EERI

Economics and Econometrics Research Institute

Wage Bargaining and Induced Technical Change in a Linear Economy: Model and Application to the US (1963-2003)

Daniele Tavani

EERI Research Paper Series No 15/2009

ISSN: 2031-4892

EERI
Economics and Econometrics Research Institute
Avenue de Beaulieu
1160 Brussels
Belgium

Tel: +322 299 3523
Fax: +322 299 3523
www.eeri.eu

Wage Bargaining and Induced Technical Change in a Linear Economy: Model and Application to the US (1963-2003)

Daniele Tavani*

April 27, 2009

Abstract

In a simple one-sector, two-class, fixed-proportions economy, wages are set through axiomatic bargaining *à la* Nash [17]. As for choice of technology, firms choose the direction of factor augmentations to maximize the rate of unit cost reduction (Kennedy [14], and more recently Funk [10]). The aggregate environment resulting by self-interested decisions made by economic agents is described by a two-dimensional dynamical system in the employment rate and output/capital ratio. The economy converges *cyclically* to a long-run equilibrium involving a Harrod-neutral profile of technical change, a constant rate of employment of labor, and constant input shares. The type of oscillations predicted by the model matches the available data on the United States (1963-2003). Finally, institutional change, as captured by variations in workers' bargaining power, has a positive effect on the rate of output growth but a negative effect on employment.

Keywords: Bargaining, Induced Technical Change, Factor Shares, Employment.
JEL Classification System: E24, E25, J52, O31.

1 Introduction

The standard literature on the direction of technical progress assumes in Neoclassical fashion that productive inputs are paid their marginal contribution to a smooth production function, so that in the absence of frictions in the labor market labor is constantly fully employed (Acemoglu [1] is an authoritative example). Foley and Michl [9] first, and more recently Basu [2], have shown using a broad cross-section of countries that the empirical support for the hypothesis of marginal productivity pricing of labor is very little, to use an euphemism.

On the other hand, wages need not to equal marginal product of labor in fixed-coefficients models of production, so that these frameworks accommodate naturally for unemployment of the labor force even when the labor market is assumed to work smoothly. However, fixed-proportions models leave typically open the determination

*Department of Economics, Colorado State University, Fort Collins, CO 80523. daniele.tavani@colostate.edu. I thank Duncan Foley, Peter Skott, Rudi Arnim, Luca Zamparelli and seminar participants at Colorado State University for valuable comments and suggestions. All errors are mine.

of factor prices, in assuming the prevailing wage to be set exogenously through social mechanisms, and not through the profit maximization or cost minimization behavior typical of the capitalist firms that economic theorists are familiar with.

Being convinced by the overwhelming evidence, for instance the one presented by Basu [2], against marginal productivity pricing of labor is one thing, hand-waving on wage determination is another, though. It is therefore surprising that not much effort has been dedicated by economists trained in linear production models into 'digging deeper' in the social features of distribution operating behind the black box labeled 'wages', so as to come up with analytical theories of wage-setting behavior. Scholars working in more mainstream traditions, on the other hand, have spent a great deal of time and brainpower in figuring out theories of wages, such as efficiency wages (Solow [27], Shapiro and Stiglitz [26]), insiders-outsiders (Lindbeck and Snower [16]), trade unions' behavior (see the survey by Oswald [20], and the famous paper by McDonald and Solow [18]), just to cite some. The problem with most of these theories is that what determines the equilibrium employment rate is still firms' labor demand schedule, so that at an equilibrium of the model wages are still equal to marginal product of labor.

The purpose of this paper is to take one step toward an analysis of social determination of wages in an otherwise standard one-sector, two-class, fixed-proportions model, and to study what are the implications of this step for choice of technology, growth and distribution in a model with induced technical progress. The reason for the choice of a fixed-coefficient technology is exactly because factor demands and factor prices can be determined without any reference to marginal products.[1] Embedding wage-setting in a framework with class-distinction and induced technological progress, on the other hand, links income distribution and class-conflict to economic growth, this way accounting for the movements of factor productivities, factor shares, and unemployment, and relating to the existing literature on the subject. Goodwin's [11] model of class struggle, for instance, is built around an assumption linking wage growth with the employment rate as a proxy for workers' bargaining power. More recently, models of distribution and growth extending Goodwin's [11] analysis of growth cycles to include induced factor-augmenting technological change have been developed by Shah and Desai [25], van der Ploeg [21], and Julius [15]. All these models feature an exogenously determined wage and an underlying fixed-proportions economy.

In contributing to the scientific production on class-struggle, growth and income distribution, I assume that two types of economic units, workers and capitalists, populate a simple one-sector economy. In the baseline version of the model, workers possess only labor power, and consume all their income. Capitalists own the means of production, hire labor and tie up capital to undertake production, and save over time in order to accumulate capital. The paper is built around two assumptions about the behavior of economic agents:

1. wage setting occurs institutionally through axiomatic bargaining *à la* Nash [17] between workers and capitalists;

2. in choosing factor-augmenting technologies, firms behave according to what Funk [10] called *hypothesis of induced innovation*, namely they choose rates of

[1] For readers familiar with such kind of issues, here's a valuable remark suggested by Duncan Foley. From a strictly mathematical point of view Leontief production functions do have 'marginal products'. What happens is that the 'gradient' one gets from the tangent plane to the isoquant is generalized to a 'subgradient', which is a set of direction vectors normal to the isoquant. In the Leontief case, of course, any direction vector between 0 and infinity is a 'marginal product'. The role of bargaining in this model is to pin down a single vector among the infinite possible ones.

factor augmentation so as to maximize the rate of unit cost reduction given the costs of technology adoption (Kennedy [14], Drandakis and Phelps [6], Samuelson [24]).

The main implications of the present analysis are that the economic decisions on wage determination, innovation and capital accumulation eventually boil down to a two-dimensional dynamical system in capital productivity and the employment rate. The dynamics of the economy evolve so as to ensure a Harrod-neutral path of technical progress, and a constant long-run employment rate which adjusts so as to ensure the constancy of factor shares at the long-run equilibrium. Convergence to the equilibrium path of growth and distribution occurs cyclically, and these oscillations are shown to be consistent with the available empirical evidence for the United States. Also, out of equilibrium both the profit rate and the growth rate of capital productivity decline with the employment rate, while the growth rate of output per worker and the real wage increase with the employment rate. Finally, variations in the relative contractual power of the bargainers have important implications for patterns of growth and distribution through induced technical change: a higher contractual weight of the workers induces a higher long-run rate of labor-augmenting innovations, coupled however with higher long-run unemployment.

The remainder of the paper is organized as follows. In Section 2, I first describe the economic environment, given by the technology for output production and the wage bargaining structure. Then, I describe a static equilibrium for the economy, in which real wage, profit rate, and factor demands are determined. I turn to the consumption and savings decisions of capitalists, and to the choice of technology adoption under the hypothesis of induced innovation. I derive the dynamical system describing the economy, characterize its long-run equilibrium, study its stability properties, analyze qualitatively the behavior of the system in the phase space, and carry comparative dynamics exercises for varying exogenous parameters. To check the relevance of the theoretical model proposed, I use annual data on output/capital ratio from the Extended Penn World Tables and annual unemployment rate from the Bureau of Labor Statistics (BLS) to show that the type of dynamic behavior predicted by the model is consistent with the available evidence on the United States (1963-2003). Finally, I extend the model to include workers' savings, and show that its conclusions are robust with respect to this different behavioral assumption. Section 5 concludes.

2 The Model

2.1 Technology

Consider a representative firm in a simple one-sector economy with two classes, capitalists and workers. Workers posses labor power only, while capitalists own the means of production, hire labor and tie up stocks of a single capital good to produce a single final good homogeneous with capital. Production takes place according to the instantaneous Leontief technique:

$$Y = \min\{AL, BK\} \tag{1}$$

where L, K denote labor and capital respectively, A and B are positive numbers summarizing the current stocks of factor-augmenting technologies.

2.2 Wage Bargains

Workers and capitalists behave in the axiomatic way first studied by Nash [17]. They face off in a market for costlessly enforceable labor contracts of length one period, in which the wage is set through bargaining. The labor market closes after a single round of negotiations so that, if a deal is not struck, the production process will be interrupted for the period. If the negotiations succeed, capitalists will earn profits per unit of capital equal to the profit rate $r = B(1-w/A)$, where w is the wage to be determined within the bargaining problem. We assume that production can be shut down at no cost, hence the capitalist's fallback position is zero. On the other hand, the gain for each worker in case of agreement is given by the difference between the wage in case a deal is struck and a fallback position $z > 0$. Denoting by η the workers' bargaining power, and assuming that both bargainers have preferences equal to their respective utility gains, we have the following problem to be solved:[2]

$$\text{Choose } w \quad \text{to maximize} \quad \eta \log(w - z) + (1 - \eta) \log(B(1 - w/A)) \tag{2}$$

The solution of the bargaining problem leads to the following equation for the wage:

$$w = \eta A + (1 - \eta)z \tag{3}$$

a linear combination of labor productivity and the outside option. If the round of negotiations is successful, the profit rate is:

$$r = (1 - \eta)B(1 - z/A) \tag{4}$$

The last two equations clarify the role of workers' bargaining power in this model. For $\eta = 0$, $w = z$ and $r = B(1 - w/A)$. Conversely, for $\eta = 1$ workers will appropriate the whole productivity of labor and therefore the profit rate will be zero.

The next step involves characterizing the outside option available for the workers. It is quite standard in the literature on unemployment (see Romer [23]) to consider the payoff for the worker in case of disagreement to be a weighted average of the wage she could earn in another firm, say w_a, and the unemployment compensation. The weight is taken to be the employment rate $v \equiv L/N \in (0, 1)$ (where $N > L$ is the total working population), this way interpreted as a proxy for the probability of finding another job. Assuming an unemployment benefit equal to zero, we have:

$$z = vw_a$$

2.3 Static Equilibrium

A *static equilibrium* for this model is: i) a wage w satisfying (3) and such that $w = w_a$; ii) an allocation of labor and capital such that profits are maximized given the wage. It is worth observing that the standard (see for instance Summers [28]) requirement on the equilibrium wages closes the model in a way alternative to the full-employment requirement typical of mainstream growth literature. At a static equilibrium, we have $w = \frac{\eta}{1-v(1-\eta)}A$, so that the equilibrium profit rate is:

$$r = (1 - \eta)B \left(\frac{1 - v}{1 - v(1 - \eta)} \right) \tag{5}$$

[2] Since the choice variable is w, it makes no difference if we consider profits, rK, or the profit rate, to appear in the capitalists' utility gain.

The equilibrium wage (profit rate) obtained above is directly (inversely) related to the workers' bargaining power and employment rate, as it is intuitve. Also, the equilibrium wage is a linear function of output per worker, with slope equal to $\frac{\eta}{1-v(1-\eta)} \in (0,1)$.

2.4 Capitalists' Consumption and Savings

As we assumed that workers consume all their incomes, capital accumulation will depend only on the economic decisions made by capitalist households. We make the hypotheses that the representative household has logarithmic preferences over consumption streams $C(t), t \in [0,\infty)$, that capital is always utilized at full capacity, and that there is no capital depreciation. Under these assumptions, the household's income will be given by profits, rK, to be allocated to consumption and investment, denoted by $\dot{K}$. Because of strict monotonicity of preferences, the household's budget constraint will be satisfied with equality at all t. Therefore, each of the capitalist households faces the following problem:

$$\begin{array}{lll} \text{Choose } C(t) & \text{to maximize} & \displaystyle\int_0^\infty \exp(-\rho t)\log(C(t))dt \\ & \text{subject to} & C(t) = rK(t) - \dot{K}(t) \\ & & K(0) = K_0 \text{ given} \\ & & \displaystyle\lim_{t\to\infty} \exp(-\rho t)K(t) = 0 \end{array} \tag{6}$$

To find a solution for (6), write down the current-value Hamiltonian

$$H = \log(C) + \mu(rK - C)$$

The first order condition on the control variable is $C = \mu^{-1}$, whereas the necessary condition for optimality on the costate variable μ is

$$(\rho - r)\mu = \dot{\mu}$$

Also, the transversality condition $\exp(-\rho t)\mu(t)K(t) = 0$ must be fulfilled. Given strict concavity of the objective function and convexity of the constraint set, the sufficient conditions for optimality will also be satisfied. Hence, the solution of the optimization problem (6) is a system of differential equations in C, K formed by the typical Euler equation $\frac{\dot{C}}{C} = r - \rho$, together with the law of motion of K over time $\frac{\dot{K}}{K} = r - \frac{C}{K}$. To solve this system, let us use a guess-and-verify strategy. Our candidate solution is $C = aK$, where a is a coefficient to be determined. It is clear that the only coefficient satisfying the two differential equations is $a = \rho$, so that $C = \rho K$. Now, consider the capitalists' propensity to save out of their profits, $\frac{rK-C}{rK} = \frac{r-\rho}{r}$. Using the capitalists' budget constraint, we have the traditional Cambridge equation

$$\frac{\dot{C}}{C} = r - \rho = \left(1 - \frac{\rho}{r}\right) r = \frac{\dot{K}}{K} \tag{7}$$

where the term in parentheses is nothing but the propensity to save, so that we have determined a balanced growth path in which capital accumulation and consumption grow at the same rate.

2.5 Choice of Direction of Technical Progress

Consider now the firm's problem of choosing rates of factor augmentation. Let $\dot{B} = \beta B, \dot{A} = \alpha A$. Assume that at each moment in time the available profiles of technological improvements (α, β) belong to the *innovation set*:

$$I = \{(\alpha, \beta) \in \mathbb{R}^2 : \alpha \leq g(\beta)\} \tag{8}$$

where $g \in \mathcal{C}^2, g' < 0, g'' < 0, g^{-1}(0) > 0$. Following Kennedy [14], call Innovation Possibility Frontier (IPF henceforth) the boundary of the innovation set. Such an IPF represents the costs of inventive activity for a given R&D budget, and it is supposed to be strictly concave to capture a notion of 'increasing complexity' in factor augmentation.

Also, we follow the traditional literature on induced bias in innovation in assuming that the firm chooses a profile of factor augmentation so as to maximize the rate of reduction in costs per unit of output. A justification for such myopic behavior can be found in Funk [10], who takes on an earlier argument by Samuelson [24]: a firm adopting a profit-increasing method of production, if successful, will be sooner or later imitated by its competitors, so that within a reasonably short amount of time such extra-profits will disappear.

In choosing rates of factor augmentation to maximize the current rate of unit-cost reduction, capitalists will clearly consider the unemployment compensation z and its variation over time as a given. The following lemma establishes that the problem of choosing rates of factor-augmentations so as to maximize the rate of unit-cost reduction and that of choosing rates of factor-augmenting technologies so as to maximize the rate of change in profits per unit of capital are dual to each other.

Lemma 1. Let G be the minimum production costs for a firm facing the technology (1). Let r be the profit rate for the same technology, and let rates of factor-augmenting technologies belong to the innovation set defined in (8). Then, the problem of choosing rates of factor-augmentation so as to maximize the rate of cost-reduction per unit of output and that of choosing rates of factor-augmentations so as to maximize the rate of change in the profit rate yield the same solution:

$$-g_\beta = \frac{1-\xi}{\xi}$$

where $\xi \equiv z/A$.

The proof of the lemma, which is just a simple exercise in duality, is provided in Appendix A.1. The reason why the result is important enough to be mentioned is that, although Kennedy [14] and his followers were thinking, and derived all their results, in terms of unit costs, differential profits as a criterion for technology adoption comes more natural for those familiar with traditional analysis of technical choice by capitalist firms in linear production models (Okishio [19], Roemer [22]). Establishing the duality of these two way of thinking about the problem is therefore meaningful.

More importantly for the present analysis, the solution of the technical choice problem identifies univocally a pair of functions $\beta(\xi), \alpha(\xi)$ respectively decreasing and increasing in their argument. This result follows immediately from $g_\beta < 0$.

An *equilibrium direction of technical change* is obtained using the equilibrium condition $z = vw$, and is characterized by:

$$\begin{aligned} -g_\beta &= \frac{1 - v\frac{\eta}{1-v(1-\eta)}}{\frac{v(\eta}{1-v(1-\eta)}} \\ &= \frac{1-v}{\eta v} \end{aligned} \tag{9}$$

to obtain which we used of course the wage corresponding to a static equilibrium. The equilibrium direction of technical progress (9) determines a pair of continuous functions $\beta(v;\eta), \alpha(v;\eta)$ representing the induced bias in innovation as a function of the employment rate v for a given value of the workers' bargaining power. Equilibrium capital-augmenting technical progress will respond negatively to the employment rate, whereas equilibrium labor-augmenting innovations will be directly related to v. The economic intuition behind this result is straightforward. The wage at a static equilibrium, and therefore the labor share in output, is a linear function of the expected value of the outside option for the workers, in turn a linear function of the employment rate. The standard argument of induced bias implies that the higher the labor share in costs, the more technical progress will be directed toward labor-augmenting blueprints for a given intensity. This feature is entirely consistent with our result, as the higher the employment rate the higher the equilibrium labor share in this model. Furthermore, an increase in the bargaining power η induces more labor-augmenting and less capital-augmenting technical progress, a result that is also in accordance with the idea of induced bias in innovation.

2.6 The Dynamical System

Consider the employment rate $v \equiv L/N$. Profit maximization under the technological constraint (1) implies that $v = \frac{BK}{AN}$. Logarithmic differentiation of v yields the dynamic equation:

$$\dot{v} = (\hat{B} + \hat{K} - \hat{A} - \hat{N})v$$

where 'hat' variables denote growth rates as usual. Assume that the model is labor-constrained: $\hat{N} \equiv n$, constant and exogenous. Plugging the equilibrium profit rate obtained in (5) in the Cambridge equation (7), together with results on the equilibrium direction of technical progress obtained above, we have the following nonlinear dynamical system in the state space (B, v):

$$\dot{B} = \beta(v;\eta)B \tag{10}$$

$$\dot{v} = \left(\beta(v;\eta) + (1-\eta)B\left(\frac{1-v}{1-v(1-\eta)}\right) - \rho - \alpha(v;\eta) - n\right)v \tag{11}$$

2.7 Long-run Equilibrium

A *long-run equilibrium* for this economy is a two-dimensional row vector (B, v) such that $\dot{B} = \dot{v} = 0$.

Proposition 1. Under the assumptions made throughout this paper, a long-run equilibrium in which $B \neq 0, v \neq 0$ exists and is unique. It features a Harrod-neutral profile of technical change, a constant unemployment rate and constant input shares.

Proof. Existence and uniqueness of a long-run equilibrium are ensured by the fact that the functions forming the dynamical system (10), (11) are both $\mathcal{C}^1$ in their arguments (Hirsch, Smale and Devaney [12], p.144). At a dynamic equilibrium,

$$\beta(v_{ss};\eta) = 0 \tag{12}$$

$$B_{ss} = \frac{\alpha(v_{ss};\eta) + n + \rho}{(1-\eta)\left(\frac{1-v_{ss}}{1-v_{ss}(1-\eta)}\right)} \tag{13}$$

so that the long-run equilibrium is of the Harrod-neutral type, with zero capital-productivity growth and a growth rate of labor-augmenting technologies equal to

$\alpha_{ss} = g(0)$. The equilibrium employment rate is constant and equal to $\beta^{-1}(0)$. Finally, once the long-run employment rate of the model is achieved, the wage share will be constant, and given by $\frac{\eta}{1-v_{ss}(1-\eta)}$. □

Proposition 1 confirms the findings typical of models of directed technical change, such as Drandakis and Phelps [6], in what basically says that our model matches the Kaldor [13] facts. Furthermore, an important characteristic of the present model is the role of the employment rate in relation to factor shares: as workers and capitalists bargain only on wages, the employment rate adjusts so as to ensure the constancy of factor shares in the long-run and to annihilate the capital-augmenting component of technical progress.[3] Also, the employment rate, and therefore the labor share, are invariant with respect to the savings decisions made by capitalist households, as in Julius [15]. As for the local stability properties of the long-run equilibrium, we have the following proposition.

Proposition 2. The long-run equilibrium of this economy is locally asymptotically stable.

Proof. Linearize the above system around its non-trivial rest point to obtain the Jacobian matrix:

$$J_{ss} = \begin{pmatrix} 0 & \beta_v \left(\frac{\alpha(v_{ss};\eta)+n+\rho}{(1-\eta)\frac{1-v_{ss}}{1-v_{ss}(1-\eta)}} \right) \\ (1-\eta)\left(\frac{1-v_{ss}}{1-v_{ss}(1-\eta)}\right) v_{ss} & \left(\beta_v - \frac{\eta}{(1-v_{ss}(1-\eta))}\frac{\alpha(v_{ss};\eta)+n+\rho}{(1-v_{ss})} - \alpha_v\right) v_{ss} \end{pmatrix}$$

This matrix has a negative trace, as $\beta_v < 0$, and a positive determinant. Therefore, its two eigenvalues have real parts that are of the same sign and sum up to a negative number, and this proves the claim. □

Also, it is easy to see that long-run output growth is equal to the Harrod rate $\hat{Y} = \alpha(v_{ss}) + n$, in turn equal to the growth rate of capital stock.

2.8 Transitional Dynamics

Let us now focus on the behavior of the dynamical system (10), (11) out of the long-run equilibrium, to show that convergence occurs cyclically. First of all, let us determine the isoclines on the phase space (B, v). The equilibrium locus for the change in output/capital ratio is a horizontal line at $v_{ss} = \beta^{-1}(0)$. In the Appendix, I show that the function $B(v;\eta)$ as defined in (13) is strictly increasing and strictly convex for any value of $v \in (0,1)$, and that it tends to ∞ as v tends to one.

We can now consider qualitatively the dynamics of the two variables (B, v) in the phase plane. A graphical representation is provided in Figure 1.

[Insert Figure 1 about here]

Clearly, B is decreasing in v when the employment rate is not at its equilibrium level, as the function $\beta(v;\cdot)$ is decreasing in its argument. Therefore, the arrows in the phase diagram point west for $v < v_{ss}$ and east for $v > v_{ss}$. As for the behavior of v outside the isocline, it is sufficient to differentiate $\dot{v}$ with respect to B in (11) to see that the employment rate increases in capital productivity. Hence,

[3] Such adjustment take place also in van Der Ploeg [21] and Julius [15], but with an exogenous wage and a reduced-form Goodwin hypothesis of wage growth being an increasing function of the employment rate.

the arrows in the phase space point north above the $\dot{v} = 0$ isocline and south below. Putting all things together, we see that the phase space is characterized by counterclockwise oscillations. Since we showed before that the long-run equilibrium is locally asymptotically stable, we conclude that the dynamics of the model describe a spiral converging to the equilibrium point. These results can be summarized in the following proposition.

Proposition 3. The long-run equilibrium of the system formed by (10), (11) is a stable spiral displaying counterclockwise oscillations in the phase space (B, v).

Next, we summarize the comparative dynamics of the long-run equilibrium of this model for varying discount rate and workers' bargaining power.

2.9 Comparative Dynamics

Proposition 4. At a long-run equilibrium, i) capital productivity is increasing in the discount rate, in the rate of population growth, and in workers' bargaining power; ii) Capital (labor)-augmenting technical change is decreasing (increasing) in workers' bargaining power.

Proof. We have

$$\frac{\partial B_{ss}}{\partial \rho} = \frac{1}{(1-\eta)\left(\frac{1-v}{1-v(1-\eta)}\right)} = \frac{\partial B_{ss}}{\partial n} > 0$$

which proves the first part of i). Also, our assumptions on the IPF ensure that $\beta_\eta < 0, \alpha_\eta > 0$, proving ii). To prove that the equilibrium output/capital ratio is increasing in workers' bargaining power, simply differentiate B with respect to η in 13 to have:

$$\frac{\partial B_{ss}}{\partial \eta} = \frac{\alpha_{ss} + n + \rho}{(1-v)(1-\eta)^2} + \frac{v(\alpha + n + \rho)}{(1-v)(1-\eta)} + \frac{(1 - v(1-\eta))\alpha_\eta}{(1-v)(1-\eta)} > 0$$

□

The only result that requires comments is the positive relation between long-run capital productivity and bargaining power. On the one hand, B_{ss} is directly related to the long-run productivity growth, in turn an increasing function of the bargaining power of workers. On the other hand, long-run capital productivity is inversely related to the savings rate, as it is standard in similar models (van der Ploeg [21], Shah and Desai [25] Julius [15]). Since capitalists are the only savings class, and their income is inversely related to the workers' bargaining power, the higher η the lower the savings rate, hence the higher B_{ss}.

The final question we ask is how does institutional change affects the long-run growth and distribution path of the model, that is how does the long-run equilibrium employment rate vary with workers' bargaining power.

Proposition 5. Assume that η increases of an amount $d\eta > 0$. Then, the corresponding variation in the equilibrium employment rate dv_{ss} must be negative.

Proof. Since $\beta(v_{ss}; \eta) = 0$, we must have that $\beta_{v_{ss}} dv_{ss} + \beta_\eta d\eta = 0$, from which:

$$\frac{dv_{ss}}{d\eta} = -\frac{\beta_\eta}{\beta_{v_{ss}}}$$

As both $\beta_\eta, \beta_{v_{ss}} < 0$, the claim is proved. □

The interpretation of the result is that, although wage bargaining occurs in an essentially cooperative way, class-conflict does not disappear from the model. In fact, once wages are set, capitalists have still full control in choosing new labor-augmenting technologies. And an increase in labor productivity reduces the demand for labor given the production technology (1).[4] To make sense of the finding from a distributional standpoint, remind that factor shares have to remain constant in the long run. Hence, the upward pressure in the labor share arising from an increase in workers' bargaining power has to be compensated by a reduction in the employment rate in order to bring back the model to its long run growth path.

3 Application: Goodwin Growth Cycles in the US Economy 1963-2003

The counterclockwise oscillations predicted by the simple model developed above match the available evidence on capital/output ratio-employment rate cycles, as shown in Figure 2. The data on output/capital ratio are taken from the Extended Penn World Tables [7] (version 3.0), and the data on employment rate are constructed taking the (complement to one of) yearly average of monthly series of the rate of unemployment from the Bureau of Labor Statistics [5].

[Insert Figure 2 about here]

A positive dependence of long-run capital productivity on the employment rate (which is guaranteed from a theoretical standpoint by Lemma 2, in the Appendix) seems confirmed by the downturn in both variables corresponding to the late 1970s and early 1980s.

The presence of growth and distribution cycles in the United States has been the object of investigation in recent applied macroeconomic literature. Barbosa-Filho and Taylor's [3] model predicts counterclockwise oscillations in the phase space featuring capital-output ratio on the horizontal axis and the wage share on the vertical axis, and these oscillations are found to be consistent with the available time-series evidence. The same kind of cycles, but in the plane *(employment rate, wage share)*, are displayed in the empirical plots appearing in Flaschel *et al* [8], so that these authors use a version of Okun's law in order to justify their findings in relation with those of Barbosa and Taylor. Both these contributions, in the Keynesian tradition, present descriptive macro models in reduced form that replicate such movements.

Here, counterclockwise growth-distribution dynamics arise in a model derived from the explicit consideration of self-interested behavior by economic agents. The only difference with the previous contributions is that the employment rate appears on the vertical axis here instead of the wage share. However, an economist familiar with Goodwin's [11] predator-prey cycles would expect a positive relation between the wage share and the rate of employment of the labor force, and such a link between the two variables is ensured in the present model through the bargaining structure leading to wage determination.

[4]What drives this result is the fact that effective inputs are complements in production. If we used a smooth production function, an increase in labor-augmenting technologies would also produce an increase in the demand for labor, with an elasticity of substitution greater than one. Such findings would not substantially change with an elasticity of substitution smaller than one, which is also supported by empirical evidence (see the brief survey in Acemoglu [1].

4 Allowing for Workers' Savings

We now relax the assumption that capitalists are the only savers in this economy to show that the main conclusions reached above are robust to this different behavioral scenario. Such an extension is entirely straightforward: it is enough to observe that all the income produced can be consumed or saved (invested). The dynamic constraint appearing in (6) becomes simply $BK = C + \dot{K}$. Hence, the Euler equation for consumption, giving also the growth rate of capital stock, is:

$$\frac{\dot{C}}{C} = B - \rho$$

Equation (11) modifies as follows:

$$\frac{\dot{v}}{v} = \beta(v;\eta) + B - \rho - \alpha(v;\eta) - n$$

and the Jacobian matrix under the new savings assumption is:

$$J = \begin{pmatrix} 0 & \beta_v(\alpha_{ss} + n + \rho) \\ v_{ss} & (\beta_v - \alpha_v)v_{ss} \end{pmatrix}$$

Again, this matrix has negative trace and positive determinant, so that the equilibrium of the system, namely $\beta(v_{ss};\eta) = 0, B_{ss} = \alpha(v_{ss};\eta) + \rho + n$ is locally asymptotically stable. As for the qualitative analysis of the out of equilibrium behavior, the counterclockwise oscillations found above carry over in this different institutional scenario. Also, Proposition 5 continues to hold trivially.

5 Concluding Remarks

In this paper, I introduced a simple axiomatic bargaining game *à la* Nash [17] as a mechanism of wage-setting into an otherwise standard one-sector, two-class, fixed-coefficients model of growth and distribution with induced technical change. Such a mechanism for wage determination is meant to emphasize, and to incorporate into the framework, the social features of distribution, which are generally left out of the analysis in such kind of models.

I showed that the economy resulting from decision-making on wages, production, savings and innovation, is completely described by a two-dimensional dynamical system in capital productivity and the employment rate. The system evolves so as to achieve a Harrod-neutral path of technical progress, and a constant employment rate. As this is a simple corn model, equilibrium unemployment is not 'natural', in the sense that its role is not to accelerate inflation. Instead, in the spirit of Goodwin [11], the role of equilibrium unemployment is to put the class-conflict between capital and labor to rest. Another feature of the model is that a long-run equilibrium of this economy features constant factor shares, increasing labor productivity, and a positive rate of capital accumulation, so that the present framework matches the Kaldor [13] facts. Finally, the dynamics of the model display cyclical behavior which replicates the available evidence on capital productivity and employment rate for post-war United States. These conclusions are qualitatively unchanged if we allow workers to save, as shown in Section 4.

Institutional change, in the form of variations in the parameter representing workers' bargaining power, has a considerable impact on economic growth, income distribution, and unemployment: I showed that an increase in η produces a higher wave of labor-augmenting technical progress, while reducing the equilibrium employment rate. This feature of the model is hard to evaluate in light of the evidence presented in Figure 2, and is related to the assumption that workers and capitalists bargain on wages only. The result would probably change in the admittedly extreme case, analyzed first by McDonald and Solow [18], in which a strong unionized labor force negotiates with capitalists on both wages and employment.

In endogenizing wages, thus linking them to labor productivity and the employment rate, the model produces considerable simplification in the dynamical description of the economy relative to the previous literature on the subject (Shah and Desai [25], van der Ploeg [21], Julius [15]), without neither altering the typical findings of Harrod-neutrality and constant input shares that characterize long-run patterns of technical change and income distribution, nor obliterating the conflictual features of income distribution between different classes.

Among the simplifying assumptions made throughout this paper, an important one is that of a full utilization of capacity. Relaxing this hypothesis would enable to address in more depth the interaction between cycles and trends in this framework. Also, Nash bargaining is only one of the several mechanisms of wage determination one can think of, and it is not immune of criticism. Exploring the implications for this model of different rules for wage-setting on the one hand, and of choice of capacity utilization on the other, appear to be fruitful areas for further research.

References

[1] Acemoglu, Daron [2003]. 'Labor and Capital Augmenting Technical Change'. *Journal of the European Economic Association*, 1: 1-37.

[2] Basu, Deepankar [2008]. 'A Simple Test of the Neoclassical View of Production, Technical Change and Distribution'. Mimeo, Colorado State University.

[3] Barbosa-Filho, N. H. and Lance Taylor, [2007]. 'Distributive and Demand Cycles in the US-Economy - A Structuralist Goodwin Model'. *Metroeconomica*, 57: 389-411.

[4] Bhaduri, A. and Stephen A. Marglin, [1990]. 'Unemployment and the real wage: the economic basis for contesting political ideologies'. *Cambridge Journal of Eonomics*, vol. 14, 375-93.

[5] Bureau of Labor Statistics [2009]. 'Unemployment Rate - Civilian Labor Force - LNS14000000'. Available online at http://www.bls.gov/data.

[6] Drandakis, E.M, and Edmund Phelps [1965]. 'A Model of Induced Invention, Growth and Distribution'. *Economic Journal*, Vol. 76, No. 304: 823-840.

[7] Extended Penn World Tables, v.3.0. Available online at http://homepage.newschool.edu/ foleyd/.

[8] Flaschel, Peter, Daniele Tavani, Lance Taylor and Timo Teuber [2009]. 'Estimated Non-linearities and Multiple Equilibria in a Distributive-Demand Model', mimeo, The New School - University of Bielefeld.

[9] Foley, Duncan K., and Thomas R. Michl, [1999]. *Growth and Distribution.* Cambridge, MA: Harvard University Press.

[10] Funk, Peter [2002]. 'Induced Innovation Revisited'. *Economica*, 68: 155-171.

[11] Goodwin, Richard [1967]. 'A growth cycle', in: Carl Feinstein, editor, *Socialism, capitalism, and economic growth*. Cambridge, UK: Cambridge University Press.

[12] Hirsh, Morris, W., Stephen Smale, and Robert L. Devaney [2004]. *Differential Equations, Dynamical Systems, and an Introduction to Chaos*. San Diego, CA. Academic Press - Elsevier.

[13] Kaldor, Nicholas [1957]. 'A Model of Economic Growth'. *Economic Journal*, 68:591-624.

[14] Kennedy, Charles [1964] 'Induced Bias in Innovation and the Theory of Distribution. *Economic Journal*, 74: 541-47.

[15] Julius, Alexander J. [2005a] 'Steady-state growth and distribution with an endogenous direction of technical change' *Metroeconomica*, 56: 1, 2005.

[16] Lindbeck, Assar, and Dennis J. Snower [1987]. 'Efficiency Wages vs. Insiders and Outsiders'. *European Economic Review*, 31: 407-416.

[17] Nash [1950]: 'The Bargaining Problem'. *Econometrica*, 18 n. 2: 155-162.

[18] McDonald, Ian M., and Robert Solow [1981]. 'Wage Bargaining and Employment' *American Economic Review* Vol. 71, No. 5: 896-908.

[19] Okishio, Nobuo [1961]. 'Technical change and the rate of profit', *Kobe University Economic Review*, 7: 86-99.

[20] Oswald, Andrew J. [1985]. 'The Economic Theory of Trade Unions: An Introductory Survey'. *The Scandinavian Journal of Economics*, Vol. 87, No. 2: 160-193.

[21] van der Ploeg, Frederick [1987]. 'Growth Cycles, Induced Technical Change, and Perpetual Conict over the Distribution of Income'. *Journal of Macroeconomics*, 9: 1-12.

[22] Roemer, John E., [1989]. *Analytical Foundations for Marxian Economic Theory*. Cambridge, UK: Cambridge University Press.

[23] Romer, David [2006]. *Advanced Macroeconomics*. McGraw-Hill.

[24] Samuelson, Paul [1965]: 'A Theory of Induced Invention along Kennedy-Weizsacker Lines'. *Review of Economics and Statistics*, 27 n. 4: 343-356.

[25] Shah, Anup, and Meghnad Desai, [1981]. 'Growth cycles with induced technical change'. *Economic Journal*, 91: 1006-10.

[26] Shapiro, Carl, and Joseph Stiglitz [1984]. 'Equilibrium Unemployment as a Worker Discipline Device'. *American Economic Review*, Vol. 74, No. 3: 433-444.

[27] Solow, Robert [1979]. 'Another Possible Source of Wage Stickiness'. *Journal of Macroeconomics*, Vol. 1, No. 1: 79-82.

[28] Summers, Lawrence H. [1988]. 'Relative Wages, Efficiency Wages, and Keynesian Unemployment'. *The American Economic Review*, Vol. 78, No. 2: 383-388.

A Proofs

A.1 Proof of Lemma 1

Consider the production technology (1). The minimum cost function corresponding to it is:

$$G\left[\frac{w}{A},\frac{r}{B}\right]Y=\left(\frac{w}{A}+\frac{r}{B}\right)Y$$

Substitute the bargaining wage(3) to obtain the minimum cost per unit of output:

$$\frac{G[\cdot]}{Y}=\eta+(1-\eta)\frac{z}{A}+\frac{r}{B}$$

Defining $\xi \equiv z/A$, $\alpha \equiv \hat{A},\beta \equiv \hat{B}$, and differentiating the above equation with respect to time yields, upon substitution of the constraint represented by the IPF ($\alpha = g(\beta)$) and of the profit rate found in (5):

$$\text{Choose } \beta \quad \text{to maximize } -\tfrac{dG/Y}{dt}=-(1-\eta)\left((\hat{z}-g(\beta))\xi+(\hat{r}-\beta)(1-\xi)\right) \tag{14}$$

whose solution is $-g_\beta = \frac{1-\xi}{\xi}$. Note that substitution of the constraint in the objective function is allowed because the maximand is linear with respect to rate of factor-augmentations and the innovation set is strictly concave, so that the solution will be unique and on the boundary of I.

On the other hand, differentiate the profit rate (4) with respect to time, and use the IPF to have:

$$\text{Choose } \beta \quad \text{to maximize } \dot{r}=(1-\eta)B(\beta(1-\xi)+(g(\beta)-\hat{z})\xi) \tag{15}$$

Differentiate with respect to β to have the claim.

A.2 Lemma 2

Lemma 2. Consider the isocline $B(v;\eta)=\frac{\alpha(v;\eta)+n+\rho}{(1-\eta)\left(\frac{1-v}{1-v(1-\eta)}\right)}$ as a function of v. We have $\lim_{v\to 1} B(v;\cdot)=\infty$. Also, this function is strictly increasing and convex for $v\in(0,1)$.

Proof. The limiting behavior of $B(v;\cdot)$ for $v\to\infty$ is obvious. To prove the second part of the claim, differentiate (13) with respect to v, factor and rearrange to obtain:

$$B_v=\frac{(\alpha+n+\rho)\eta}{(1-v)^2(1-\eta)}+\frac{1-v(1-\eta)\alpha_v}{(1-v)(1-\eta)}$$

The term in the RHS is positive since $\alpha_v>0$, so that $B(v;\cdot)$ is increasing in v. Differentiate once again the above expression, factor, rearrange terms, to see that the inequality

$$2(n+\rho)+(2-(1-\eta)(3v-1-2v^2))\alpha(v)+\alpha_v(1-\eta)(1-2v+2v^2-v^3)>0$$

must hold for convexity. As $\alpha(v),\alpha_v>0$, a sufficient condition for the above restriction to be fulfilled is the system of inequalities

$$\begin{aligned}2-(1-\eta)(3v-1-2v^2)&>0\\ 1-2v+2v^2-v^3&>0\end{aligned}$$

to have a solution for any $v\in(0,1)$. As $\eta\in(0,1)$ the sufficient condition is easily checked. □

B Figures

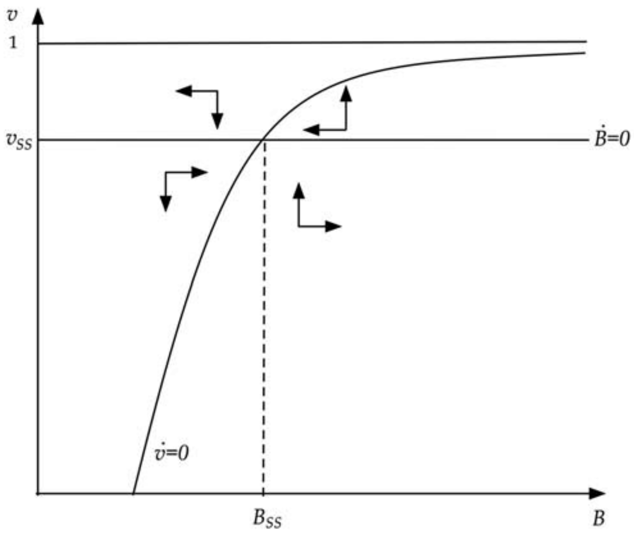

Figure 1: Phase Diagram for the Dynamical System (10), (11).

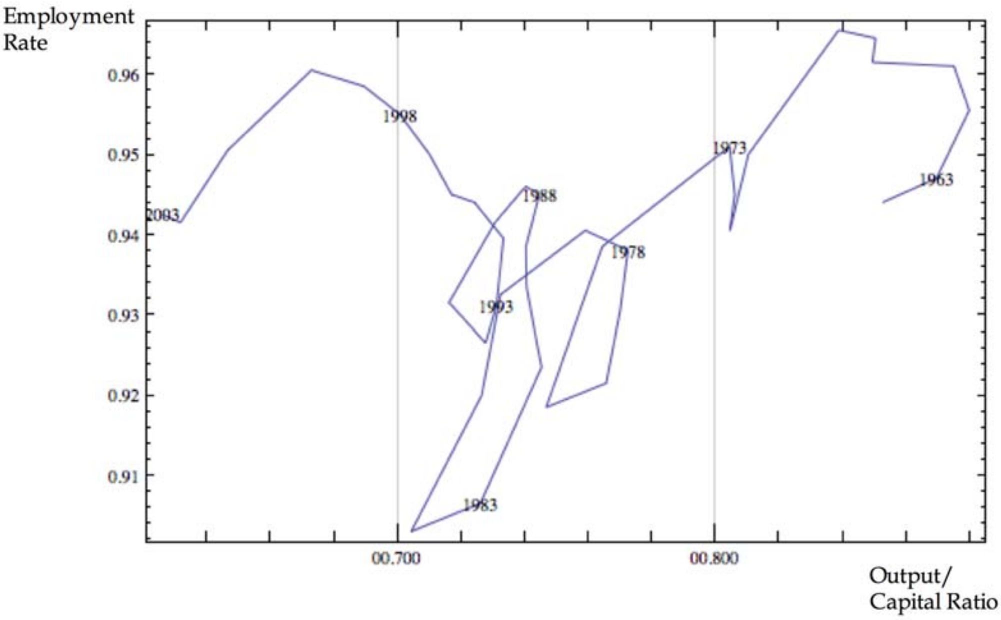

Figure 2: Counterclockwise growth cycles in the phase plane (B, v) in the US. Source: Extended Penn World Table (Output/Capital Ratio), BLS (Employment Rate).