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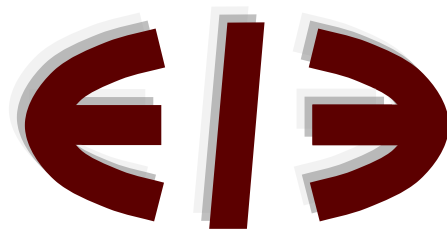
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## **Modelling Inter-Regional Trade Flows: Data and Methodological Issues in Rhomolo**

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**MODELLING INTER-REGIONAL TRADE FLOWS:  
DATA AND METHODOLOGICAL ISSUES IN RHOMOLO**

*Olga Ivanova<sup>1</sup>, d'Artis Kancs<sup>2</sup> and Dirk Stelder<sup>3</sup>*

**Abstract:** The present paper studies the modelling of inter-regional trade flows in the newly developed general equilibrium model of European Commission, Rhomolo. In contrast to international trade, usually, trade of goods and services between regions is not recorded statistically. As a result, no statistical data exists for inter-regional trade, which could be readily used for modelling purposes. We propose an augmented entropy approach, which combines the Transtools model's inter-regional trade and inter-regional transport cost data with data from regional and national statistics to obtain a full set of balanced bilateral trade data for all inter-regional transactions.

**Keywords:** Inter-regional trade, general equilibrium, trade costs, entropy.

**JEL Classification:** C68, F10, F15, F17, R11, R41.

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## 1 Introduction

Regions and countries are linked with each other through trade of goods and services, factor, and income flows. Regions import and export goods and services from other regions in the same country and from other countries. The pattern of intensity of trade differs, however, between regions and their trading partners.

In contrast to international trade, usually, trade of goods and services between regions is not recorded statistically. As a result, no statistical data exists for inter-regional trade, which could be readily used for modelling purposes. Such data, however, is necessary for empirical implementation of inter-regional general equilibrium models, such as Rhomolo.

The objective of this paper is to discuss the data and methodological issues of modelling inter-regional trade flows and, based on the gained insights, to develop a strategy for implementing the inter-regional trade in Rhomolo.

## 2 Estimating inter-regional trade

The intensity and pattern of trade between regions depends on many factors, such as, geographic and cultural proximity of the trading partners, consumer preferences, trade costs, and the structure and size of trading economies. The relative importance of each factor driving or hampering the inter-regional trade can be estimated econometrically, but as mentioned above, only for trade between countries.

Conceptually, trade flows can be estimated in many different methodological and empirical specifications, which differ by the data requirements. Due to the easiness of implementation and its empirical success, often, trade flows are modelled in a gravity framework.

Different specifications and micro-foundations of the gravity model have been proposed in the literature (Feenstra 2002). For example, Anderson and van Wincoop (2003) show that in a one-sector economy, when consumers have CES preferences, trade between regions  $r$  and  $s$  can be expressed as:

$$(1) \quad XDDE_{r,s} = (XD_r * XD_s / XD) * (trm_{r,s})^{1-\sigma} / (MTR_r * MTR_s)^{1-\sigma}$$

where  $XD_r$  and  $XD_s$  are the levels of GDP,  $XD$  is world GDP,  $trm_{r,s}$  are transport costs between  $r$  and  $s$ , and  $MTR_r$  and  $MTR_s$  are multilateral trade resistance terms of exporting and importing regions. They summarise the average trade resistance between a region and its trading partners in sense of ideal aggregation (Anderson and van Wincoop 2003). According

to equation (1), the bilateral trade flows between  $r$  and  $s$  depend on bilateral costs,  $trm_{r,s}$ , divided by the product of their multilateral resistance indices for  $s$ 's imports and for  $r$ 's exports. Hence, according to equation (1), the bilateral trade between  $r$  and  $s$  depends also on the relative trade barriers.

In the gravity model of trade (1) all variables but the bilateral trade cost and the elasticity of substitution parameters are available in the data. These two parameters – trade costs and the elasticity of substitution – are not observable; usually they are estimated or calibrated.

The use of equation (1) for the estimation of these two parameters is subject to several issues. First, the identification of trade costs,  $trm_{r,s}$ , and the elasticity of substitution,  $\sigma$ , is not straightforward, as trade flows are decreasing in both the bilateral trade costs and the elasticity of substitution. As a result, the two missing parameters are usually calibrated. The identification issue in the estimation/calibration exercise is not important in the Rhomolo model, because a very detailed generalised transport cost data by region and sector are available.

Second, the estimation of equation (1) requires data on inter-regional trade flows,  $XDDE_{r,s}$  (bilateral flows for the base year or times series). However, as noted above, no statistical data exists for inter-regional trade flows,  $XDDE_{r,s}$ , because firm sales are not recorded by region. Moreover, given that the upstream industry consists of many agents (wholesalers, transport, storage, retailers, etc), even producers cannot always track the spatial flow of their output. Again, this issue is not important for Rhomolo, because inter-regional trade flows are not used for model's parameterisation.

## **4 Modelling inter-regional trade in CGE**

### **4.1 The maximum entropy approach**

In general equilibrium models of trade, the trade of goods and services is determined endogenously as an optimisation outcome of the optimal strategy of firms, households and other economic 'actors'. Solving the model for the equilibrium prices and quantities, yields also inter-regional trade flows which reflect the underlying theoretical trade relationships and behavioural parameters.

In a general equilibrium model the predicted trade flows need to satisfy also the macroeconomic data constraints for the base year. The macroeconomic constraints can be

imposed e.g. via the maximum entropy method. The maximum entropy method allows to infer inter-regional trade flows such that, on the one hand, they satisfy the imposed data constraints and, on the other hand, the distance to the initial (unconstrained) estimates of trade flows is minimised.

In the context of our study the maximum entropy approach has several advantages. First, the maximum entropy approach is flexible in sense that it allows to take into account different data sources for inter-regional trade flows as well as to put economic restrictions on the data, e.g. by the production and consumption accounts in regions. Second, the maximum entropy approach employs a nonlinear programming technique and, hence, can handle non-linear problems. Third, it can be implemented in GAMS, which is the programming language used in Rhomolo.

#### 4.2 Objective function

The main component of the maximum entropy problem is the objective function. The objective function represents the distance between the estimated inter-regional trade flows, which satisfy the set of restrictions, and the initial (unrestricted) estimates of inter-regional trade flows.

In the GAMS code (see Appendix) the objective function of the entropy problem is represented by EQentropy:

$$\begin{aligned}
 \text{entropy} = E = & \text{sum}((\text{sec}, \text{reg}, \text{regg}) \$ \text{XDDE}_o(\text{sec}, \text{reg}, \text{regg}), \text{XDDE}_V(\text{sec}, \text{reg}, \text{regg})) \\
 & * \log(\text{XDDE}_V(\text{sec}, \text{reg}, \text{regg}) / \text{XDDE}_o(\text{sec}, \text{reg}, \text{regg})) \\
 & + \text{sum}((\text{sec}, \text{reg}, \text{regg}) \$ \text{transpm}(\text{sec}, \text{reg}, \text{regg}), \text{transpmV}(\text{sec}, \text{reg}, \text{regg})) \\
 & * \log(\text{transpmV}(\text{sec}, \text{reg}, \text{regg}) / \text{transpm}(\text{sec}, \text{reg}, \text{regg})) \\
 & + \text{sum}((\text{sec}, \text{cnt}) \$ \text{shRoEUtrade}(\text{cnt}, \text{sec}), \text{shRoEUtradeV}(\text{cnt}, \text{sec})) \\
 & * \log(\text{shRoEUtradeV}(\text{cnt}, \text{sec}) / \text{shRoEUtrade}(\text{cnt}, \text{sec})) \\
 & + \text{sum}((\text{sec}, \text{cnt}) \$ \text{shRoEUtradeE}(\text{cnt}, \text{sec}), \text{shRoEUtradeEV}(\text{cnt}, \text{sec})) \\
 & * \log(\text{shRoEUtradeEV}(\text{cnt}, \text{sec}) / \text{shRoEUtradeE}(\text{cnt}, \text{sec}))
 \end{aligned}$$

#### 4.3 Data constraints

The second component of the maximum entropy problem is satisfying the constraints. One set of constraints represents the equality between demands for goods and services in each region and the total inflow of trade. Another set of constraints represents the equality between supplies of goods and services in each region and the total outflow of trade. Regional specific demands and supplies in each region are derived based on the regional production and consumption statistics. Another important constraint is that the aggregate inter-regional trade flows should be consistent with the inter-national trade statistics at the national level. Finally, regional trade cost margins should sum up to the transport services in national accounts.

Hence, data on regional production; regional consumption; bilateral trade of goods and services between countries; and inter-regional trade costs (transport cost margins) provide constraints, which have to be satisfied when estimating inter-regional trade flows. These four constraints envelope the potential set of trade flows estimations. The more constraints can be derived from the theory and the available data, the smaller is the potential estimation error of inter-regional trade flows.

#### 4.3.1 Regional production

The first constraint which needs to be satisfied is regional production. Region  $r$  produces goods of value  $XD_r$ . The regional production constraint says that the total sales from region  $r$ ,  $\sum_{r,s}XDDE_{r,s}$ , to all regions (intra-regional plus inter-regional) must be equal to regional production,  $XD_r$ .

$$(2) \quad XD_r = \sum_s XDDE_{r,s}$$

For the purpose of Rhomolo, production data,  $XD_r$ , by region and sector are readily available in statistics.

A general GAMS notation of equation (2) is as follows:

$$XDZ(sec,reg) = E = \text{sum}(regg, XDDE\_V(sec,reg,regg))$$

In the GAMS code (see Appendix) equation (2) is represented by EQ11 and EQ12:

$$XDZ(sec,reg) - EEU25Z(sec,reg) - EROWZ(sec,reg) = E =$$

$$\text{sum}(regg, XDDE\_V(sec,reg,regg))$$

$$XDZ(sec,reg) = E = \text{sum}(regg, XDDE\_o(sec,reg,regg), XDDE\_V(sec,reg,regg))$$

$$+ EEU25Z(sec,reg) * \text{sum}(\text{mapCR}(cnt,reg), \text{shRoEUtradeEV}(cnt,sec))$$

$$+ EROWZ(sec,reg)$$

### 4.3.2 International trade flows

The second constraint which needs to be satisfied is international trade flows. Part of goods and services produced in region  $r$  are sold in  $r$ , the rest is exported to other regions (within the same country and to other countries). The international trade constraint says that the total amount of goods and services shipped from all regions in a country to each foreign country must sum up to the international trade flows between the two countries.

$$(3) \quad XDDE_{c,cc} = \sum_r XDDE_{r,s} \quad r \in c, s \in cc$$

For the purpose of Rhomolo, bilateral trade flows at country level,  $XDDE_{c,cc}$ , are readily available in international trade statistics.

In the GAMS code (see Appendix) equation (3) is represented by EQ3.

$$\begin{aligned} & \text{sum}((\text{sec}, \text{reg}) \$ \text{mapCR}(\text{cnt}, \text{reg}), XDDEZ(\text{sec}, \text{reg}) * \text{scal}) \\ & = E = \text{sum}(\text{sec}, \text{sum}((\text{reg}, \text{reg}) \$ \text{mapCR}(\text{cnt}, \text{reg}), \\ & \quad XDDE\_V(\text{sec}, \text{reg}, \text{reg}))) \end{aligned}$$

### 4.3.3 Regional consumption

Part of goods consumed,  $X_s$ , in region  $s$  stems from local production, the rest is imported from other regions. Both local sales (within the region) and inter-regional sales (to other regions) of all goods and services are subject to trade costs,  $\text{trm}_{r,s}$ . Trade costs,  $\text{trm}_{r,s} > 0$ , increase the price of consumption goods, when they reach consumers. Hence, the transportation of goods  $XDDE_{r,s}$  from region  $r$  increases goods' value in destination region  $s$  to  $XDDE_{r,s} * (\text{PDC}_s + \text{PTM}_{r,s} * \text{trm}_{r,s})$ .

The regional consumption constraint says that the value of goods consumed,  $P_s * X_s$ , in region  $s$  is equal to the total value of goods,  $\sum_s XDDE_{r,s}$ , sold in region  $s$  valued at producer prices and transportation costs,  $\text{PDC}_s + \text{PTM}_{r,s} * \text{trm}_{r,s}$ .

$$(4) \quad P_s * X_s = \sum_s XDDE_{r,s} * (\text{PDC}_s + \text{PTM}_{r,s} * \text{trm}_{r,s})$$

For the purpose of Rhomolo, consumption data (value of goods consumed),  $P_s * X_s$ , by region and sector are readily available in statistics.

The inter-regional trade cost data is available from the Transtools model, which provides bilateral trade costs by NUTS2 region and sector for the base year. In order to assure the quality of the Transtools data, the transportation margins from the Transtools model have undergone extensive checks of their quality, reliability and consistency. The new

transportation cost data from the Transtools model satisfy all four criteria. Therefore, it is suitable for being used in the Rhomolo model.

A general GAMS notation of equation (4) is as follows:

$$XZ(sec,reg) = E = \text{sum}(regg, XDDE\_V(sec,regg,reg) * (1 + \text{transpmV}(sec,regg,reg)))$$

In the GAMS code (see Appendix) equation (4) is represented by EQ21 and EQ22:

$$\begin{aligned} XZ(sec,reg) - MEU25Z(sec,reg) - MROWZ(sec,reg) = E = \\ \text{sum}(regg, XDDE\_V(sec,regg,reg) * (1 + \text{transpmV}(sec,regg,reg))) \\ XZ(sec,reg) = E = \text{sum}(regg, XDDE\_o(sec,regg,reg), \\ XDDE\_V(sec,regg,reg) * (1 + \text{transpmV}(sec,regg,reg))) \\ + MEU25Z(sec,reg) * \text{sum}(\text{mapCR}(cnt,reg), \text{shRoEUtradeV}(cnt,sec)) \\ + MROWZ(sec,reg) \end{aligned}$$

#### 4.3.4 Inter-regional trade costs

In order to transport goods and services between and within regions, transport services need to be purchased. The trade cost constraint says that the sum of transport services purchased from all regions  $r$  in country  $c$ ,  $\sum_r XDDE_{r,s} * \text{trm}_{r,s}$ , is equal to the amount of transport services,  $TMX_c$ , produced in country  $c$ .

$$(5) \quad TMX_r = \sum_s XDDE_{r,s} * \text{trm}_{r,s} \quad r \in c$$

For the purpose of Rhomolo, the total amount of transport services,  $TMX_c$ , produced in country  $c$  is available from the national accounts data which enters the national Social Accounting Matrices.

In the GAMS code (see Appendix) equation (5) is represented by EQ4:

$$\begin{aligned} \text{sum}((sec,reg) \$ \text{mapCR}(cnt,reg), TMXZ(sec,reg) * \text{scal}) \\ = E = \text{sum}(sec, \text{sum}((reg,regg) \$ \text{mapCR}(cnt,reg), \\ XDDE\_V(sec,reg,regg) * \text{transpmV}(sec,reg,regg))) \end{aligned}$$

#### 4.4 The estimation procedure



In a first step the gravity model of trade (1) is estimated. This yields unrestricted estimates of inter-regional trade flows. Given that no restrictions are imposed on the estimates at this stage, the unrestricted trade flows are not consistent with regional and national accounts.

Alternatively, the unrestricted estimates of inter-regional trade flows can be taken directly from data, if available. In such a case, no gravity models need to be estimated. Given that the Transtools model provides data also on inter-regional trade flows, both approaches will be explored in Rhomolo. This will increase the robustness of model's results.

In a second step, in order to impose the consistency between trade flows and the rest of the model variables, the constraints (2)-(5) are imposed by applying the maximum entropy approach. This means that starting from the unrestricted trade flow estimates, the maximum entropy approach searches for the smallest deviations such that satisfying the constraints (2)-(5): the regional production constraint; the international trade constraint; the regional consumption constraint; and the trade cost constraint. The obtained trade flow estimates are consistent with both the national and regional accounts data.

## **5 Data sources**

### **5.1 Trade cost data**

The inter-regional trade cost data is available from the Transtools model, which provides bilateral trade costs by NUTS2 region and sector for the base year. In order to assure the quality of the Transtools data, the transportation margins from the Transtools model have undergone extensive checks of their quality, reliability and consistency. In particular, all bilateral trade cost margins have been analysed according to the following four criteria:

- The share of transportation margin in the goods value. We expect that this margin would not exceed 0.2-0.3 for most types of goods and regions (Anderson and von Woncoop 2003).
- The ratio of intra-regional trade costs to inter-regional trade costs. Normally, this ratio should be lower than 1.0 for most sectors and regions, because usually it is less costly to sell a good in the same region it is produced than to export it to other regions.
- Symmetry of trade costs between trade flows of opposite direction for the same pair of regions. We expect the coefficient of correlation between the bilateral trade cost matrix and its inverse is larger than 0.5.

- Trade cost outliers. We expect that the share of outliers within each sector and geographically proximate regions exceeding one standard deviation would not be larger than 0.1.

The new transportation cost data from the Transtools model satisfy all four criteria. Therefore, it is suitable for being used in the Rhomolo model.

## References

Anderson, James E. and Eric van Wincoop (2003), "Gravity with Gravitas: A Solution to the Border Puzzle," *American Economic Review*, 93, 170-192.

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Feenstra, Robert C. (2002), "The Gravity Equation in International Economics: Theory and Evidence," *Scottish Journal of Political Economy*, 49(5), 491-506.

Kapur, J.N., H.K. Kesavan (1992), "Entropy Optimization Principles with Applications." Academic Press, New York, 1992.

## Appendix: GAMS code of the entropy problem

*\$TITLE:RHOMOLOENTROPY*

*\$eolcom #*

*Option LIMROW = 0, LIMCOL = 0;*

*option decimals = 8 ;*

*Sets*

*b numbers of columns and rows in the SAM /1\*65/*

*cact(b) sectors + commodities - numeric /1\*46/*

*cm(cact) sectors - numeric - commodities /1\*23/*

*act(cact) sectors - numeric - activities /24\*46/*

*count for reading in data /1\*44/*

*sec sectors /sec1\*sec23/*

*pubs(sec) public sectors /sec10,sec20\*sec23/*

*transp(sec) transport services /sec15/*

*goods(sec) goods /sec1\*sec9/*

*cnt countries /DE,PL,CZ,HU,SK/*

*reg regions /*

*\$include Data/NUTS2/PL.txt*

*\$include Data/NUTS2/DE.txt*

*\$include Data/NUTS2/HU.txt*

*\$include Data/NUTS2/SK.txt*

*\$include Data/NUTS2/CZ.txt*

*/*

*regPL(reg) Polish regions*

*/*

*\$include Data/NUTS2/PL.txt*

*/*

*regDE(reg) German regions*

*/*

*\$include Data/NUTS2/DE.txt*

*/*

*regHU(reg) Hungarian regions*

*/*

*\$include Data/NUTS2/HU.txt*

*/*

*regSK(reg) Slovak regions*

*/*

*\$include Data/NUTS2/SK.txt*

*/*

*regCZ(reg) Czech regions*

*/*

*\$include Data/NUTS2/CZ.txt*

*/*

*mapCR(cnt,reg) mapping between NUTS regions and countries*

*mapRR(reg,reg) mapping between regions of the same country*

*;*

*mapCR("DE",reg)\$regDE(reg) = yes ;*

*mapCR("PL",reg)\$regPL(reg) = yes ;*

*mapCR("HU",reg)\$regHU(reg) = yes ;*

$mapCR("SK",reg)\$regSK(reg) = yes ;$

$mapCR("CZ",reg)\$regCZ(reg) = yes ;$

### *Alias*

$(b,bb)$

$(cact,cactt)$

$(cm,cmm)$

$(act,actt)$

$(reg,regg)$

$(reg,reggg)$

$(reg,regggg)$

$(sec,secc)$

$(sec,seccc)$

$(cnt,cntt)$

$(cnt,cnttt)$

;

### *Parameters*

$XZ(sec,reg)$       *initial domestic sales*

$XDZ(sec,reg)$       *initial domestic gross production (output) level*

$TMXZ(sec,reg)$       *initial consumption for prod of transport margins*

$EEU25Z(sec,reg)$       *initial exports to EU25*

$EROWZ(sec,reg)$       *initial exports to RoW*

$MEU25Z(sec,reg)$       *initial imports from EU25*

$MROWZ(sec,reg)$       *initial imports from RoW*

$shRoEUtrade(cnt,sec)$       *share of RoEU trade in total EU imports*

*shRoEUtradeE(cnt,sec) share of RoEU trade in total EU exports*

*XDDEZ(sec,reg) international trade between countries*

*XDDE\_o(sec,reg,regg) initial split according to origin indic (TRANSTOOLS)*

*transpm(sec,reg,regg) transport margins (TRANSTOOLS)*

;

*\$libinclude xlexport XZ Data/Entropydata.xls XZ! ;*

*\$libinclude xlexport XDZ Data/Entropydata.xls XDZ! ;*

*\$libinclude xlexport TMXZ Data/Entropydata.xls TMXZ! ;*

*\$libinclude xlexport EEU25Z Data/Entropydata.xls EEU25Z! ;*

*\$libinclude xlexport EROWZ Data/Entropydata.xls EROWZ! ;*

*\$libinclude xlexport MEU25Z Data/Entropydata.xls MEU25Z! ;*

*\$libinclude xlexport MROWZ Data/Entropydata.xls MROWZ! ;*

*\$libinclude xlexport shRoEUtrade Data/Entropydata.xls shRoEUtrade! ;*

*\$libinclude xlexport shRoEUtradeE Data/Entropydata.xls shRoEUtradeE! ;*

*\$libinclude xlexport XDDEZ Data/Entropydata.xls XDDEZ! ;*

*\$libinclude xlexport XDDE\_o Data/TranstoolsData.xls Tonnes!C1..O35926 ;*

*\$libinclude xlexport transpm Data/TranstoolsData.xls GCostUnit!C1..O35926 ;*

*Scalar*

*scal scaling parameter monetary values*

;

*scal = 1000 ;*

*Positive variables*

*XDDE\_V(sec,reg,regg)*

*transpmV(sec,regg,reg)*

*shRoEUtradeV(cnt,sec)*

*shRoEUtradeEV(cnt,sec)*

;

*Variables*

*entropy*

;

*Equations*

*EQ11(sec,reg)*

*EQ12(sec,reg)*

*EQ21(sec,reg)*

*EQ22(sec,reg)*

*EQ3(cnt)*

*EQ4(cnt)*

*EQentropy*

;

*EQ11(sec,reg)\$((XDZ(sec,reg)- EEU25Z(sec,reg) - EROWZ(sec,reg)) and (not goods(sec)) )..*

*(XDZ(sec,reg)- EEU25Z(sec,reg) - EROWZ(sec,reg)) =E=*

*sum(regg, XDDE\_V(sec,reg,regg)) ;*

*EQ12(sec,reg)\$ (XDZ(sec,reg) and goods(sec))..*

*XDZ(sec,reg) =E= sum(regg\$XDDE\_o(sec,reg,regg), XDDE\_V(sec,reg,regg))*



$$\begin{aligned}
& + EEU25Z(sec,reg)*sum(mapCR(cnt,reg),shRoETradeEV(cnt,sec)) \\
& + EROWZ(sec,reg) ;
\end{aligned}$$

$$\begin{aligned}
EQ21(sec,reg) & \$((XZ(sec,reg) - MEU25Z(sec,reg) - MROWZ(sec,reg)) \text{ and } (not\ goods(sec)) ).. \\
& (XZ(sec,reg) - MEU25Z(sec,reg) - MROWZ(sec,reg)) =E= \\
& sum(regg, XDDE\_V(sec,regg,reg)*(1+transpmV(sec,regg,reg))) ;
\end{aligned}$$

$$EQ22(sec,reg) \$ (XZ(sec,reg) \text{ and } goods(sec))..$$

$$\begin{aligned}
XZ(sec,reg) & =E= sum(regg \$ XDDE\_o(sec,regg,reg), \\
& XDDE\_V(sec,regg,reg)*(1+transpmV(sec,regg,reg))) \\
& + MEU25Z(sec,reg)*sum(mapCR(cnt,reg),shRoETradeV(cnt,sec)) \\
& + MROWZ(sec,reg) ;
\end{aligned}$$

$$\begin{aligned}
EQ3(cnt).. & sum((sec,reg) \$ mapCR(cnt,reg),XDDEZ(sec,reg)*scal) \\
& =E= sum(sec, sum((reg,regg) \$ mapCR(cnt,reg), \\
& XDDE\_V(sec,reg,regg))) ;
\end{aligned}$$

$$\begin{aligned}
EQ4(cnt).. & sum((sec,reg) \$ mapCR(cnt,reg),TMXZ(sec,reg)*scal) \\
& =E= sum(sec, sum((reg,regg) \$ mapCR(cnt,reg), \\
& XDDE\_V(sec,reg,regg)*transpmV(sec,reg,regg))) ;
\end{aligned}$$

$$\begin{aligned}
EQentropy.. & entropy =E= sum((sec,reg,regg) \$ XDDE\_o(sec,reg,regg),XDDE\_V(sec,reg,regg) \\
& *log(XDDE\_V(sec,reg,regg)/XDDE\_o(sec,reg,regg))) \\
& + sum((sec,reg,regg) \$ transpm(sec,reg,regg),transpmV(sec,reg,regg) \\
& *log(transpmV(sec,reg,regg)/transpm(sec,reg,regg))) \\
& + sum((sec,cnt) \$ shRoETrade(cnt,sec),shRoETradeV(cnt,sec))
\end{aligned}$$

```

*log(shRoETradeV(cnt,sec)/shRoETrade(cnt,sec))
+ sum((sec,cnt)$shRoETradeE(cnt,sec),shRoETradeEV(cnt,sec)
*log(shRoETradeEV(cnt,sec)/shRoETradeE(cnt,sec))) ;

```

*Model entropy\_min /EQ11,EQ12,EQ21,EQ22,EQ3,EQ4,EQentropy/ ;*

```

XDDE_V.L(sec,reg,regg) = XDDE_o(sec,reg,regg) ;

```

```

transpmV.L(sec,reg,regg) = transpm(sec,reg,regg);

```

```

shRoETradeV.L(cnt,sec) = shRoETrade(cnt,sec) ;

```

```

shRoETradeEV.L(cnt,sec) = shRoETradeE(cnt,sec) ;

```

```

XDDE_V.FX(sec,reg,regg)$ (XDDE_o(sec,reg,regg) eq 0) = XDDE_o(sec,reg,regg) ;

```

```

transpmV.FX(sec,reg,regg)$ (transpm(sec,reg,regg) eq 0) = transpm(sec,reg,regg);

```

```

shRoETradeV.FX(cnt,sec)$ (shRoETrade(cnt,sec) eq 0) = shRoETrade(cnt,sec) ;

```

```

shRoETradeEV.FX(cnt,sec)$ (shRoETradeE(cnt,sec) eq 0) = shRoETradeE(cnt,sec);

```

```

Option iterlim = 1000000 ;

```

```

Option reslim = 1000000 ;

```

```

option nlp = conopt3 ;

```

*Solve entropy\_min using nlp minimizing entropy ;*

```

Display XDDE_V.L, transpmV.L,shRoETradeEV.L,shRoETradeV.L ;

```

```

loop ((sec,reg),

```

```

loop(regg,

```

```
if ((XDDE_V.L(sec,reg,regg) lt 0),  
    abort "check XDDE(sec,reg,regg)"  
);  
);  
);
```