$ is a linear trend and $G_t$ is a $(m \times 1)$ vector of regressors. The symbol $\Delta$ is the difference operator and $\phi$ is a matrix of parameters for $\Delta G_t$.

Annual data for the period from 1975 to 2008 is used. It is taken from International Financial statistics (IMF) CD-ROM, World Bank Development Indicator (WDI) CD-ROM and 50 years of Statistics of Pakistan.

### 5. Empirical Findings

The long run demand equation derived in equation (13) implies that $m_t \ln(Y_t - P_tM_t)$, $P_t$ and $Z_t$ are cointegrated under the assumption that the random preference shocks $b_t$ and $c_t$ are strictly stationary. We use the following specifications for the preference shocks $b_t$ and $c_t$:

$$
b_t = b_0 + e^{b_t}, \quad c_t = c_0 + e^{c_t}
$$

where $e^{b_t}$ and $e^{c_t}$ have zero mean and constant variance. The composite preference shock $\xi_t$ can be rewritten as

$$
\xi_t = \frac{1}{\eta}[(b_0 - c_0) + (e^{b_t} - e^{c_t})] = \pi_0 + \epsilon_t.
$$

Combining this with equation (13) we get the final estimating equation for the long run import demand function:

$$
m_t = \pi_0 + \pi_1 \ln(Y_t - P_tM_t) + \pi_2 P_t + \pi_3 Z_t + \epsilon_t
$$

(17)
We estimate equation (14), which forms the basis of our empirical analysis, for Pakistan using annual data over the period from 1975 to 2008. As suggested by well-known econometric literature, there are two main issues in the empirical analysis: (i) the validity of the cointegration or stationary restriction embodies in equation (14), (ii) estimation of the cointegrating vector (s). To test the existence number of the long run relation (s), we use the bonds “F” test developed by Pesaran, Shin and Smith (2001) along with the widely used Johansen approach to the determination of the cointegration rank.

To estimate the elasticities, the following two alternative approaches are used: (i) ARDL approach, and (ii) Dynamic Ordinal Least Square (DOLS) method developed by Stock and Watson (1993). The alternative methods are used to test the sensitivity of the results with respect to different estimation techniques. For ARDL approach, we adopt the two-step procedure suggested by Pesaran and Shin (1999) where the specification of the ARDL model is chosen by Schwartz Bayesian Criterion (SBC) and then in second-step the ARDL equation is estimated by OLS.

The Monte-Carlo evidence of Pesaran and Shin (1999) provides significance evidence that this two-step procedure effectively corrects for endogeneity of explanatory variables and the estimates exhibit good small sample properties. Finally, the stability of the estimated parameters is tested by using Chow test, CUSUM and CUSUMSQ tests.

The first step involved in applying cointegration is to determine the order of integration of each variable/series. To do this, we performed the ADF test to test the null of unit root against the alternative of stationary both at level and first differences of real imports (LM), domestic consumption (LH), relative prices (LP) and foreign exchange reserve (LF). The estimated ADF statistics are reported in Table 1. Akaike Information Criterion (AIC) is used to identify the optimal lag length for ADF equation. The optimal lag lengths are given in parentheses.

It can be observed from the table that the estimated ADF test statistics (both without and with trend) are less than critical value at 5 percent level of significance for all the series at
their levels. It implies that the null hypothesis of a unit root in the level series cannot be rejected. Therefore, it can be said that the series neither drift nor trend stationary at their levels over examined period. However, the first difference of all the variables appeared stationary.

**Table 1: Unit Root Test Estimates**

<table>
<thead>
<tr>
<th>Variables</th>
<th>At levels $t_{ADF(c)}$</th>
<th>At first-difference $t_{ADF(c+1)}$</th>
<th>$t_{ADF(c)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Imports</td>
<td>-1.149(4)</td>
<td>-1.738(5)</td>
<td>-3.984(4)</td>
</tr>
<tr>
<td>Domestic consumption</td>
<td>-0.743(5)</td>
<td>-1.247(3)</td>
<td>-3.548(0)</td>
</tr>
<tr>
<td>Relative prices</td>
<td>-0.986(5)</td>
<td>-1.407(4)</td>
<td>-4.635(0)</td>
</tr>
<tr>
<td>Foreign exchange reserve</td>
<td>-1.639(1)</td>
<td>-1.596(-1)</td>
<td>-5.633(1)</td>
</tr>
</tbody>
</table>

*Notes:* $t_{ADF(c)}$ and $t_{ADF(c+1)}$ are the standard ADF test statistics for the null of nonstationary of the variable in the study without and with a trend, respectively, in the model for testing. The 10% and 5% asymptotic critical values are -2.57 and -2.86 for $t_{ADF(c)}$ respectively, and are -3.12 and -3.41 for $t_{ADF(c+1)}$, respectively.

### 5.1 Estimates of the Long Run Import Model

The next step to estimating the import demand model is to explore a long-run relationship. As mentioned earlier, the bounds tests suggested by Pesaran and Shin (1999) and the rank tests for cointegration developed by Johansen (1995) are used. The specifications of the ARDL and VAR models (lag order and deterministic part) for the tests of cointegration are determined on the basis of the AIC. To proceed with this, the AIC statistics are calculated for lags ranging from one to four for all possible cointegration vectors form models with no intercept and no trend, with intercept and no trend and with intercept and a linear trend. The maximum absolute value of the criterion suggests that an optimal lag length for Model I and II is 3 and for Model III is 2.

Table 2 presents the Johansen trace test results to determine the number of cointegration vectors for the optimal lag length suggested by the selection criteria. Log values of
import prices, log values of domestic consumption, log values of relative prices and scarcity premium are included in cointegrating vector. The null and alternative hypotheses are given in first and second columns of the table. The estimated F-statistics with their critical values are given in last three columns of the table. The results provide strong evidence of existing cointegrating relationship among the said variables. In general, these findings are robust to model specifications. However, the numbers of cointegration vectors are vary with model specifications. For example, the results using a specification with only intercept indicate one-cointegration vector for the said variables. Whereas, when the cointegration equation includes both intercept and a linear trend the two-cointegration vectors appear statistically significant.

The presence of the cointegration in the said variables implies that these variables have co-movement in the long run. The existence of the long-run equilibrium relationship indicating that the level of domestic consumption, relative prices and the level of foreign exchange reserve are simultaneously playing important role to determine the demand for imports in Pakistan.

Table 2: Johansen Cointegration Results based on Trace of the Stochastic Matrix: LM, LH, LP and Z are included in Cointegration Vector

<table>
<thead>
<tr>
<th>Hypotheses</th>
<th>F-Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Intercept, No Trend</td>
</tr>
<tr>
<td></td>
<td>Test Statistics</td>
</tr>
<tr>
<td>$r = 0$</td>
<td>51.889</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>26.380</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>11.735</td>
</tr>
<tr>
<td>$r \leq 3$</td>
<td>4.239</td>
</tr>
</tbody>
</table>

The results of the bounds tests are given in Table 3. The F-statistics are calculated by estimating the Model I to Model III with specifications of no intercept and no trend, with intercept and no trend and finally by including both intercept and a linear time trend. For
estimating the bounds “F” tests, the lag length, selected by AIC is two when the model includes neither intercept nor trend and when includes only intercept. However, the criterion suggests the optimum lag length one when the model includes both intercept and a linear time trend. The main objective behind to estimate the bounds “F” tests using different specifications is to test the robustness of the results with respect to different specifications.

It can be seen from the table that results of the bounds “F” tests show that the null hypothesis of no cointegration can be rejected at 5% or less significance level for all different specification. The overall results from the Johansen’s cointegration tests and bounds tests provide strong evidence in favor of a significant long run relationship among the variables included in the import demand model.

**Table 3: Bound Tests for Long-run Relationship in an ARDL Framework**

<table>
<thead>
<tr>
<th>Empirical Models</th>
<th>F-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Intercept, No Trend</td>
</tr>
<tr>
<td>Model I:</td>
<td>69.184*</td>
</tr>
<tr>
<td>(LM = f(LH, LP, Z))</td>
<td>(58.089*)</td>
</tr>
<tr>
<td>Model II:</td>
<td>(78.158*)</td>
</tr>
<tr>
<td>(LM = f(LH, LP))</td>
<td>(LM = f(LH, LP, LF))</td>
</tr>
</tbody>
</table>

where \(LM = \text{log value of imports, LH = log value of domestic consumption, LP = log value of relative prices, LF = log value of foreign exchange reserve, F = foreign exchange reserve and Z = scarcity premium, } \left((\text{GDP + Imports - Exports}/\text{CPI})/F\right)\text{ multiplied by trade liberalization dummy. } * \text{ denotes significant at one percent level of significant.}

Since there are strong evidence of the existence of a long run relationship among the variables included in the long run import demand model, we estimate the long-run cointegration relation (long-run coefficients) for import using the ARLD and DOLS single equation estimation methods. The optimal lag length for the ARDL model was chosen by SBC starting from 4 lags. In the case of DOLS estimation, sufficient lags and leads of first difference terms are included in the regression in order to eliminate the problem of serial correlation. The DOLS model involves two lags in case of Model I. The
results from the ARDL and DOLS estimation of the long run demand relationship are reported in Table 4.

**Table 4: Estimates of Long-run Relationships**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Long-run Estimates</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ARDL</td>
<td>DOLS</td>
</tr>
<tr>
<td>H</td>
<td>1.065 (7.01)</td>
<td>0.98 (8.13)</td>
</tr>
<tr>
<td>P</td>
<td>-0.918 (-4.87)</td>
<td>-0.948 (-1.05)</td>
</tr>
<tr>
<td>Z</td>
<td>-0.219 (-2.23)</td>
<td>-0.014 (-1.08)</td>
</tr>
<tr>
<td>Intercept</td>
<td>-2.258 (-1.43)</td>
<td>3.456 (2.73)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Diagnostic Tests</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Serial Correlation Test</td>
<td>3.563 [0.18]</td>
</tr>
<tr>
<td>Normality Test</td>
<td>1.364 [0.50]</td>
</tr>
</tbody>
</table>

It can be seen from the bottom panel in Table 4, the regression diagnostic tests show that the residuals from the estimated regressions display no problem of serial correlation and/or non-normality in the case of ARDL and DOLS estimated methods. The estimated coefficient for income and relative price satisfy the theoretical sign restrictions over the examined sample period regardless of estimation methods. The estimated coefficients are highly statistically significant at 5% level of significance in case ARDL and DOLS as well. For income coefficient, the magnitude of ARDL estimate is lightly higher than that of DOLS. The estimates of income coefficient vary from 1.065 (ARDL) to 0.98 (DOLS).

However, the ARDL estimate of relative price coefficient is slightly lower in absolute magnitude as compared with the DOLS estimate over the examined period. The ARDL and DOLS estimates of relative price coefficient are -0.918 and -0.948, respectively. The ARDL and DOLS estimates of coefficients of scarcity premium variable have correct

---

2 The values are given in the brackets below the test statistics are p-values.
3 The estimated t-statistics are reported in the parentheses.
negative sign; however, it appears only statistically significance in case of ARDL. This piece of evidence confirms the existence of a binding foreign exchange constraint on aggregate imports before the economic liberalization in Pakistan.

5.2 Stability of the Estimated Parameters

Instability of the estimated elasticity parameters is a major issue in the policy analysis. For instance, Marquez (2003) reports evidence of parameter instability in the case of income elasticity for U.S. imports. Such parameter instability could result from misspecification of the long run import relationship particularly when span over a very long time horizon. Therefore, we test for the stability of the estimated parameters from both ARDL and DOLS by using Chow break point tests and CUSUN and CUSUMSQ tests. According to the Chow breakpoint tests, the ARDL estimates of the parameter are stable over the time and do not show any instability (the estimated F-statistic is 1.78 with P-value (0.15)). The results from CUSUM and CUSUMSQ tests for ARDL estimations are presented in Figure 1a and 1b. It can be observed from the figures that both of the tests (CUSUM and CUSUMQS) do not provide any evidence of instability in the estimated parameters at 5 percent level of significance for ARDL estimation method. The results from CUSUM and CUSUMSQ tests for DOLS estimations are given in Figure 2a and Figure 2b. Since the plot of CUSUM of recursive residuals lies within the critical bound at 5% level of significance, there is no evidence of instability in the estimated parameters for DOLS estimation method. However, as can be observed from the figure, the plot of CUSUMSQ of recursive residuals is crossing the critical lower bound at 5% level of significance. This implies that the estimated parameters are not stable over the time. Overall, the results from ARDL estimation are relatively better than the DOSL estimations.
5.3 Comparison with Alternative Models

5.3.1 Modified Traditional Model

In this sub-section, we present the results of the empirical analysis of the modified traditional model (in our case it called Model II). Model II is derived from equation (17) by excluding $Z_t^*$. We also estimate the Model III which incorporates the foreign exchange availability. It is derived from equation (17) with log of real foreign exchange availability replacing $Z_t^*$. The general empirical strategy is the same as that followed above.

The AIC are used to decide on the number of lags to be included in the empirical models. The prime objective here is to select the optimal lag-length that eliminates any
autocorrelation present in the residuals. Initially, the three VAR models i.e., first neither includes intercept nor trend, second includes only intercept and third one includes both intercept and a linear trend in cointegration equation, are estimated with four lags for both of the bounds “F” tests and Johansen’s cointegration technique.

The estimated AIC statistics suggest three lags for first model and two lags for second and third models. The estimated trace statistics for the modified traditional model with their critical values are presented in Table 5.

**Table 5: Johansen Cointegration Results based on Trace of the Stochastic Matrix: LM, LH and LP are included in Cointegrating Vector**

<table>
<thead>
<tr>
<th>Hypotheses</th>
<th>F-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Intercept, No Trend</td>
</tr>
<tr>
<td>Null</td>
<td>Test Statistics</td>
</tr>
<tr>
<td>$r = 0$</td>
<td>43.484</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>15.431</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>3.991</td>
</tr>
</tbody>
</table>

As can be observed from the table, there are strong evidences for the existence of the long run association among the said variable over the examined period. The estimated trace statistics are significantly greater than the critical values at five percent level of significance for all specifications.

The long-run parameters of the modified traditional model are estimated by the ARDL and the DOLS methods and are given in Table 6. The results show that the estimates have correct sign when the import equation is estimated from an ARDL model. Both the estimates (income elasticity and price elasticity) are also statistically significant at 5 percent level of significance. It is interesting to note that the magnitude of income elasticity is very close to one. However, the magnitude of price elasticity (-0.658) is significantly less than one in absolute term.
Table 6: Estimates of Long-run Relationship in Traditional Modified Model

<table>
<thead>
<tr>
<th>Variables</th>
<th>Long-run Estimates</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ARDL</td>
<td>DOLS</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>1.0015</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7.467)</td>
<td>(1.235)</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>-0.658</td>
<td>0.89</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-4.573)</td>
<td>(1.035)</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-1.631</td>
<td>-2.342</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-1.167)</td>
<td>(-1.765)</td>
<td></td>
</tr>
</tbody>
</table>

Diagnostic Tests

| Serial Correlation Test | 3.480 | 2.760 |
|                        | [0.062]| [0.154]| 
| Normality Test         | 1.328 | 1.234 |
|                        | [0.515]| [0.768]| 

Note: t-statistics are reported in parentheses and p-values are given in brackets.

Although the DOLS estimates of income elasticity have the correct positive sign but they are statistically insignificant. The magnitudes of income elasticity, according to the DOLS estimates, are also implausibly small (0.05). Regarding price elasticity in case of DOLS estimations, the estimates provide evidence that the price coefficient has a positive sign and is statistically insignificance at the 5% level of significance. By doing the comparison between the both estimation methods, we find that the results from ARDL are relatively better as both the price and income elasticity have the correct signs.

5.3.2 Foreign Exchange Rate Availability Formulation

Finally, we estimated the Model III which incorporates the foreign exchange availability. It is derived from equation (17) with log of real foreign exchange availability replacing \( Z_t^* \).

Initially, the three VAR models i.e., first neither includes intercept nor trend, second includes only intercept and third one includes both intercept and a linear trend in cointegration equation, are estimated with four lags for both of the bounds “F” tests\(^4\) and Johansen’s cointegration technique. To estimate the Johansen’s cointegration test

\(^4\) The bounds F-test results are presented in Table 3.
statistics, we used as suggested by AIC two lags for first model and one lag for second and third models. The estimated trace statistics with their critical values are presented in Table 7.

**Table 7: Johansen Cointegration Results based on Trace of the Stochastic Matrix:**
LM, LH, LP and LF are included in Cointegrating Vector

<table>
<thead>
<tr>
<th>Hypotheses</th>
<th>F-Statistic</th>
<th>No Intercept, No Trend</th>
<th>With Intercept, No Trend</th>
<th>With Intercept, With Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test Statistics</td>
<td>Critical Value</td>
<td>Test Statistics</td>
<td>Critical Value</td>
</tr>
<tr>
<td>$r = 0$</td>
<td>101.086</td>
<td>39.810</td>
<td>151.281</td>
<td>53.480</td>
</tr>
<tr>
<td>$r = 1$</td>
<td>23.100</td>
<td>24.050</td>
<td>29.355</td>
<td>34.870</td>
</tr>
</tbody>
</table>

The estimated trace statistics are significantly greater than the critical values at five percent level of significance for all specifications in case of at least one cointegrating vector. Thus, we can conclude that there is a unique long-run statistically significant association among the variables included in cointegration regression. However, as can be observed from table, the estimates with specification of both intercept and linear trend provide evidence of the significance of second cointegrating vector as well. Since the first cointegrating vector has the highest eigenvalue, we consider the only first one to estimate the long-run coefficient.

The long-run parameters with foreign exchange availability formulation are estimated by using the two alternative methods (ARDL and the DOLS). The estimates are reported in Table 8. The income and price elasticity estimates for ARDL estimation method bear the sign according to described by theory (positive in case of income elasticity and negative for price elasticity) and are statistically significant at the 5% level of significance. The income and price elasticity magnitudes are 1.018 and -1.197. The income elasticity is
close to one which clearly shows the strength of the near identity problem. One the other hand, the estimate of price elasticity is significantly higher than one. The ARDL estimate of the coefficient of foreign exchange availability is relatively small however, it has correct sign. It is highly statistically insignificant at the 5% level of significance. Finally, the estimates of diagnostic tests provide evidence that the residuals for ARDL estimation are normally distributed and free from the problem of serial correlation.

**Table 8: Estimates of Long-run Relationship in Foreign Exchange Availability Model**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Long-run Estimates</th>
<th>ARDL</th>
<th>DOLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>1.018</td>
<td>(7.224)</td>
<td>0.779</td>
</tr>
<tr>
<td>P</td>
<td>-1.197</td>
<td>(-6.847)</td>
<td>-0.945</td>
</tr>
<tr>
<td>F</td>
<td>0.472</td>
<td>(2.935)</td>
<td>-0.239</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.318</td>
<td>(0.191)</td>
<td>-0.506</td>
</tr>
</tbody>
</table>

**Diagnostic Tests**

<table>
<thead>
<tr>
<th></th>
<th>Serial Correlation Test</th>
<th>Normality Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.180</td>
<td>0.328</td>
</tr>
<tr>
<td></td>
<td>[0.156]</td>
<td>[0.786]</td>
</tr>
<tr>
<td></td>
<td>3.170</td>
<td>0.543</td>
</tr>
<tr>
<td></td>
<td>[0.189]</td>
<td>[0.762]</td>
</tr>
</tbody>
</table>

Note: t-statistics are reported in parentheses and p-values are given in brackets.

The DOLS estimates of income and price elasticity have right signs and are statistically significance at conventional level of significance. However, both estimates are significantly lower as compared to ARDL estimates. The income elasticity is 0.779 which is less than one as well as than the ARDL estimate of income elasticity. Similarly, the estimate of price elasticity (-0.945) is considerably less than the ARDL estimate of price elasticity in absolute term. Quite contrary to the ARDL estimates, the DOLS estimate of the coefficient of foreign exchange availability is relatively small and has also implausibly negative sign which does not match with the theory. It is, however, statistically insignificant at the 5% level of significance.
6. Conclusions

In this paper we test the model of aggregate imports for Pakistan. The empirical results from both ARDL and Johansen’s method show strong evidence of the existence of a long-run relationship among the variables included in the long run import demand models. The long-run estimates of the activity variables (GDP-exports) and price elasticities are highly significant and follow the sign restriction embodied in the theoretical and empirical model. The mean of activity variable (GDP-exports) is 1.065. This variable shows a renewed form of income elasticity. The neoclassical economic theory implies that long run income elasticity should be equal to one, and if it is slightly higher than one than it is supported by new trade theory. As the activity variable in our selected model shows unitary income elasticity, for the improvement of trade balance it is required to adopt certain measures that cause a reduction in income elasticity.

The mean of relative price elasticity is -0.918. It is closer to one and is greater than all previous studies presented in Pakistan (Arize (2004) and Zehra (2002)). Importance of relative price elasticities is confirmed from the previous literature, because increase in world trade each year has been caused by price related factors, such as reduction in tariff rates as a result of trade liberalization efforts, exchange rate policy, the reduction in long run transportation cost or pricing strategies at firm and industry level.

The ARDL estimate of the coefficient of scarcity premium is also significant with correct sign. It confirms the presence of a binding foreign exchange constraint on aggregate import demand, before the period of trade liberalization. In the stability analyses, all the variables are appeared stable between the lower and upper bound.

In general, the results confirm the validity of modified form of traditional model. However, when we remove the variable of scarcity premium, the elasticity estimates receive lesser values as compared to our original empirical equation (14). Our findings are important for policy analyses in the number of areas, such as exchange rate policy, tariff reduction programs and calculation of optimal taxes.
References


