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Nuno Garoupa and Ana Paula Martins

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EERI
Economics and Econometrics Research Institute
Avenue de Beaulieu
1160 Brussels
Belgium

Tel: +322 298 8491
Fax: +322 298 8490
www.eeri.eu

UNIVERSIDADE CATÓLICA PORTUGUESA
Faculdade de Ciências Económicas e Empresariais

Crime and Punishment Re-Awakened – Insights on a Risky Business from the Worker´s Perspective †

Nuno Garoupa *

Ana Paula Martins **

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* Faculdade de Economia da Universidade Nova de Lisboa, Travessa Estevão Pinto, 1099 – 32, Lisbon, Portugal. Phone: 351 21 3801600. Fax: 351 21 3886073. Email: ngaroupa@fe.unl.pt. (Former: Department d´Economia i Empresa, Universitat Pompeu Fabra, Ramon Trias Fargas 25-27, 08005 Barcelona, Spain. Phone: 34-93-542 2639. Fax: 34-93-542 1746.)

** Faculdade de Ciências Económicas e Empresariais, Universidade Católica Portuguesa, Cam. Palma de Cima, 1600 Lisboa, Portugal. Phone: 351 217214248. Fax: 351 217270252. Email: apm@europa.ucp.pt

ABSTRACT

Crime and Punishment Re-Awakened – Insights on a Risky Business from the Worker´s Perspective.

Nuno Garoupa

Ana Paula Martins

This research inspects the general implications of considering duration of confinement as a deduction to the convicted consumer-worker time endowment. Even if analytically simple, the model is able to shed some light on the expected wage profile of criminals, and the pattern of their preferences. The methodology used relies on the analysis of the crime-deterrent composite scheme, establishing the relation between required fines, duration of imprisonment, the monetary size of the offence, arrest probabilities and individual preferences and endowments.

The main conclusions stem from the fact, proven below, that dissuasive seclusion or fines may decrease with the individuals´ wage rate under a much wider range of consumer preferences than with non-labor earnings – for crimes for which seclusion length is high relative to fines. The intuition behind such conclusion lies on the fact that the money value of the opportunity costs of imprisonment is proportionally indexed to the individual´s wage rate.

Sensitivity of the deterrent sanction to uncertainty in the endowments distribution, as well as of dispersion in the sanction itself, was also inspected. It was found dependent on the pattern of individual preferences, particularly on how the response to a risk behaves in the presence of others. Corresponding risk measures were applied.

JEL Classification: K42, J22, J28, D81, D91.

Keywords: Risk and Uncertainty and Labor Supply, Criminal Behavior, Illegal Behavior, Crime Deterrence, Risk Aversion, Multivariate Risks.

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"Lord of Hosts, forever defend my Mind, my Heart and my Soul..."

Introduction.

Even if there has been a long-run trend of substitution of fines for imprisonment, incarceration is an important component of the penalty system and has risen sharply over the last decade; in 1998, 0,147% of the Portuguese population was locked in common prisons. This paper analyses the implications of interpreting imprisonment as a deduction to the time endowment of the convicted consumer-worker – of "taking away years of a convict´s life".

The analysis is quite simple and obvious once the problem has been set in such terms. However, it involves additional complications and refinements relative to the textbook example of consumer behaviour under uncertainty, because endogenous choice of a special two-good consumer basket is now implicit.

Two branches of economic literature are akin to the problem. On the one hand, we draw instruments from individual labor supply theory, including the analysis of the impact of time costs on desired hours or participation - that we can find in Heckman (1974), Wales (1980) and Cogan (1980) and (1981) [1].

On the other, aspects of the theory of economic decisions under uncertainty, relying on von Neumann-Morgenstern expected utility formula, usually interpretable in insurance terms, that include inspection of how the minimal risk premium responds to changes in the exogenous environment. Firstly, non-labor income, wage and price levels; that led us to the role of measures of risk-aversion in the Arrow (1965) and Pratt (1964) sense, and subsequent extensions to the analysis of one risk in the presence of multiple decisions - Deschamps (1973) and Hanoch (1977) – or presence of multiple risks - Ross (1981), Kihlstrom, Romer and Williams (1981), Pratt and Zeckhauser (1987), Kimball (1990 and 1993) and Gollier and Pratt (1996); and to the general study of multivariate risks – Kihlstrom and Mirman (1974), Keeney (1973), Duncan (1977) and

[1] See also more complicated structures in Hanoch (1980) and Oi (1976). Of related interest, also Heckman (1980). Martins (2002) surveys the impact of changes in the time endowment on labor supply.

Martins (2002). Secondly, response of the premium to o r under uncertainty in the exogenous endowments, which subscribes reasoning that can be traced back to Rothschild and Stiglitz (1970) work on the definition and effects of risk on expected utility, and Diamond and Stiglitz (1974) on the relation between such definitions and risk aversion measures [2].

The application of uncertainty theory to criminal analysis seems to have been quite active in the 70´s and 80´s [3]. Subjects like the relation of criminality with wealth and preferences toward wealth, and the rel ative effectiveness of monetary sanctions versus detection probabilities were treated by Becker (1968), Brown and Reynolds (1973), Polinsky and Shavell (1979), Pyle (1983) – Garoupa (1997), Polinsky and Shavell (2000) and Garoupa (2000) contain thorough recent surveys on the theory of optimal law enforcement. In most of these models, results were indistinguishably applicable to robbery, larceny, bribery, embezzlement, extorsion, or simple parking infractions, and sanctions were generally modelled as deducti ons to wealth. Probability of detection, arrest, conviction or punishment were uniformly treated.

Mingles of time allocation theory with criminal analysis can also be found in other research where the loot of the crime is the output of a time -consuming activity - see for example Block and Heineke (1975). The treatment of tax evasion– and the determinants of untaxed work - , intersecting this type of problem, was also incorporated in the analysis – see Cowell (1992) for a survey. These models, even if relyin g, unlike the former, on the consumer -worker problem and, hence, on the mechanics of labor supply theory, were not aimed at the study of the problem of imprisonment [4].

Seclusion was previously contemplated in the literature – by Block and Lind (1975a), Sjoquist (1971) and Polinsky and Shavell (1984). Polinsky and Shavell (1984) discuss the subject of optimal use of fines and imprisonment. They model incarceration as a monetarily evaluated imprisonment term imposed on individuals that adds to a money fine: b ut they do not link the unit cost of the former to the individual wage rate. They rely on the existence of costs of operating the jail system and costs of detection; they discuss fines against imprisonment for different wealth groups, concluding that for w ealthier groups it is optimal to use the fine and then the imprisonment term regardless of risk aversion. Some of their results depend on standard measures of risk aversion with respect to general wealth.

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[2] Alternative uncertainty theories have been applied to the problem. See Garoupa (2001) for some examples and recent references.

[3] Cameron (1988) surveys the pertinent research. See also Heineke (1978).

Our framework is far simpler and highlights another aspect: that the individual´s "full income" cost, or opportunity cost, of incarceration may actually be indexed to the wage rate. As such, non-labor earnings or wealth may be independent of such costs, but they will be proportional to the wage rate the individual faces - hence for a given sanction much larger for high-wage workers. In general, a homogeneous penalty system is, thus, expected to be heavier, that is, more crime dissuasive, for this group – or from the dual perspective, increasing wages may decrease criminality.

The assumption that seclusion costs are proportional to the wage rate may be acceptable for two reasons. On the one-hand, time spent in prison is actually time an individual cannot use in the standard labor market; work done in confinement is poorly or non-paid, the product of which is used to pay for jail maintenance. On the other, incarceration is related to penitence and isolation; it is not "standard leisure" that the individual can consume with the same satisfaction as if free; any non-work time the individual spends there may actually be more unpleasant than if he used it in non-paid standard work. This will be the status we will attribute to it in our model – akin to non-stop forced but "free" – non-paid and compulsory but undertaken out of prison – labor [5].

That costs associated with imprisonment depend upon the length of the sentence served and the wage rate has been previously advanced – see for example Sjoquist (1971) [6]. However, he translates it into a relation between costs and wages and the time spent in illegal activity and does not pursue the former argument. Block and Lind (1975a) suggest the existence of a "wealth equivalent" to a particular sentence and specifically note that if legal earnings foregone are "the primary cost of crime" then an increase in the wage rate will increase the cost of crime, but model it in a standard utility framework, assigning the relation to the cost of crime through the time required to perpetrate the crime; instead, we translate the implications of lost time due to the application of a prison sentence within the consumer-worker problem, which (also) allows us to decompose wage from non-labor earnings effects.

We consider no (additional) psychic costs of imprisonment imposed upon the convict – that would require us to contemplate separately time spent in prison in the utility function. Any crime-aversion or proneness attitudes will stem from properties of the utility function, which are kept completely general in most of the research, only constrained by rationality pre-requisites.

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[4] Even if they treated the trade-off between penalties and probability of detection – see Sandmo (1981), for example.

[5] Imprisonment has, thus, a worse status than slavery in the model - slavery would allow for some leisure consumption.

[6] And implicitly stated by Becker (1968), associating costs of imprisonment with foregone earnings.

And we will concentrate on property crimes – the time required to plan and complete the theft is considered negligible and independent of the size of the infraction; or consider, in general, the subjective (net) monetary value of an infraction to the individual. This allows us to isolate the pure effect of gross time loss of confinement.

No costs of implementation of the penalty scheme are involved or called for – we shall be analysing what would deter crime: the "supply of offences" [7], or, more precisely, the "reservation offence" – taking the enforcement point of view, we will analyse the impact of exogenous conditions on the "reservation sanction", the one below which the individual will find it worthwhile to commit the crime. Nor additional benefits arising from impossibility of committing crimes while in prison – subject dealt with in Shavell (1987), for example. Justice is also assumed to be immediate, or speed of execution affecting the results only through the probability of conviction parameter or the perceived sanction [8].

Finally, imprisonment may be an important component of the actual sentences for individuals of very low non-labor income or earnings, that cannot afford to pay the fine. In any case, crime analysis should include the wage rate as an important signal of the toughness of the penalty system. If real wages are pro-cyclical - and available loots are not -, crimes punished by imprisonment will most likely be counter-cyclical, that is, increase in troughs, decrease in booms of the economic activity; the (average) size of observed infractions - under a constant penalty system – would exhibit the opposite pattern. The same may not apply for crimes only punishable with fines. Also, the long-run trend of rising wages would explain why there has been, in developed economies, an increase in the importance of fines relative to imprisonment - a pattern already noted in Becker (1968): with higher wages, the latter would have become more deterrent; as more costly to society [9], it was subsequently lowered, both in length required for a given infraction, as in the range of infractions it covers.

For the sake of completeness, we also inspect the effects of wage and non-labor income dispersion on criminality. Uncertainty with respect to the arrest rate statistical distribution was contemplated in previous literature – Becker (1968), Block and Heineke (1975), and others. Volatility in endowments does not seem to have been related to the subject. The last two decades assisted to an increase – not only in the average wage level, which has followed a long-run trend, but particularly - in wage dispersion, even if controlling for individual characteristics as skill or age

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[7] Becker (1968), Stigler (1970).

[8] Davis (1988) analyses the effects of delay in punishment in an intertemporal context.

[9] Even if enforcement and collection of fines may also be costly and interrelated with the societies ability to exert imprisonment, as noted by Block and Lind (1975).

[10]; it is thus a relevant question to inquire if such environment is more prone to crime or not. The inclusion of uncertainty in labor supply models led in some cases to the inspection of cross or substitution effects in utility functions, as in Block and Heineke (1972, 1973 and 1975) or Tressler and Menezes (1980). We are to find the same type of effects, also analysed in the theory of multi-attribute uncertainty theory [11] cited above, and analogous multi-period applications – that date back to Leland (1968), Epstein (1975). Uncertainty in the penalty - in the degree of enforcement in case of arrest -, is also modelled; this allows us to answer if and when a discretionary judicial system is crime–deterrent.

Overall, the main thrust of the paper is to position crime in an empirically tractable problem within economic – labor economics – methodology. Research on the subject has previously been conducted, but most is attributed to criminologists – as Freeman´s (1999) survey concludes. Even if "crime is closely related to poverty, social exclusion and other economic problems" [12], offenders are looked at as irrational individuals, whose deviant behavior does not show conformity with economic assumptions. Empirical proof that this is the (general) case is definitely lacking.

The paper is organized as follows: Notation and the general properties of the standard labor supply functions required for the analysis are summarized in section I. In section II, the criminal´s supply and demand functions are derived. Analysis of the crime-deterrent composite and its relation with the individual´s non-labor income, wage rate and preferences (e.g., attitude towards risk) are inspected in section III. Section IV illustrates how crime opportunity affects the marginal trespasser labor supply. Section V deals with the effect of uncertainty in the endowment distribution on the critical penance scheme. Some analytical examples are forwarded in section VI, involving the specification of utility functions. In section VII, applications of the principles involved in the analysis of individual´s willingness to participate in hazardous professions and in the study of health insurance are briefly outlined. Section VIII summarizes some Portuguese evidence on criminal and judicial activities. The exposition ends with a summary of the conclusions and main results in section IX.

I. Notation: Standard Consumer-Worker.

1. Let us write the worker's problem in standard notation [13]:

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[10] See a thorough survey in Katz and Autor (1999).

[11] A survey can be found in Karni and Schmeidler (1991).

(I.1) $$\max_{Y,O} \; U(Y,O) \quad \text{s.t.: } pY = WH + V \;;\quad T = O + H$$

where Y denotes income, O is leisure, H hours of work supplied; W refers to the hourly wage, p a price index, V is exogenous non-labor income and T denotes time endowment. U is the well-behaved quasi-concave utility function. If F denotes full-income,

(I.2) $$F = WT + V$$

the two restrictions can collapse into:

(I.3) $$\max_{Y,O} \; U(Y,O) \quad \text{s.t.: } pY + WO = F$$

The solution of the problem yields uncompensated demands for leisure and income, which can be defined without direct reference to (I.1) as "full-income" demands [14]:

(I.4) $$O = O^F(p, W, F)\;;\quad \frac{\partial O^F}{\partial T} = 0\;;\quad \frac{\partial O^F}{\partial F} > 0\;;\quad \frac{\partial O^F}{\partial W} < 0$$ [15]

$$Y = Y^F(p, W, F)\;;\quad \frac{\partial Y^F}{\partial T} = 0\;;\quad \frac{\partial Y^F}{\partial F} > 0\;;\quad \frac{\partial Y^F}{\partial p} < 0$$

These functions have the same properties as standard consumer demands. We can consider H^F, full-income labor supply, defined as:

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[12] Freeman (1999), p. 3532.

[13] The response to changes to the time endowment can be found in Martins (2002), implicit from standard analysis of the consumer-worker problem.

[14] See, for example, Deaton and Muellbauer (1980), p. 8993.

(I.5) $$H = H^F(p, W, F, T) = T - O^F(p, W, F)$$

$$\frac{\partial H^F}{\partial T} = 1\ ;\ \frac{\partial H^F}{\partial F} = -\ \frac{\partial O^F}{\partial F} < 0\ ;\ \frac{\partial H^F}{\partial W} = -\ \frac{\partial O^F}{\partial W} > 0$$

Replacing (I.2) in (I.4), we get the demands for leisure and income and labor supply:

(I.6) $$O = O^F[p, W, (W\,T + V)] = O(p, W, V, T)$$
$$Y = Y^F[p, W, (W\,T + V)] = Y(p, W, V, T)$$

(I.7) $$H = H^F[p, W, (W\,T + V), T] = T - O^F[p, W, (W\,T + V)]$$

where

(I.8) $$H = H(p, W, V, T) = T - O(p, W, V, T)$$

We can immediately see that:

(I.9) $$\frac{\partial J}{\partial V} = \frac{\partial J^F}{\partial F},\quad J = O, Y, H$$
$$\frac{\partial J}{\partial T} = W\,\frac{\partial J^F}{\partial F},\quad J = O, Y$$
$$\frac{\partial J}{\partial W} = \frac{\partial J^F}{\partial W} + T\,\frac{\partial J^F}{\partial F},\quad J = O, Y, H$$
$$\frac{\partial H}{\partial T} = 1 - W\,\frac{\partial O^F}{\partial F} = 1 + W\,\frac{\partial H^F}{\partial F} = 1 + W\,\frac{\partial H}{\partial V}$$

And

(I.10) $$0 < \frac{\partial H}{\partial T} < 1\ ;\ \frac{\partial H}{\partial V} < 0$$

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[15] From Slutsky decomposition, $\partial O^F/\partial W < 0$ (and $\partial H^F/\partial W > 0$) if (but not only if) leisure is normal, i.e., income effect on leisure is positive: $\partial O^F/\partial V > 0$. Even if not required by first nor second order conditions, it is usually assumed. In any case, either leisure or income must not be an inferior good.

As is well -known, the response of H to W can be either positive or negative. Indirect "full income" utility function

$$\text{(I.11)} \qquad v^F(p, W, F) = U[Y^F(p, W, F), O^F(p, W, F)]$$

$$\frac{\partial v^F}{\partial T} = 0\,; \quad \frac{\partial v^F}{\partial F} > 0\,; \quad \frac{\partial v^F}{\partial p} < 0; \quad \frac{\partial v^F}{\partial W} < 0$$

It only depends on T indirectly through F. The standard indirect utility function:

$$\text{(I.12)} \quad v(p, W, T, V) = v^F[p, W, W\,T + V] = U[Y(p, W, V, T), O(p, W, V, T)]$$

$$\frac{\partial v}{\partial T} = W\,\frac{\partial v^F}{\partial F} = W\,\frac{\partial v}{\partial V} > 0\,; \quad \frac{\partial v}{\partial V} > 0\,; \quad \frac{\partial v}{\partial p} < 0;$$

It will be the case that:

$$\text{(I.13)} \qquad \frac{\partial v(p, W, V, T)}{\partial W} = \frac{\partial v^F(p, W, F)}{\partial W} + T\,\frac{\partial v^F(p, W, F)}{\partial F} > 0$$

Roy´s identity with respect to leisure, can be stated as:

$$\text{(I.14)} \qquad O^F(p, W, F) = -\,\frac{v^F{}_W(p, W, F)}{v^F{}_F(p, W, F)}$$

and with respect to labor supply as:

$$\text{(I.15)} \quad H(p, W, V, T) = \frac{v_W(p, W, V, T)}{v_V(p, W, V, T)} = T + \left.\frac{v^F{}_W(p, W, F)}{v^F{}_F(p, W, F)}\right|_{F = V + WT}$$

Remark: 1. Responses indexed to "full income" are, in fact, indistinguishable from responses to non-labor earnings, not to earnings of individuals.
2. Consumption responses to changes in the time endowment, T, will have the same sign as responses to non-labor earnings; they will equate the former multiplied by the wage rate. Labor supply will react to time availability and non-labor earnings in the opposite direction.
3. The responses cited above are not directly related to effects of price or wage changes – these involve substitution and income effects. Only the latter shows

up in cases invoked in 1. and 2.

2. The reservation wage - the wage below which people will not participate in the labor force - may be defined either from:

$$(I.16) \qquad \left.\frac{\frac{\partial U}{\partial O}}{\frac{\partial U}{\partial Y}}\right|_{O = T,\, p\, Y = V} = \frac{W_R}{p}$$

or W_R that solves

$$(I.17) \qquad H = H(p, W_R, V, T) = T - O(p, W_R, V, T) = 0$$

Hence,

$$(I.18) \qquad W_R = W_R(p, V, T) \;\; ; \;\; \frac{\partial W_R}{\partial T} < 0 \; ; \; \frac{\partial W_R}{\partial V} > 0$$

As long as income is not an inferior good, and labor supply is positively sloped around the reservation wage, the reservation wage decreases with the time endowment.

II. Criminal Behavior.

1. Assume the following scenario: an individual commits a felony which yields, without any loss of time, a reward R. He faces two possibilities: with probability q, he is arrested: he has to return R, pay a fine of size M, and goes to prison for time P. With probability (1 -q), he evades punishment and enjoys R.

It is clear that if he is not caught, he solves problem (I.3) with (I.2) replaced by

$$(II.1) \qquad F = F^1 = W\,T + V + R$$

His labor supply will be:

$$(II.2) \qquad H^F(p, W, F + R, T) = H(p, W, V + R, T)$$

It will be smaller than if he did not commit the crime, once $\frac{\partial H}{\partial R} = \frac{\partial H}{\partial V} = \frac{\partial H^F}{\partial F} < 0$. The reservation wage function relates to (I.18) according to:

(II.3) $$W_R = W_R^1 = W_R(p, V + R, T)$$

and $\frac{\partial W_R}{\partial R} = \frac{\partial W_R}{\partial V} > 0$.

If the crime is detected, the worker is confronted with problem (I.3) with

(II.4) $$F = F^2 = W(T - P) + V - M$$

Labor supply will be described by:

(II.5) $$H^F[p, W, W(T - P) + V - M, T - P] = H(p, W, V - M, T - P)$$

$\frac{\partial H}{\partial P} = -1 - W \frac{\partial H^F}{\partial F} = - \frac{\partial H}{\partial T} < 0$: at M = 0, the convict will exhibit lower labor supply than a good citizen. However, $\frac{\partial H}{\partial P} > -1$ (provided income is not an inferior good): (even at M = 0) the difference in labor supply of a complying citizen relative to the one of a convict is less than the duration of the prison sentence.

(II.6) $$W_R = W_R^2 = W_R(p, V - M, T - P)$$

And $\frac{\partial W_R}{\partial P} = - \frac{\partial W_R}{\partial T} > 0$. A rise in imprisonment increases the reservation wage, decreasing participation.

The effect of M on labor supply is opposite to the one of F or V: $\frac{\partial H}{\partial M} = - \frac{\partial H^F}{\partial F} > 0$. $\frac{\partial W_R}{\partial M} = - \frac{\partial W_R}{\partial V} < 0$. An increase in M increases labor supply and participation.

Hence, if (but not only if) M is small, a convict may have a smaller labor supply than a complying citizen.

2. The expected labor supply of a criminal is, thus,

(II.7) $(1\text{-}q)\, H^F(p, W, F^1, T) + q\, H^F(p, W, F^2, T - P) =$

$= (1\text{-}q)\, H(p, W, V + R, T) + q\, H(p, W, V - M, T\text{-} P)$

iff $W > \text{Max}[\, W_R(p, V + R, T),\; W_R(p, V - M, T - P)]$

(II.8) $H = 0$ iff $W < \text{Min}[\, W_R(p, V + R, T), W_R(p, V - M, T - P)]$.

If $\text{Min}[W_R(p, V + R, T),\; W_R(p, V - M, T - P)] < W <$

$< \text{Max}[\, W_R(p, V + R, T),\; W_R(p, V - M, T - P)]$, either:

$\text{Min}[W_R(p, V + R, T), W_R(p, V - M, T - P)] = W_R(p, V - M, T - P)$ and

$H = q\, H(p, W, V - M, T - P)$. This is likely to occur if R is large, P is small and M is large. Then, labor supply has a kink at the reservation wage of the successful trespasser.

Or

$\text{Min}[W_R(p, V + R, T), W_R(p, V - M, T - P)] = W_R(p, V + R, T)$ and

$H = (1\text{-}q)\, H(p, W, V + R, T)$.

Proposition 1: Expected labor supply of criminals

1. at the same wage, time endowment and exogenous income, will be smaller than for a non-offender, provided the fine is small relative to imprisonment.

2. will have a kink at the reservation wag e of a convicted criminal if the money fine is small relative to imprisonment and the loot is small. Otherwise, it will have a kink at the reservation wage of an escapee.

3. will respond positively to the fine, negatively to the size of the theft, negatively to time in prison.

4. will respond positively to the probability of detection if labor supply of a convict is larger than that of an escapee. The larger R, the smaller P and the larger M, the more likely this is to occur. It will necessarily if $P = 0$.

Other properties of the labor supply function will prevail: the effect of an increase in V is negative, the effect of a change in W is ambiguous.

III. Crime Deterrence.

1. The expected utility of an individual that engages in crime will be:

$$\text{(III.1)} \qquad (1-q)\, v^F(p, W, F^1) + q\, v^F(p, W, F^2) = $$
$$= (1-q)\, v^F(p, W, W\,T + V + R) + q\, v^F[p, W, W\,(T - P) + V - M]$$

It is non-decreasing in F^1 and F^2, non-increasing in p, and, for fixed F^1 and F^2, in W. However, the standard indirect utility of the worker´s problem, v(p, W, V, T), is non-decreasing in W. Hence, the indirect utility function of the criminal will be increasing (non-decreasing) in W, V, T and R, decreasing (non-increasing) in q, P and M.

Crime deterrence will be achieved iff P and M are set, for given R, such that:

$$\text{(III.2)} \qquad (1-q)\, v^F(p, W, F^1) + q\, v^F(p, W, F^2) < v^F(p, W, F)$$

or

$$\text{(III.3)} \qquad (1-q)\, v^F(p, W, W\,T + V + R) + q\, v^F[p, W, W\,(T - P) + V - M] <$$
$$< v^F(p, W, W\,T + V)$$

Define, thus, the minimal crime deterrent relation between q, P and M as arising from:

$$\text{(III.4)} \qquad (1-q^*)\, v^F(p, W, F^1) + q^*\, v^F(p, W, F^{2*}) = v^F(p, W, F)$$

or

$$\text{(III.5)} \qquad (1-q^*)\, v^F(p, W, W\,T + V + R) + q^*\, v^F[p, W, W\,(T - P^*) + V - M^*] =$$
$$= v^F(p, W, W\,T + V)$$

2. For an individual who is risk averse with respect to F - i.e., v^F is concave in F or $v^F_{FF}(p, W, F) < 0$ -, as F is (always) between F^{2*} and F^1, (III.4) requires

$$\text{(III.6)} \qquad (1-q)\ F^1 + q\, F^{2*} > F$$

that is, to deter risk averse individuals, the expected value of punishment can be smaller than the expected value of the loot:

$$\text{(III.7)} \qquad (1-q^*)\, R > q^*\, (W\,P^* + M^*) \quad \text{or}$$

$$\text{(III.8)} \qquad P^* < \frac{(1 - q^*)\, R - q^* M^*}{q^* W}$$

This is a necessary condition for a deterrence scheme aimed at risk-averse individuals. The inequality signs in (III.6)-(III.8) would be reversed for risk lovers – that is, if $v^F(p, W, F)$ is convex in F, that is, $v^F_{FF}(p, W, F) > 0$.

From (III.5) we can derive a general deterrent relation:

$$\text{(III.9)} \qquad G(P^*, R, M^*, q^*, V, W, T) = 0$$

That is, we can talk about an efficient penalty aggregate M* + W P* = M(R, V, W, T, q*), which is just derived from function G(.) = 0. If with respect to fines, it might be unimportant to study the relations – we may just choose the highest M*, for the most likely criminal and eventually for the highest loot (again, assuming that it is not interpersonal justice that is at stake but adequate prevention) -, imprisonment has large implementation costs[16] (Moreover, fines can be used to compensate victims for other inflicted damages and pay for judicial procedures; eventual prison labor surpluses do not even pay for jail maintenance). As such, the study of the adequate profile of the minimal imprisonment requirements to insure safety may be of large relevance.

3. For rational and general preferences, adequate punishment or detection probability can deter crime, and, provided that hitting the lower bound of the utility function is not required for deterrence[17], - that is, in "interior deterrent schemes", with sufficiently high detention rates - there will be a one-to-one trade-off between the several crime deterrent parameters:

$$\text{(III.10)} \qquad \frac{\partial M^*}{\partial R} = \frac{(1 - q^*)\ v^F{}_F(p, W, F^1)}{q^*\ v^F{}_F(p, W, F^{2}*)} > 0$$

$$\text{(III.11)} \qquad \frac{\partial M^*}{\partial P^*} = -W < 0$$

[16] Yet, see footnote 7.

[17] Block and Lind (1975) show that, assuming boundedness from below of the utility function, there is no sanction that can prevent any crime, but there will always be a probability of punishment that will deter any crime.

$$\text{(III.12)} \qquad \frac{\partial M^*}{\partial q^*} = - \frac{v^F(p, W, F^1) - v^F(p, W, F^2*)}{q^* v^F{}_F(p, W, F^2*)} < 0$$

If individuals are risk averse, $v^F(p, W, F)$ is concave in F and $v^F{}_F(p, W, F^1) < v^F{}_F(p, W, F^2*)$. Then:

$$\text{(III.13)} \qquad \frac{\partial M^*}{\partial R} < \frac{1 - q^*}{q^*}$$

(III.13) reproduces a consequence of standard statements in the analysis of risk-aversion: if an individual is risk-averse, the increase in the required fine per unit increase in the output of theft is smaller than the inverse of the odds ratio. The reverse will be true for risk-lovers. Notice that, provided R > 0, M* + WP* > 0 always, once expected utility decreases in M* and P* regardless of risk-aversion: in general, there will be a positive composite-deterrent scheme – as implied by (III.10) – (III.12).

We can also derive that,

$$\text{(III.14)} \qquad \frac{\partial P^*}{\partial J} = \frac{\frac{\partial M^*}{\partial J}}{W}, \quad J = R, W, T, V, q^*$$

and

$$\text{(III.15)} \qquad \text{sign of } \frac{\partial q^*}{\partial J} = \text{sign } \frac{\partial M^*}{\partial J} = \text{sign } \frac{\partial P^*}{\partial J}, \quad J = R, W, T, V$$

Equations (III.14) -(III.15) state the equivalent sign response of any of the deterrent parameters to changes in the individual´s endowments.

Additionally, (III.14) – and (III.11) - states that the impact on the required P* over the equivalent impact on the, alternatively, required M* of any change in an exogenous parameter is inversely related to the wage rate. They state an equivalence amplified by the wage rate in the treatment of seclusion time and fines in what concerns consumer-worker choices.

Proposition 2: Whatever consumer preferences are, for any given individual:

1. there will always be some trade-off between the parameters of the minimal crime deterrent scheme. That is, if R increases, there must be an increase in q*, P* or M* to

guarantee deterrence. In "interior deterrent schemes", at given R, if either q*, P* or M* increases, one or more of the others will decrease.

2. Unilateral responses of the minimal crime deterrent parameters to changes in exogenous endowments are equivalent in sign (in interior solutions).

3. The response of the minimal required fine to any other parameter or exogenous change is always equal to the wage rate times the analogous reaction of the minimal required seclusion.

Point 2. of Proposition 2 tells us that it is indifferent to inquire about sign effects of exogenous conditions on any of the judicial parameters, while keeping the others constant. We can therefore concentrate on one, which, for simplicity, will be M*.

Point 3. implies that the trade-off between the deterrent capacity of imprisonment and fines favours the first at higher wage rates.

4. A note can be added highlighting the trade-off between M* and q*: As is well known, decreases in F^2 are more crime deterrent for risk averse individuals, increases in q are more effective in deterring crime of risk lovers [18]. That was Becker´s (1968) conclusion [19].

A 1% decrease in q* and a $\frac{\partial M^*}{R+M^*}$ increase in M* of 1% at P* = 0 will leave the expected full income endowment of a criminal unchanged. That is, at a given expected "full income" of the marginal criminal – or of a risk neutral agent -, $-\frac{\partial M^*}{\partial q^*}\frac{q^*}{R+M^*} = 1$. We can thus compute under (III.4) or (III.5):

$$-\frac{\partial M^*}{\partial q^*}\frac{q^*}{R+M^*} = \frac{v^F(p, W, F^1) - v^F(p, W, F^2*)}{(R+M^*)\, v^F{}_F(p, W, F^2*)}$$

It will be smaller than one if fines are more effective than the probability of detection in crime deterrence: a 1% decrease in q* requires an increase in M* $\frac{\partial M^*}{R+M^*}$ smaller than 1%.

$$-\frac{\partial M^*}{\partial q^*}\frac{q^*}{R+M^*} < 1 \text{ iff } v^F(p, W, F^1) - v^F(p, W, F^2*) < v^F_F(p, W, F^2*)\,(R+M^*)$$

[18] See also Layard and Walters, Ex. 13-4.

[19] Also discussed in Brown and Reynolds (1973).

As $F^1 - F^{2*} = R + M^*$, that occurs if $v^F(p, W, F)$ is concave in F – in which case, also $v^F_F(p, W, F^{2*}) > v^F_F(p, W, F^1)$ - and individuals are risk averse. The opposite result would show up if individuals are risk lovers: raising apprehension probabilities would be more effective than to raise fines.

If P* > 0, the increase in M* that will leave the expected value of income unchanged when q* decreases by $\frac{\partial q^*}{q^*}$ will be $\frac{\partial M^*}{R + M^* + W P^*}$. Hence, the correct ratio to use will be

$$(III.16) \qquad -\frac{\partial M^*}{\partial q^*} \frac{q^*}{R + M^* + W P^*} = \frac{v^F(p, W, F^1) - v^F(p, W, F^{2*})}{(R + M^* + W P^*)\, v^F{}_F(p, W, F^{2*})}$$

Fines will be more effective than probability of detection if $v^F(p, W, F)$ is concave in F – individuals are risk-averse.

Analogous considerations can be made about the relation between $\frac{\partial q^*}{q^*}$ and $\frac{W\, \partial P^*}{R + M^* + W P^*}$ to infer about the relative efficiency between the apprehension probability and imprisonment. One can show that, under (III.4) or (III.5):

$$(III.17) \qquad -W \frac{\partial P^*}{\partial q^*} \frac{q^*}{R + M^* + W P^*} = -\frac{\partial M^*}{\partial q^*} \frac{q^*}{R + M^* + W P^*}$$

(III.17) equals (III.16). If individuals are risk averse, the expression is smaller than one and raising imprisonment length is more effective than increasing the arrest rate. The reverse occurs for risk lovers.

The probability of detection / sanctions trade-off was stated in Becker (1968) with respect to fines. We see that it is, in this framework, enlarged to imprisonment and stays qualitatively unchallenged in corner solutions of each penalty. Notice, however, that if M* and P* > 0, under the deterrent constraint,

$$(III.18) \qquad -\frac{\partial M^*}{\partial q^*} \frac{q^*}{M^*} > -\frac{\partial M^*}{\partial q^*} \frac{q^*}{R + M^*} > -\frac{\partial M^*}{\partial q^*} \frac{q^*}{R + M^* + W P^*}$$

$$(III.19) \qquad -\frac{\partial P^*}{\partial q^*} \frac{q^*}{P^*} > -W \frac{\partial P^*}{\partial q^*} \frac{q^*}{R + W P^*} > -W \frac{\partial P^*}{\partial q^*} \frac{q^*}{R + M^* + W P^*}$$

The left hand-side elasticities are not appropriate for our framework, nor the middle ones: all these will favor sanctions less often than detection than the "correct" comparisons. This sheds some light on the sanction-probability of apprehension controversy and why empirically, raising the probability of apprehension seems more effective than raising sanctions (by the same percentage) [20].

In Becker, as in other literature, the criminal is assumed to detain the output of the crime in case of detection – in real life examples one or the other may be more appropriate. In such circumstances, we replace - M by - $M^{\#}$ + R in our framework, implying that, other things constant, $M^{\#*} = M^* + R$: "the punishment would include confiscation of the gain from committing the crime" [21]; F^{2*} would stay unaltered. In general, for any parameter J, inspected above or below, other than R, $\frac{\partial M^{\#*}}{\partial J} = \frac{\partial M^*}{\partial J}$; $\frac{\partial M^{\#*}}{\partial R} = \frac{\partial M^*}{\partial R} + 1 > 0$ and the trade-off possibility between the penalty parameters remains. When analyzing efficiency, we would end up with (III.16) and (III.17), hence, with same right hand-side of the last expressions, but replacing R + M* by $M^{\#*}$ - equivalent (for P* = 0) to the concept used in Becker.

It is not adequate to use the same type of requirement to analyze the relative efficiency of fines versus imprisonment, once no relation to the features of expected utility is found; $-\frac{\frac{\partial M^*}{\partial P^*}}{W} = 1$ always: technically, their efficiency is the same. Rather, an interpretation based on (III.11) may be more enlightening.

Finally, the measurement of effectiveness may have a more adequate meaning in a context where implementation costs are present – that is, however, outside the scope of this research.

5. If $v^F(p, W, F)$ is linear in F – say, $v^F(p, W, F) = K(p, W) F + M(p, W)$ -, then:

$$\text{(III.20)} \qquad P^* = \frac{(1 - q^*) R - q^* M^*}{q^* W} = \left(\frac{1}{q^*} - 1\right) \frac{R}{W} - \frac{M^*}{W}$$

Proposition 3: If the "full income" indirect utility function is linear in F, then

[20] See Brown and Reynolds (1973), Block and Lind (1975a), Pyle (1983). Another reason – see Block and Lind (1975) – is that for some felonies the loot is so high that the range of sanctions being applied cannot guarantee an "interior" deterrent scheme.

1. The crime-deterrent scheme equates the expected value of full income of the criminal to the full income of a non-offender - and (III.20) holds.
2. The equilibrating equation will be independent of non-labor earnings and of the time endowment (and other parameters of the utility function)
3. Other things fixed, the deterrent confinement decreases with the wage rate. Alternatively, all else constant, when confinement is also being applied, the required monetary penalty decreases with the wage rate.
4. An increase in W is compatible with an increase in R and a decrease in q*, the probability of detection if P* is strictly positive.

Point 4. implies that individuals of higher wages require a larger product of the crime or a smaller probability of detection for the crime to be worthwhile. Hence:

Corollary 1: If the "full income" indirect utility function is linear in F, consumer-workers have the same preferences (even if not the same non-labor income nor time endowment), and fines or imprisonment are only dependent on the size of the infraction, individuals of higher wages (or, when wages are higher, individuals) will:
1. less likely commit crimes punished by imprisonment
2. commit crimes punishable by imprisonment of higher money value or more difficult to detect
than individuals of lower wages (than when wages are low).

6. The effects derived from (III.20) arise in the presence of an analog to neutrality towards risk of a consumer. Firstly, it leads us to conclude that it is likely that on G(.), both $\frac{\partial M^*}{\partial W}$ and $\frac{\partial P^*}{\partial W} < 0$ in cases of risk-lovers as of risk-averse consumers for crimes punished with prison sentences. Secondly, notice that an increase in seclusion time, P*, will originate a "full income" loss variable with and directly proportional to the wage rate; hence, the direct deterrent effect of a given seclusion time is always larger for high wage earners; only if for these individuals $\frac{\partial v^F(p, W, F)}{\partial F}$ is much lower than for individuals of higher wages can the "total" deterrent effect of an increase in P be weaker for latter. Thirdly, in general, G(.) will not be independent of V nor T; we can compute:

[21] Pyle (1983), p. 101.

(III.21) $$\frac{\partial M^*}{\partial V} = \frac{(1-q^*)\, v^F{}_F(p, W, F^1) + q^*\, v^F{}_F(p, W, F^{2*}) - v^F{}_F(p, W, F)}{q^*\, v^F{}_F(p, W, F^{2*})}$$

=

$$= \frac{\frac{\partial M^*}{\partial T}}{W}$$

An increase in V is an exogenous increase in certain income. It would thus be reasonable to find that a positive $\frac{\partial M^*}{\partial V}$ will be associated to an individual´s utility function exhibiting decreasing risk aversion in full income or non-labor earnings – as his endowment increases, a larger fine is necessary to deter him from engaging in risk; if $\frac{\partial M^*}{\partial V} < 0$, the individual´s aversion to risk increases with V. It is straight-forward to show that if (but not only if) $v^F{}_F(p, W, F)$ is linear or concave in F – regardless of whether the individual is risk averse or risk lover -, $\frac{\partial M^*}{\partial V} < 0$. For $\frac{\partial M^*}{\partial V} > 0$, the marginal utility function $v^F{}_F(p, W, F)$ must be convex in F – but that may not be sufficient; by Jensen´s inequality, for any F close to $(1-q^*)\, F^1 + q^*\, F^{2*}$, $\frac{\partial M^*}{\partial V} > 0$ if v_F is convex (the expected value of the function is larger than the function of the expected value). That is, the sign of the relation depends on the sign of the third derivative of v^F with respect to the argument F – a result which is not uncommon in the uncertainty literature[22].

The Arrow-Pratt measure of absolute risk aversion, a measure of the concavity of a function v(.), $-\frac{v''(.)}{v'(.)}$, increases with the argument if but not only if the third derivative of the utility function is negative, i.e. if $v^F{}_F(p, W, F)$ is linear or concave in F. The standard textbook illustration of how that measure works usually proves or implies that, around R = 0, the trade-off between (M* + W P*) and R that equalizes the expected utility to the certain case increases with R symmetrically and approximately proportionally to that measure, $\rho^F(p, W, F)$:

(III.22) $$\rho^F(p, W, F) = -\frac{v^F{}_{FF}(p, W, F)}{v^F{}_F(p, W, F)}$$

[22] See Rothenberg and Smith (1971) for an early remark on the subject.

$M^* + W P^*$ varies with F in the opposite direction of $\rho(p, W, F)$. For small loots, the sign of $\frac{\partial M^*}{\partial V}$ will be symmetric to the one of $\frac{\partial \rho^F (p, W, F)}{\partial F}$. If risk aversion increases with F, a smaller fine is necessary to deter a given crime as the individual´s full income base rises. [23]

In (III.22) we stated the equivalent sign response of M* to V and T. Even if more adequately studied in a lifecycle model, we can visualize aging in a decreasing ly available time endowment [24]. Then, decreasing risk aversion with respect to F would imply $\frac{\partial M^*}{\partial T} > 0$: criminality would increase with T, hence, decrease with individuals´ age. (Yet, it would increase with life expectancy). The reverse would occur if $\rho^F(p, W, F)$ increases with full income.

The effect of changes in W relate to $v_W(p, W, V + R, T)$ in an analogous way to as V relates to $v^F_F(p, W, F)$:

$$\text{(III.23)} \qquad \frac{\partial M^*}{\partial W} = \\ = \frac{(1 - q^*)\, v_w(p, W, V + R, T) + q^* v_w(p, W, V - M^*, T - P^*) - v_w(p, W, V, T)}{q^* v^F{}_F(p, W, F^2{}^*)}$$

We can decompose, the effect under $v^F(p, W, F)$, and deduct that:

$$\text{(III.24)} \frac{\partial M^*}{\partial W} = T \frac{(1 - q^*)\; v^F{}_F(p, W, F^1) + q^* v^F{}_F(p, W, F^2{}^*) \frac{T - P}{T} - v^F{}_F(p, W, F)}{q^* v^F{}_F(p, W, F^2{}^*)} + \\ + \frac{(1 - q^*)\, v^F{}_w(p, W, F^1) + q^* v^F{}_w(p, W, F^2{}^*) - v^F{}_w(p, W, F)}{q^* v^F{}_F(p, W, F^2{}^*)} = \\ = T \frac{\partial M^*}{\partial V} - P^* + \frac{(1 - q^*)\; v^F{}_w(p, W, F^1) + q^* v^F{}_w(p, W, F^2{}^*) - v^F{}_w(p, W, F)}{q^* v^F{}_F(p, W, F^2{}^*)}$$

Two considerations can be forwarded:

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[23] See Polinsky and Shavell (1979), p. 888. They note that same mechanism, in a different model, with respect to wealth.

[24] The static model is more appropriate to investigate an overall lifetime or a finite horizon decision plan. Obviously, the implied behavior when we interpret T as lifetime left to live and admit decisions on H and Y to be continually updated, leads to time inconsistent results. We are just suggesting a simplistic analogic explanation of forward-looking agents´ responses.

Even if the full income effect is positive with respect to F, we now have an extra negative effect coming from the fact that an increase in W induces an extra negative effect in F^{2*}, contained in the term $-$ P*. The larger the length of seclusion, the more likely is $\frac{\partial M^*}{\partial W}$ to be negative [25].

On the other hand, $v^F_w(p, W, F^1)$ is non-increasing in W, i.e., $v^F_w(p, W, F^1) < 0$, but it can move in either direction with F. For an F around $(1-q^*)\, F^1 + q^*\, F^{2*}$, the expression suggests that if v^F_w is concave in F, the last term will be negative; if convex, it will be positive [26].

Uncertainty theory recognizes that individuals may react more negatively to increases in income uncertainty than to price uncertainty [27]. Technically, this derives from the fact that indirect utility functions of format $v^F(p, W, F)$ are required to be quasi-convex in (p, W) – which, at least, rules out concavity in (p, W) that would be required for risk-aversion in terms of prices [28]. However, that is not the effect in question here: the issue is not of how v^F_w varies with W, but how it varies with F. If risk aversion behaves with v^F_w as it behaves with v^F, plain risk aversion in v with respect to F, i.e., concavity of v in F, is sufficient to guarantee that the last term is negative around any F close to $(1-q^*)\, F^1 + q^*\, F^{2*}$.

By analogy with the remarks put forward with respect to the measure of risk aversion and $\frac{\partial M^*}{\partial V}$, we can hypothesise that the sign of $\frac{(1-q^*)\, v^F{}_w(p, W, F^1) + q^*\, v^F{}_w(p, W, F^{2*}) - v^F{}_w(p, W, F)}{q^*\, v^F{}_F(p, W, F^{2*})}$ will be (also symmetrically) related to how a measure of the rate of decline of v^F_w with F (or v^F_F with W)

$$\text{(III.25)} \qquad \kappa^F(p, W, F) = -\frac{v^F{}_{wF}(p, W, F)}{v^F{}_F(p, W, F)}$$

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[25] Another explanation for higher delinquency among youth would be provided by the fact that wages increase with age. Nevertheless, the comments of footnote 24 apply to this conclusion.

[26] The term reminds Roy´s identity. It can be close to certain labor supply, minus expected labor supply of the marginal offender, minus q* P* for R close to 0 or $v^F_F(p, W, F)$ roughly invariant with F. See (IV.8) and (IV.9) below.

[27] See Hirshleifer and Riley (1992), p. 41.

[28] Nevertheless, any monotonic function of a single variable is both quasi-concave and quasi-convex - see Chiang (1974), p. 732. Single-variable concavity of $v^F(p, W, F)$ with respect to W, i.e., $v^F_{ww} < 0$ may arise and be compatible with quasi-convexity of $v^F(p, W, F)$ in (p, W).

responds to F. It is straight-forward to show that:

$$\text{(III.26)} \qquad \frac{\partial \rho^{F}(p, W, F)}{\partial W} = \frac{\partial \kappa^{F}(p, W, F)}{\partial F} =$$

$$= - \frac{v^{F}{}_{wFF}(p, W, F)\, v^{F}{}_{F}(p, W, F) - v^{F}{}_{wF}(p, W, F)\, v^{F}{}_{FF}(p, W, F)}{v^{F}{}_{F}(p, W, F)^{2}}$$

and in the sense of Keeney (1973), $\frac{\partial \rho^{F}(p, W, F)}{\partial W}$ would be related to the notion of conditional risk dependence or utility dependence between F and W – defined as 0 in case of independence. (The risk premium equivalent (M) is here defined in the full income metric; that conditions the denominator of the risk aversion measures used above. And the risk at stake, that determines the pattern of the numerator, is also in "full income". There will be complementary definitions for risks in wages and prices but they may not be equivalent to these ones, if they apply to risk premiums defined in the wages or price metric. For instance, the correct Arrow-Pratt measure of aversion to a risk in price, for a risk premium defined in the "full income metric", should be $- \frac{v^{F}{}_{pp}(p, W, F)}{v^{F}{}_{F}(p, W, F)}$. See section V. B. and C. below.)

$\kappa^F(p, W, F) < 0$ if there is some sort of wage-full income complementarity in the indirect utility function and the marginal utility derived from one of them increases with the level at which the other is. A mathematical interpretation of $\kappa^F(p, W, F)$ is provided in the Martins (2002). We found reference to this type of measures in Duncan (1977). The features of $\frac{\partial \kappa^{F}(p, W, F)}{\partial F}$ in (III.26), can be found in Diamond and Stiglitz´s (1974) Theorem 2, conditioning the sign of a mean utility preserving increase in risk on the optimal level of a control variable.

$\kappa^F(p, W, F)$ will be independent of F if we can write the utility function as $v^F(p, W, F) = G(p, W)\, Q(F, p) + S(p, W)$. It will be for the case for homogeneity of the direct utility function in (O, Y) – if α is the degree of homogeneity, once in these cases the expenditure function can be written as $F = u^{1/\alpha} N(p, W)$, $v^F(p, W, F) = F^{\alpha} N(p, W)^{-\alpha}$. For homogeneous preferences, labor supply will be linear in F; $\rho^F(p, W, F) = \frac{1-\alpha}{F}$ and (directly) independent of W [29].

[29] See Deschamps´ (1973) Proposition 1 and 2 and Hanoch´s (1977) Corollary 3 for requirements of utility, expenditure and standard consumer demand functions when relative risk aversion with respect to income is constant.

If $v^F(p, W, F) = Q(F, p) + S(p, W)$, $v^F_{wF}(p, W, F) = 0$ and $\kappa^F(p, W, F) = 0$.

It is easy to visualise graphically that if for risk-averse individuals (in F), v^F_w increases with F and for risk lovers decreases with F, convexity of v^F_w in F is sufficient for the last term of (III.24) to be positive; in those cases, necessarily $\frac{\partial \kappa^F(p, W, F)}{\partial F} < 0$. On the other hand, if for risk-averse individuals, v^F_w decreases with F and for risk lovers increases with F, concavity of v^F_w in F is sufficient for the term to be negative – and also for $\frac{\partial \kappa^F(p, W, F)}{\partial F} > 0$. By analogy with the standard treatment of the insurance premium, it is, thus, reasonable to assume that the sign will be opposite to the one of $\frac{\partial \kappa^F(p, W, F)}{\partial F}$.

Finally, increasing risk aversion of $v^F(p, W, F)$ with F is necessary and sufficient to guarantee that $\frac{\partial M^*}{\partial V} < 0$. In the presence of a positive seclusion time, if $v^F(p, W, F)$ is additively separable in the partition [(p, W), F] in such a way that $v^F(p, W, F) = G(p, W)\, Q(F) + S(p, W)$, it is sufficient but not necessary to ensure that $\frac{\partial M^*}{\partial W} < 0$.

Proposition 4:

1. Criminality increases with non-labor earnings if individual preferences or indirect utility functions exhibit decreasing (absolute) risk aversion with respect to F. Under those preferences, it would decrease with the individuals´ age.
2. If the "full income" indirect utility function is additively separable in the partition [(p, W), F], such that $v^F(p, W, F) = G(p, W)\, Q(F) + S(p, W)$, even if individuals´ preferences exhibit decreasing risk aversion, criminality may decrease with the wage rate (it will for that type of utility functions if risk aversion increases with "full income").
3. If preferences exhibit increasing risk aversion, criminality will decrease with the wage if the Arrow-Pratt (conditional) measure of risk aversion in "full income" increases with the wage rate - or the measure of "full income"-wage substitutability increases with income. However, that is not a necessary condition.

Going back to (III.23), we can advance the following: around $P^* = 0$, the sign of $\frac{\partial M^*}{\partial W}$ will be symmetric to the one of the derivative with respect to V of the measure:

(III.27) $\kappa(p, W, V, T) = -\dfrac{v_{wV}(p, W, V, T)}{v_V(p, W, V, T)}$

We can decompose:

(III.28) $\kappa(p, W, V, T) = \kappa^F(p, W, W\,T + V) + T\ \rho^F(p, W, W\,T + V)$

Obviously,

(III.29) $$\frac{\partial \kappa\,(p, W, V, T)}{\partial V} = \frac{\partial \rho^{\,F}(p, W, W\,T + V)}{\partial W} =$$
$$= \frac{\partial \rho^{\,F}(p, W, F)}{\partial W}\Bigg|_{F = W\,T + V} + T\,\frac{\partial \rho^{\,F}(p, W, F)}{\partial F}\Bigg|_{F = W\,T + V}$$

Decreasing risk aversion with respect to full income - that is, $\dfrac{\partial \rho^{\,F}(p, W, F)}{\partial F} < 0$ - renders $\dfrac{\partial \kappa\,(p, W, V, T)}{\partial V} < 0$ more likely.

Nevertheless, if $P^* > 0$ and sizeable, $\kappa(p, W, V, T)$ can decrease with V and still $\dfrac{\partial M^*}{\partial W}$ be negative. In any case, if $\kappa(p, W, V, T)$ increases with V, $\dfrac{\partial M^*}{\partial W} < 0$ necessarily.

7. It is worth noting a relation of the measures above with the labor supply response. Using Roy´s identity one can show that:

(III.30) $$\frac{\partial H^F}{\partial F} = \frac{\partial H}{\partial V} = -\,[\rho^F(p, W, F)\,(T - H) + \kappa^F(p, W, F)] < 0$$

(III.31) $$\frac{\partial H}{\partial W} = H\,[\,\kappa^V(p, W, V, T) - \phi^w(p, W, V, T)] =$$
$$= -\,[\kappa^F(p, W, F) + T\ \rho^F(p, W, F) - \phi^F(p, W, F)\,]\,(T - H) - T\,\kappa^F(p, W, F)$$
$$= (T - H)\,[\phi^F(p, W, F) - T\,\rho^F(p, W, F)] - (2\,T - H)\,\kappa^F(p, W, F)$$

where $\phi^F(p, W, F) = - \frac{v^F_{ww}(p, W, F)}{v^F_w(p, W, F)}$ and $\phi^W(p, W, V, T) = - \frac{v_{ww}(p, W, V, T)}{v_w(p, W, V, T)}$.

$\frac{\partial H}{\partial W}$ will be positive if substitutability outweighs risk aversion with respect to the wage – or in symmetric terms, preference for risk in wages outweighs complementarity.

Hence the Arrow-Pratt measure of risk aversion and the proposed version of risk substitutability in the utility function can be related to the effect of wage and income on labor supply. The former is related to the "returns" to scale of the direct utility function with respect to (O,Y) [30]; the second one will eventually be related with the degree of substitutability between O and Y.

The effect of F or V and W on these measures will, thus, be more directly related to de concavity of H with respect to each of the arguments [31].

IV. Labor Supply.

1. Considerations about labor supply in section II apply for labor supply of criminals but were derived without the requirement that he is an optimizer.

Expected utility increases with R and decreases with q. Obviously, if R is large enough (or q small enough, provided $H^F(p, W, F^2, T - P) > H^F(p, W, F^1, T)$), the more likely is the individual to commit the crime – hence, as labor supply decreases with R, there will always be a larger chance of an individual that commits the crime to lower his expected labor supply.

Also, if M=0, an individual that commits the crime will necessarily have a lower expected supply than if he did not – both R and P will insure such result. In such cases or if M is small, necessarily:

(IV.1) $\quad (1\text{-}q)\, v^F(p, W, F^1) + q\, v^F(p, W, F^2) > v^F(p, W, F)$

implies

[30] In production theory, decreasing (increasing) returns to scale in the production function with respect to inputs imply convexity (concavity) of cost-functions with respect to output. Indirect utility functions are inverse expenditure functions that have towards utility analogous properties cost functions have towards output.

[31] See Carroll and Kimball (1996) for a recent research on the subject but with respect to standard consumption functions.

(IV.2) $$(1-q)\, H^F(p, W, F^1, T) + q\, H^F(p, W, F^2, T - P) =$$
$$= (1-q)\, H(p, W, V+R, T) + q\, H(p, W, V-M, T-P) < H(p, W, V, T)$$

or, in terms of demand for leisure:

(IV.3) $$(1-q)\, O^F(p, W, F^1) + q\, O^F(p, W, F^2) + q\, P > O^F(p, W, F)$$

But for the (near) "marginal criminal", if $M > 0$ or sizeable, that is not necessarily the case and, we can inquire whether an individual that committed the crime has forcefully, a smaller labor supply than if he did not.

2. Let us study the most unfavourable situation for the hypothesis of a lower expected labor supply of a criminal relative to a non-offender: the marginal criminal.

At the deterrent basket:

(IV.4) $$(1-q^*)\, U[O^F(p, W, F^1), Y^F(p, W, F^1)] + q^*\, U[O^F(p,W, F^{2*}), Y^F(p, W, F^{2*})]$$
$$= U[O^F(p, W, F), Y^F(p, W, F)]$$

If U is concave in (O,Y), then, either

(IV.5) $$(1-q^*)\, O^F(p, W, F^1) + q^*\, O^F(p, W, F^{2*}) > O^F(p, W, F)$$

or

(IV.6) $$(1-q^*)\, Y^F(p, W, F^1) + q\, Y^F(p, W, F^{2*}) > Y^F(p, W, F)$$

would be required for the equality to hold: the average basket is larger than the certain basket that yields the same utility. (IV.5) is equivalent to

(IV.7) $$(1-q^*)\, H(p, W, V+R, T) + q\, H(p, W, V-M^*, T - P^*) + q^*\, P < H(p, W, V, T)$$

in which case labor supply decreases with the possibility to commit the crime even if $P^* = 0$.

However, (IV.5) may not hold and U may not be concave and still expected labor supply decreases with the criminal opportunity.

3. Using Roy´s identity, an increase in expected leisure of the marginal consumer, i.e., (IV.5) would require

$$\text{(IV.8)} \qquad -(1\text{-}q^*)\ \frac{v^F{}_W(p, W, F^1)}{v^F{}_F(p, W, F^1)}\ -q^*\ \frac{v^F{}_W(p, W, F^2{*})}{v^F{}_F(p, W, F^2{*})}\ >\ -\ \frac{v^F{}_W(p, W, F)}{v^F{}_F(p, W, F)}$$

Using the fact that $v^F{}_w(p, W, F) + T\, v^F{}_F(p, W, F) = v_w(p, W, V, T)$:

$$\text{(IV.9)} \qquad (1\text{-}q^*)\ \frac{v_W(p, W, V+R, T)}{v^F{}_F(p, W, F^1)}\ +\ q^*\ \frac{v_W(p, W, V-M^*, T-P^*)}{v^F{}_F(p, W, F^2{*})}\ + q^*\ P$$

<

$$<\ \frac{v_W(p, W, V, T)}{v^F{}_F(p, W, F)}$$

If M* = 0, we know that necessarily:

$$\text{(IV.10)} \qquad (1\text{-}q^*)\ H(p, W, V+R, T) + q^*\ H(p, W, V\text{-}M^*, T\text{-}P^*)\ =$$

$$=\ (1\text{-}q^*)\ \frac{v_W(p, W, V+R, T)}{v^F{}_F(p, W, F^1)}\ +\ q^*\ \frac{v_W(p, W, V-M^*, T-P^*)}{v^F{}_F(p, W, F^2{*})}\ <$$

$$<\ H(p, W, V, T) = \frac{v_W(p, W, V, T)}{v^F{}_F(p, W, F)}$$

Hence, regardless of the preferences, (IV.4) and (IV.9) also hold – expected leisure and labor supply decrease for the marginal criminal.

If P* = 0 (IV.10), (IV.5) and (IV.9) are interchangeable and they may or may not hold.

4. The relation with expected labor supply can be visualized in the following terms. Let P* = 0. Assume a change in R, dR and the corre sponding change dM* that insures (keeps insuring) deterrence. Then

$$\text{(IV.11)} \qquad \frac{\partial R}{\partial M^*}\ =\ \frac{q^*\ v^F{}_F(p, W, F-M^*)}{(1-q^*)\ v^F{}_F(p, W, F+R)}$$

The compounded effect on expected labor supply of the simultaneous change is:

$$\text{(IV.12)}\quad \frac{\partial E(H)}{\partial M^*} = (1-q^*)\,\frac{\partial H(p, W, V+R, T)}{\partial V}\,\frac{\partial R}{\partial M^*} - q^*\,\frac{\partial H(p, W, V-M^*, T)}{\partial V} =$$

$$= q^*\left[\frac{\partial H(p, W, V+R, T)}{\partial V}\,\frac{v^F{}_F(p, W, F-M^*)}{v^F{}_F(p, W, F+R)} - \frac{\partial H(p, W, V-M^*, T)}{\partial V}\right]$$

If the income effect is relatively constant, i.e., $\frac{\partial H(p, W, V+R, T)}{\partial V} = \frac{\partial H(p, W, V-M^*, T)}{\partial V}$, or H(p, W, V, T) is linear or concave in V, risk aversion guarantees that $\frac{\partial E(H)}{\partial M^*} < 0$: expected labor supply is always smaller when R and M increase above 0. Then with or no imprisonment, expected labor supply decreases with crime opp ortunity.

If $\frac{\partial H(p, W, V+R, T)}{\partial V} = \frac{\partial H(p, W, V-M^*, T)}{\partial V}$, or H(p, W, V, T) is linear or convex in V, and individuals are risk -lovers, $\frac{\partial E(H)}{\partial M^*} > 0$. Then, (for P* small) it is possible that the marginal criminal exhibits larger expected labor supply than non-offenders.

5. Finally, if $v^F(p, W, F) = K(p, W) + Q(F)$ and $\kappa^F(p, W, F) = 0$, (IV.8) reduces to

$$\text{(IV.13)}\qquad (1-q^*)\,\frac{1}{Q'(F^1)} + q^*\,\frac{1}{Q'(F^2{}^*)} < \frac{1}{Q'(F)}$$

As $(1-q^*)\,Q(F^1) + q^*\,Q(F^{2*}) = Q(F)$, and under the reasoning of $\frac{\partial M^*}{\partial V}$, with decreasing risk aversion with respect to F, $(1-q^*)\,Q'(F^1) + q^*\,Q'(F^{2*}) > Q'(F)$ and the inequality holds: expected labor supply of the marginal criminal will be smaller than if he did not have the opportunity to commit the crime. With increasing risk aversion with F, expected labor supply of the marginal trespasser will be larger than if he did not have the opportunity to commit the crime.

If $v^F(p, W, F) = G(p, W)\,Q(F)$ and $\kappa^F(p, W, F)$ is independent of F:

$$\text{(IV.14)}\qquad (1-q^*)\,\frac{Q(F^1)}{Q'(F^1)} + q^*\,\frac{Q(F^2{}^*)}{Q'(F^2{}^*)} < \frac{Q(F)}{Q'(F)}$$

$$\text{(IV.15)}\qquad (1-q^*)\,Q(F^1)\,\frac{Q'(F)}{Q'(F^1)} + q^*\,Q(F^{2*})\,\frac{Q'(F)}{Q'(F^2{}^*)} < Q(F)$$

V. Income and Wage Uncertainty.

A. Non-labor Earnings.

1. Uncertainty with respect to exogenous endowment was not considered. Let us introduce uncertainty with respect to non labor earnings in the spirit of a Rothschild and Stiglitz´s (1970) "mean preserving spread". With probability r, the individual receives non-labor earnings V + s: With probability (1 – r) he gets $V - s\frac{r}{1-r}$. Expected non-labor earnings are always V but the variance (hence uncertainty) increases with s, the "shift parameter", provided s > 0; if s < 0, an increase in uncertainty is represented by a decrease in s.

It is clear that under such circumstances expected utility decreases with uncertainty (increases with s and s > 0) iff:

$$\text{(V.1)} \qquad v^F_F(p, W, F + s) - v^F_F(p, W, F - s\frac{r}{1-r}) < 0$$

Expected utility increases with uncertainty when s < 0, that is, increases with –s (decreases with s) iff the inequality sign in (V.1) is reversed. A necessary and sufficient condition for both inequalities to hold is that the indirect utility function is concave in F, i.e., individuals are risk-averse in full income. Convexity in F leads to the opposite result: risk lovers expected utility increases with uncertainty.

An increase in non-labor income uncertainty will increase (general) expected labor supply iff labor supply is convex in "full income". In that case, an increase in uncertainty increases income relative to leisure consumption. If it is concave, the opposite occurs[32].

[32] Block and Heineke (1973) for risk-averse individuals in income of direct utility conclude that a rise in non-labor earnings uncertainty, in general, increases labor supply. They rely on direct utility maximization, hence considering an optimal ex ante or "unconditional" labor supply decision. By relying on indirect utility maximization, we allow the individual to adjust the labor supply decision to circumstances – that is more convenient to our scenario: if convicted individuals time endowment changes; therefore, our results are not comparable to theirs.

2. Let us define S as the amount of non -labor earnings the individual is willing to give up to avoid the uncertainty. That is, S is the standard risk premium, defined in the same metric – non-labor earnings or full income - as the lottery. Then, S solves:

$$\text{(V.2)} \qquad r\, v^F(p, W, W\,T + V + s) + (1 - r)\, v^F(p, W, W\,T + V - s\,\frac{r}{1-r}) = \\ = v^F(p, W, W\,T + V - S)$$

$$\text{(V.3)} \qquad \frac{\partial S}{\partial s} = r\,\frac{v^F{}_F(p, W, F - s\frac{r}{1-r}) - v^F{}_F(p, W, F + s)}{v^F{}_F(p, W, F - S)}$$

$v^F{}_F(p, W, F) > 0$. If $v^F(p, W, W\,T + V + s)$ is concave, i.e., $\rho^F(p, W, F) > 0$, $s \neq 0$ implies $S > 0$; then, if $s > 0$, $\frac{\partial S}{\partial s} > 0$ (if $s < 0$, a decrease in s increases uncertainty; then $\frac{\partial S}{\partial s} < 0$.) The risk premium rises with uncertainty for risk -averse individuals; it decreases for risk-lovers.

Exemplifying the sensitivity of the risk premium to exogenous endowments:

$$\text{(V.4)} \qquad \frac{\partial S}{\partial V} = \\ \frac{v^F{}_F(p, W, F - S) - r\, v^F{}_F(p, W, F + s) - (1 - r)\, v^F{}_F(p, W, F - s\frac{r}{1-r})}{v^F{}_F(p, W, F - S)}$$

As is well known, $\frac{\partial S}{\partial V}$ will be related to how the Arrow-Pratt measure of risk-aversion $\rho^F(p, W, F)$ responds to F (or V). If risk -aversion increases with V, $\frac{\partial S}{\partial V} > 0$.

$$\text{(V.5)} \qquad \frac{\partial S}{\partial W} = \\ = \frac{v_w(p, W, V - S, T) - r\, v_w(p, W, V + s, T) - (1 - r)\, v_w(p, W, V - s\frac{r}{1-r}, T)}{v^F{}_F(p, W, F - S)}$$

$\frac{\partial S}{\partial W}$ will be related to how the measure of substitutability between W and V, $-\frac{v_{wV}(p, W, V, T)}{v_V(p, W, V, T)}$, responds to V; alternatively, how $-\frac{v^F{}_{FF}(p, W, V + W\,T)}{v^F{}_F(p, W, W\,T + V)}$ responds to W. If substitutability between W and V increases with V, or equivalently if $\rho(p, W, V, T) = \rho^F(p, W, W T + V)$ increases with W, $\frac{\partial S}{\partial W} > 0$.

B. Wage Rate Uncertainty.

1. With respect to wages, we can, however, use two kinds of measures of risk aversion. On the one hand, we can, as before, define the risk premium as how much the individual is willing to pay, in terms of lump-sum income, to get rid of uncertainty. That is, the S such that:

$$\text{(V.6)} \qquad r\, v(p, W + s', V, T) + (1 - r)\, v(p, W - s' \frac{r}{1-r}, V, T) = \\ = v(p, W, W T + V - S)$$

$$\text{(V.7)} \qquad \frac{\partial S}{\partial s'} = r \frac{v_w(p, W - s'\frac{r}{1-r}, V, T) - v_w(p, W + s', V, T)}{v_V(p, W, V - S, T)}$$

If preferences v(p, W, V, T) are convex in W, $s \neq 0$ implies $S < 0$ and the individual prefers the lottery to its expected value. In that case – of $v_{ww}(p, W, V, T) > 0$ –, once $v_V(p, W, V, T) > 0$, if $s' > 0$, $\frac{\partial S}{\partial s'} < 0$; if $s' < 0$, $\frac{\partial S}{\partial s'} > 0$. Hence, S will decrease with uncertainty; or –S, how much the consumer must be given to give it up, increases with uncertainty. If preferences are concave in W, $v_{ww}(p, W, V, T) < 0$, $S > 0$, if $s' > 0$, $\frac{\partial S}{\partial s'} > 0$; if $s' < 0$, $\frac{\partial S}{\partial s'} < 0$: S increases with uncertainty.

The risk premium would be related to $-\frac{v_{ww}(p, W, V, T)}{v_V(p, W, V, T)}$. If this measure increases (decreases) with V, also $\frac{\partial S}{\partial V} > 0\ (< 0)$; if it increases with W, $\frac{\partial S}{\partial W} > 0$. If $v_{ww}(p, W, V, T) > 0$, and that measure increases with V, the premium the individual is willing to pay for the lottery, -S, decreases with non-labor earnings. If $v_{ww}(p, W, V, T) < 0$ and the measure increases with V, the premium the individual is willing to pay to get rid of uncertainty increases with non-labor earnings.

Consumer theory requires quasi-convexity of $v^F(p, W, F)$ in (p, W). However, that is compatible with $v(p, W, V, T) = v^F(p, W, W T + V)$ concave in W – more likely for risk averse consumers with respect to full income. See section VI below.

2. An alternative question would be what reduction in wages, S′ would the consumer be willing to accept to get rid of the uncertainty in wages. That is, redefine the risk premium in the wages metric.

$$\text{(V.8)} \qquad r\, v(p, W + s', V, T) + (1 - r)\, v\left(p, W - s' \frac{r}{1-r}, V, T\right) = \\ = v(p, W - S', V, T)$$

$$\text{(V.9)} \qquad \frac{\partial S'}{\partial s'} = r\, \frac{v_w\left(p, W - s'\frac{r}{1-r}, V, T\right) - v_w(p, W + s', V, T) -}{v_w(p, W - S', V, T)}$$

Being preferences $v\left(p, W - s' \frac{r}{1-r}, V, T\right)$ convex in W, once $v_{w}(p, W, V, T) > 0$, $S' < 0$ – if the consumer-worker is risk-lover with respect to wages when he can adjust hours - and $\frac{\partial S'}{\partial s'} < 0$. If $v_{ww}(p, W, V, T) < 0$, $S' > 0$; $s' > 0$ implies $\frac{\partial S'}{\partial s'} > 0$.

A traditional Arrow-Pratt measure of risk aversion applied to how v is related to W behaves as the premium S′.

3. An increase in wage rate uncertainty will increase (general) expected labor supply iff labor supply is convex in the wage rate – regardless of whether positive or negatively sloped. In that case, an increase in uncertainty increases income relative to leisure consumption. If it is concave, the opposite occurs.[33]

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[33] Block and Heineke (1973) conclude that the effect of increased uncertainty on labor supply is ambiguous, even for a restricted set of consumer preferences. Tressler and Menezes (1980) found a negative effect. Both studies depart from expected direct utility maximization and not directly comparable to ours- see footnote 32.

C. Price Uncertainty.

With respect to prices, things work out differently than wages because, while $v_w(p, W, V, T) > 0$, $v_p(p, W, V, T) = v^F_p(p, W, W T + V) < 0$.

1. In terms of lump-sum income premium, we can define S such that:

$$\text{(V.10)} \qquad r\, v^F(p + s'', W, F) + (1 - r)\, v^F\left(p - s''\frac{r}{1-r}, W, F\right) = \\ = v^F(p, W, F - S)$$

$$\text{(V.11)} \qquad \frac{\partial S}{\partial s''} = r\, \frac{v^F{}_p\left(p - \frac{r}{1-r}s'', W, F\right) - v^F{}_p(p + s'', W, F)}{v^F{}_F(p, W, F - S)}$$

If preferences $v^F(p, W, F)$ are convex in p, $S < 0$. Once $v^F{}_F(p, W, F) > 0$, $\frac{\partial S}{\partial s''} < 0$ ($s'' > 0$) in that case. Being "risk-lover" with respect to price, what the individual must be paid to forego the lottery, $-S$, increases with uncertainty. The risk premium would be related to $-\frac{v^F{}_{pp}(p, W, F)}{v^F{}_F(p, W, F)}$. If this measure increases (decreases) with F (or V), also $\frac{\partial S}{\partial V} > 0\ (< 0)$; if it increases with W – being F replaced by W T + V -, $\frac{\partial S}{\partial W} > 0$.

2. An alternative question would be what increase in prices, S" would the consumer be willing to accept to get rid of the uncertainty in prices.

$$\text{(V.12)} \qquad r\; v^F(p + s'', W, F) + (1 - r)\; v^F\left(p - s''\frac{r}{1-r}, W, F\right) = \\ = v^F(p + S'', W, F)$$

$$\text{(V.13)} \qquad \frac{\partial S''}{\partial s''} = r\, \frac{v^F{}_p(p + s'', W, F) - v^F{}_p\left(p - \frac{r}{1-r}s'', W, F\right)}{v^F{}_p(p + S'', W, F)}$$

Being preferences $v^F(p, W, F)$ convex in p, once $v^F_p(p, W, F) < 0$, $S" < 0$ – i.e., the consumer prefers uncertainty; he will have to be given a larger reduction in prices to accept less variability: $\frac{\partial S"}{\partial s"} < 0$.

When we are considering a negatively valued commodity, we may still associate aversion to uncertainty in that commodity to cases where the utility of the expected value of the lottery is larger than the expected utility from facing the lottery – the individual prefers the expected value to the lottery. This will still occur for concave functions. As utility is negatively related to the commodity, the risk premium should be defined - as S" - as how much the individual is willing to additionally accept of that commodity to get rid of uncertainty; then in this cases, the Arrow-Pratt measure of risk aversion should be defined without the minus sign, $\frac{v^F{}_{pp}(p, W, F)}{v^F{}_p(p, W, F)}$. Interpretation of this measure is then standard: an increase in it denotes a higher aversion to risk in prices. If $v^F_p(p, W, F)$ is convex, the premium is negative and individuals are risk-lovers with respect to prices, preferring the lottery to the expected value of the lottery.

3. An increase in price uncertainty will increase (general) expected labor supply iff labor supply is convex in the price. In that case, an increase in uncertainty increases income relative to leisure consumption. If it is concave, the opposite occurs.

D. Criminal Deterrence.

1. To analyse the effect of uncertainty on criminal deterrence we have to consider that:

- we have two risks involved
- we shall be interested only in effects on the "premium" of one of the risks, crime,
- the metric considered is measured in full income terms, the metric of the (negative) crime "premium" M*.

In this context, we apply implications from Ross (1981), Kihlstrom, Romer and Williams (1981) and Pratt (1988) – Pratt and Zeckhauser (1987), Kimball (1993) – and Gollier and Pratt (1996) that study aversion to one risk in the presence of independent uncertainty from other sources.

In general, when the other risk changes, there will be a change in the crime deterrent sanction, M´* - for simplicity, we will consider only a general deterrent fine in this section. ρ(p, W, V, T) measures how the (one-directional) risk premium of

$$(1-q)\ v^F(p, W, F+R)\ +\ q\ v^F(p, W, F-R\ \frac{1-q}{q})$$

responds to the levels of p, W, V, and T. But not how the risk premium responds to uncertainty in those variables.

Abstractly, criminality inserts risk in full income. Consider an added noise Z to full income with E[Z] = 0. In general, the risk premium to uncertainty associated with Z conditional on another variable X is u satisfying

$$E_Z\ U(F+Z, X) = U(F-u, X)$$

From Arrow (1965) and Pratt (1964), the behavior of u is associated with $\rho(F, X) = -U_{FF}(F, X) / U_F(F, X)$. If we are studying the risk premium under general uncertainty of X, we concentrate on u′ arising from

$$E_Z\ E_X\ U(F+Z, X \mid Z) = E_X\ U(F-u', X)$$

Assume independence between Z and X. Then, by analogy, the behavior of u′ will be associated to the measure of absolute measure of risk aversion of the utility function $W(F) = E_X\ U(F, X)$. We can relate u′ with the measure $R(F) = -\frac{E_X\ U_{FF}(F, X)}{E_X\ U_F(F, X)}$ [34].

We are not interested in how this measure responds to the risk in F [35]. That is, we do not expect interesting highlights by analyzing, for instance, $\frac{\partial M'^*}{\partial V}$ in this more complex setting than before. Rather, we are interested in how M′* – hence, R(F) – reacts to changes in the distribution of X.

2.1. We consider the following scenario.

$$\text{(V.14)}\qquad r\,v^F(p, W, F+s) + (1-r)\,v^F(p, W, F-s\ \frac{r}{1-r}) =$$

$$= (1-q^*)\,[r\,v^F(p, W, F+R+s) + (1-r)\,v^F(p, W, F+R-s\ \frac{r}{1-r})] +$$

$$+\ q^*\,[r\,v^F(p, W, F-M'^*+s) + (1-r)\,v^F(p, W, F-M'^*-s\ \frac{r}{1-r})]$$

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[34] See Gollier and Pratt (1996). Also Pratt (1988).

[35] Pratt (1988) in Theorem 3. states that if U(F,X) is decreasingly risk-averse in F for every X, then E_X U(F, X) is decreasingly risk-averse.

The individual faces uncertainty with respect to "full income" whether he commits the crime or not. If he dislikes risk, as he is already facing it, he will be, on average, even worse. The issue is whether the additional uncertainty compounds the problem or not.

$$\text{(V.15)} \qquad \frac{\partial M'^*}{\partial s} = r\ \{(1 - q^*)\ [v^F{}_F(p,W,F + R + s) - v^F{}_F(p, W, F + R - s\ \frac{r}{1 - r})] + \\ + q^*\ [v^F{}_F(p, W, F - M'^* + s) - v^F{}_F(p, W, F - M'^* - s\ \frac{r}{1 - r})] - \\ - [v^F{}_F(p, W, F + s) - v^F{}_F(p, W, F - s\ \frac{r}{1 - r})]\ \}\ / \\ / \{q^*\ [r\ v^F{}_F(p, W, F - M'^* + s) + (1 - r)\ v^F{}_F(p, W, F - M'^* - s\ \frac{r}{1 - r})]\}$$

The denominator is positive; $\frac{\partial M'^*}{\partial s}$ will be negative iff

$$(1 - q^*)\ v^F{}_F(p, W, F + R + s) + q^*\ v^F{}_F(p, W, F - M'^* + s) - v^F{}_F(p, W, F + s) < \\ < (1 - q^*)\ v^F{}_F(p, W, F + R - s\ \frac{r}{1 - r}) + q^*\ v^F{}_F(p, W, F - M'^* - s\ \frac{r}{1 - r}) - \\ - v^F{}_F(p, W, F - s\ \frac{r}{1 - r})$$

For any s larger than 0, the left hand-side measures the difference between the expected value of marginal utility of the primary lottery and the approximate "certain equivalent" at a certain full income base F + s. The right hand-side, the same difference but at a smaller implied expected level: $s + s\ \frac{r}{1 - r}$ lower. If risk aversion – concavity of v in F - increases with F, the expression on each side evaluated at s = 0 represents the numerator of $\frac{\partial M^*}{\partial V}$, the sign of which was related to how concavity of v^F, measured by the Arrow-Pratt index of risk aversion, evolves with F. Then, intuitively, the expression would be signed according to "concavity of concavity" of v^F – to how the response of concavity of v^F to F is further enhanced by F, being $\frac{\partial M'^*}{\partial s}$ negative if it is. A measure of risk aversion to additional noise in the same risk was derived and justified in Martins (2002) – and named a measure of providence - as:

$$\text{(V.16)} \qquad N^F(p, W, F) = - \frac{v^F{}_{FFF}(p, W, F)}{v^F{}_F(p, W, F)}$$

If $\frac{\partial N^F(p, W, F)}{\partial F} > 0$:

(V.17) $$v^F_{FFFF}(p, W, F) < v^F_{FFF}(p, W, F)\ \frac{v^F{}_{FF}(p, W, F)}{v^F{}_F(p, W, F)}$$

Then, the risk premium to F increases with additional uncertainty or noise in F and $\frac{\partial M'^*}{\partial s} < 0$. When $\frac{\partial N^F(p, W, F)}{\partial F} > 0$, the individual is averse to additional or uninsurable uncertainty in F; when this uncertainty rises, he is less willing to assume a (the) risk that he can decide upon (here, commit the crime). The reverse happens if $\frac{\partial N^F(p, W, F)}{\partial F} < 0$.

2.2. It would be possible that conviction does not entail background noise. This hypothesis may apply if the sanction is mainly composed of imprisonment. Then:

(V.18) $$\begin{aligned} & r\, v^F(p, W, F + s) + (1 - r)\, v^F(p, W, F - s\ \frac{r}{1 - r}) = \\ & = (1 - q^*)\ [r\, v^F(p, W, F + R + s) + (1 - r)\, v^F(p, W, F + R - s\ \frac{r}{1 - r})] + \\ & + q^*\, v^F(p, W, F - M'^*) \end{aligned}$$

(V.19) $$\begin{aligned} \frac{\partial M'^*}{\partial s} = & \ r\ \{(1 - q^*)\ [v^F_F(p,W,F + R + s) - v^F_F(p, W, F + R - s\ \frac{r}{1 - r})] - \\ & - [v^F_F(p, W, F + s) - v^F_F(p, W, F - s\ \frac{r}{1 - r})]\ \}\ /\ [q^*\, v^F_F(p, W, F - M'^*)] \end{aligned}$$

If q* is close to 0, $\frac{\partial M'^*}{\partial s} > 0$ if

$$\begin{aligned} & v^F_F(p,W,F + R + s) - v^F_F(p, W, F + R - s\ \frac{r}{1 - r}) > \\ & > v^F_F(p, W, F + s) - v^F_F(p, W, F - s\ \frac{r}{1 - r}) \end{aligned}$$

That occurs ($s > 0$) for v^F_{FF} increasing with F ($-v^F_{FF}$ decreasing with F), that is, $v^F_{FFF} > 0$. (For $s < 0$, $\frac{\partial M'^*}{\partial s} < 0$ under the same circumstances – a rise in uncertainty increases criminality.)

If q* is close to 1 ($s > 0$) $\frac{\partial M'^*}{\partial s} > 0$, when $v^F_{FF} < 0$. Risk-averse individuals will be detained less easily if free-life uncertainty rises – the criminal risk does not involve that uncertainty and becomes relatively more attractive.

One can show – Martins (2002) - that a risk-premium defined in the dimension of V (F) increases – M′* would decrease - with out-of-prison uncertainty in non-labor earnings under the previous circumstances iff

$$\text{(V.20)} \qquad q^*\, v^F_{FFFF}(p, W, F) + (1 - q^*)\, 2\, v^F_{FFF}(p, W, F)\, \frac{v^F_{FF}(p, W, F)}{v^F_{F}(p, W, F)} < 0$$

This expression involves the same terms that (V.17). Yet, while in (V.17) we balance the terms $v^F_{FFFF}(p, W, F)$ and $v^F_{FFF}(p, W, F)\, \frac{v^F_{FF}(p, W, F)}{v^F_{F}(p, W, F)}$, (V.20) requires their weighted average of the first one and twice the second one to be negative for criminality to decrease with out-of-prison uncertainty. (V.20) has the sign of a weighted average of temperance ($-\frac{v^F_{FFFF}(p, W, F)}{v^F_{FFF}(p, W, F)}$, the risk aversion measure exhibited by the second derivative $v^F_{FF}(p, W, F)$ [36]) and the measure of risk aversion, multiplied by providence ($-\frac{v^F_{FFF}(p, W, F)}{v^F_{F}(p, W, F)}$). (However, the result implied in (V.20) is not so robust as (V.17).)

2.3. A final aspect is the correlation between "good" and "bad" states. Let us consider the following:

$$\text{(V.21)} \qquad (1 - q)\, v^F(p, W, F + s) + q\, v^F\!\left(p, W, F - s\, \frac{1-q}{q}\right) =$$
$$= (1 - q)\, v^F(p, W, F + R + s) + q\, v^F\!\left(p, W, F - M'^* - s\, \frac{1-q}{q}\right)$$

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[36] See Gollier and Pratt (1996).

If s > 0, there is positive correlation between the "good" states (and positive correlation between "bad states"): when the crime is successful, the background noise assumes its positive value; when it fails, it assumes its negative state. If s < 0, the opposite occurs.

$$(V.22)\qquad \frac{\partial M'^*}{\partial s} = \frac{1-q}{q}\left[\frac{v^F{}_F(p, W, F+R+s) - v^F{}_F(p, W, F - M'^* - s\frac{1-q}{q})}{v^F{}_F(p, W, F - M'^* - s\frac{1-q}{q})} - \frac{v^F{}_F(p, W, F+s) - v^F{}_F(p, W, F - s\frac{1-q}{q})}{v^F{}_F(p, W, F - M'^* - s\frac{1-q}{q})}\right]$$

The numerator will be negative iff

$$v^F{}_F(p, W, F+R+s) - v^F{}_F(p, W, F - M'^* - s\,\frac{1-q}{q}) < < v^F{}_F(p, W, F+s) - v^F{}_F(p, W, F - s\,\frac{1-q}{q})$$

Regardless of whether s is larger or smaller than 0:

- If $v^F{}_{FF}(p, W, F) < 0$, $v^F{}_F(p, W, F+R+s) < v^F{}_F(p, W, F+s)$ and $v^F{}_F(p, W, F - M'^* - s\,\frac{1-q}{q}) > v^F{}_F(p, W, F - s\,\frac{1-q}{q})$. Then, $\frac{\partial M'^*}{\partial s} < 0$
- If $v^F{}_{FF}(p, W, F) > 0$, $v^F{}_F(p, W, F+R+s) > v^F{}_F(p, W, F+s)$ and $v^F{}_F(p, W, F - M'^* - s\,\frac{1-q}{q}) < v^F{}_F(p, W, F - s\,\frac{1-q}{q})$. Then, $\frac{\partial M'^*}{\partial s} > 0$

But an increase in s has opposite meanings: if s > 0, it means an increase in income dispersion; if s < 0, an increase in s implies a decrease in income dispersion (in – s). (See Martins (2002) for a formal interpretation of the effect of correlation in risks on a particular risk premium.)

Proposition 5:

1. Criminality increases with spurious uncertainty in non-labor income if individuals´ preferences or indirect utility functions exhibit decreasing risk aversion to additional noise in F (V).
2. Criminality increases with out-of-prison uncertainty in non-labor income if individuals´ are risk averse with respect to full income (if probability of detection is high) and/or

marginal indirect utility functions exhibit preference towards risk (added to) in F (V) (if probability of detection is low).

3. If individuals are risk averse with respect to full income: criminality decreases with uncertainty for positively correlated "good states" - by committing the crime, they compound dispersion which is generally disliked. For negatively correlated "good states" the opposite occurs – by committing the crime, they reduce overall dispersion – and some risk-diversification effect is achieved.

4. If individuals are risk lovers with respect to full income, criminality increases with volatility in non-labor income for positively correlated "good states"; it decreases if they move in the opposite direction.

Positively correlated "good states" implies that when detected or convicted, the individual may have less opportunities to obtain non-labor earnings. And/or that the crime can only be committed in "good states" of non-labor earnings [37]. That the former is a reasonable assumption, the second may not. But if acceptable, it implies that if individuals are risk averse, criminality decreases with income uncertainty or non-labor earnings volatility; if individuals are risk-lovers, it increases.

3. As modelled, s could indistinguishably (only multiplied by W) represent uncertainty in T – that only shows in F = W T + V. As such:

Proposition 6: In general, criminality reacts to uncertainty to lifetime endowment in the same way as to uncertainty in non-labor earnings:

1. Criminality increases with spurious uncertainty in lifetime endowment if individuals´ preferences or indirect utility functions exhibit decreasing risk aversion to additional noise in F (V).

2. Criminality increases with out-of-prison uncertainty in lifetime endowment if individuals´ are risk averse with respect to full income (if the probability of detection is high) and/or marginal indirect utility functions exhibit preference towards risk (added to) in F (V) (if the probability of detection is low).

[37] See Kihlstrom, Romer and Williams (1981), p. 913. They hypothesise this type of result for a general risk premium under correlated uncertainty. Note, however, that our results refer to a "partial premium" to uncertainty of one of the variables to general uncertainty in the other.

3. If individuals are risk averse with respect to full income: criminality decreases with uncertainty for positively correlated "good states". For negatively correlated "good states" the opposite occurs.

4. If individuals are risk lovers with respect to full income, criminality increases with lifetime uncertainty for positively correlated "good states"; it decreases if they move in the opposite direction.

If reduced lifetime expectations are consonant with conviction (we have positively correlated states) - even under a given mean level of time endowment – risk-averse individuals with respect to F (V) will refrain from criminal activities when such type of uncertainty rises. The opposite occurs for risk lovers.

4.1. Considerations about wage uncertainty would stem from equivalent equations but where $v^F(p, W, F)$ is replaced by $v(p, W, V, T)$. The deterrent sanctions P* and M* solve:

$$\text{(V.23)} \qquad r\, v(p, W+s, V, T) + (1-r)\, v\!\left(p, W - s\frac{r}{1-r}, V, T\right) =$$
$$= (1-q^*)\left[r\, v(p, W+s, V+R, T) + (1-r)\, v\!\left(p, W - s\frac{r}{1-r}, V+R, T\right)\right] +$$
$$+ q^*\left[r\, v(p, W+s, V-M^*, T-P^*) + (1-r)\, v\!\left(p, W - s\frac{r}{1-r}, V-M^*, T-P^*\right)\right]$$

The individual faces wage rate uncertainty whether he commits the crime or not. Whether he dislikes risk in "full income" or not, if indirect utility functions are convex in W, uncertainty towards the wage leaves him better off; but that is not the relevant effect. He will prefer less risk in F – and require a lower M* - when s rises, if some sort of preference towards risk substitution between the two assets exists.

$$\text{(V.24)} \qquad \frac{\partial M^*}{\partial s} = \frac{r}{q^*}\,[(1-q^*)$$
$$\frac{v_w(p, W+s, V+R, T) - v_w\!\left(p, W - s\frac{r}{1-r}, V+R, T\right)}{r\, v_F(p, W+s, F-M'^*) + (1-r)\, v_F\!\left(p, W - s\frac{r}{1-r}, F-M'^*\right)} +$$
$$+ q^* \frac{v_w(p, W+s, V-M^*, T-P^*) - v_w\!\left(p, W - s\frac{r}{1-r}, V-M^*, T-P^*\right)}{r\, v_F(p, W+s, F-M'^*) + (1-r)\, v_F\!\left(p, W - s\frac{r}{1-r}, F-M'^*\right)} -$$

$$- \frac{v_w(p, W+s, V, T) - v_w(p, W - s\frac{r}{1-r}, V, T)}{r\, v_F(p, W+s, F-M'^*) + (1-r)\, v_F(p, W - s\frac{r}{1-r}, F-M'^*)}]$$

The denominator is positive; $\frac{\partial M^*}{\partial s}$ will be negative iff

$$(1 - q^*)\, v_W(p, W+s, V+R, T) + q^*\, v_W(p, W+s, V - M^*, T-P^*) - v_W(p, W+s, V, T) <$$
$$(1 - q^*)\, v_W(p, W - s\frac{r}{1-r}, V+R, T) + q^*\, v_W(p, W - s\frac{r}{1-r}, V - M^*, T - P^*) -$$
$$- v_W(p, W - s\frac{r}{1-r}, V, T)$$

For any s larger than 0, the left hand -side measures the difference between the expected value of marginal utility with respect to the wage and th e marginal utility of the certain equivalent at wage W + s. The right hand -side, the same difference but at a smaller implied expected level of W: $s + s\frac{r}{1-r}$ lower. By analogy to the reason above for F – noticing the similarity of each s ide of the expression with the numerator of $\frac{\partial M^*}{\partial W}$ without uncertainty -, the sign of $\frac{\partial M^*}{\partial s}$ would be related to how the concavity of v in V is concave in W – or how the substitutability between W and V increases with V (a ssociated to the sign of $\frac{\partial M^*}{\partial W}$ in the absence of other noises) is reinforced when W rises. A measure of how the aversion to uncertainty in V responds to (the additional) risk in W was also derived and formally justified in Martins (2002); applied here:

$$\text{(V.25)} \qquad Z^V(p, W, V, T) = -\frac{v_{VWW}(p, W, V, T)}{v_V(p, W, V, T)}$$

It may be, thus, interpreted as a measure of how substitutability between V and W responds to W. If it increases with V, then the risk premium to V increases with uncertainty in W. In this case, $\frac{\partial M^*}{\partial s} < 0$ for $s > 0$ ($\frac{\partial M^*}{\partial s} > 0$ for $s < 0$) and criminality decreases with uncertainty in wages. That is, when $\frac{\partial Z^V(p, W, V, T)}{\partial V} > 0$, and

$$(V.26) \qquad v_{VVww}(p, W, V, T) < v_{Vww}(p, W, V, T) \frac{v_{VV}(p, W, V, T)}{v_V(p, W, V, T)},$$

the individual´s averseness to a risk in V is enhanced by uninsurable uncertainty in W; when this uncertainty rises, he is less willing to assume a (the) primary risk - that he can decide upon: commit the crime. The reverse happens if $\frac{\partial Z^V(p, W, V, T)}{\partial V} < 0$.

Define:

$$(V.27) \quad Z^F(p, W, F) = - \frac{v^F{}_{FWW}(p, W, F)}{v^F{}_F(p, W, F)} \; ; \quad T^F(p, W, F) = - \frac{v^F{}_{FFW}(p, W, F)}{v^F{}_F(p, W, F)}$$

Then:

$$(V.28) \qquad Z^V(p, W, V, T) = \\ = [Z^F(p, W, F) + 2\, T^F(p, W, F) + T^2 N^F(p, W, F)]_{\,|\, F = W T + V}$$

Obviously,

$$(V.29) \qquad \frac{\partial Z^V(p, W, V, T)}{\partial V} = \\ = \frac{\partial Z^F(p, W, F)}{\partial F}\Big|_{F = W T + V} + 2\, T \frac{\partial T^F(p, W, F)}{\partial F}\Big|_{F = W T + V} + \\ + T^2 \frac{\partial N^F(p, W, F)}{\partial F}\Big|_{F = W T + V}$$

A positive (negative) response of $N^F(p, W, F)$ to F enhances the likelihood that $Z^V(p, W, V, T)$ increas es (decreases) with V - and criminality decreases (increases) with uncertainty in wages.

4.2. If conviction does not entail background noise in wage :

$$(V.30) \qquad r\, v(p, W + s, V, T) + (1 - r)\, v(p, W - s \frac{r}{1 - r}, V, T) = \\ = (1 - q^*) [r\, v(p, W + s, V + R, T) + (1 - r)\, v(p, W - s \frac{r}{1 - r}, V+R, T)] +$$

$$+ q^* v(p, W, V - M^*, T - P^*)$$

(V.31) $$\frac{\partial M^*}{\partial s} = \frac{r}{q^*} \left[(1-q^*) \frac{v_w(p, W+s, V+R, T) - v_w(p, W - s\frac{r}{1-r}, V+R, T)}{v_F(p, W, F - M'^*)} - \right.$$
$$\left. - \frac{v_w(p, W+s, V, T) - v_w(p, W - s\frac{r}{1-r}, V, T)}{v_F(p, W+s, F - M'^*)} \right]$$

If q* is close to 0, $\frac{\partial M'^*}{\partial s} > 0$ if

$$v_w(p, W+s, V+R, T) - v_w(p, W - s\,\frac{r}{1-r}, V+R, T) >$$
$$> v_w(p, W+s, V, T) - v_w(p, W - s\,\frac{r}{1-r}, V, T)$$

That occurs (s > 0) for v_{ww} increasing with V ($-v_{ww}$ decreasing with V) or $v_{Vww} > 0$ – marginal utility with respect to non-labor earnings is convex in wages. (For s < 0, $\frac{\partial M'^*}{\partial s} < 0$ under the same circumstances – a rise in uncertainty increases criminality.)

If q* is close to 1 (s > 0) $\frac{\partial M'^*}{\partial s} > 0$, when $v_{ww} < 0$. If individuals are averse to risk in wages, they will be detained less easily if free-life wage uncertainty rises.

One can show – Martins (2001) (However, the result is not so robust as (V.26).) - that a risk-premium defined in the dimension of V (F) increases with uncertainty in W under the previous circumstances if

(V.32) $$q^* v_{VVww}(p,W,V,T) + (1-q^*)\, 2\, v_{Vww}(p,W,V,T)\, \frac{v_{VV}(p,W,V,T)}{v_V(p,W,V,T)} < 0$$

4.3. With respect to the correlation between “good” and “bad” states:

(V.33) $$(1-q)\, v(p, W+s, V, T) + q\, v(p, W - s\,\frac{1-q}{q}, V, T) =$$

$$(1-q)\, v(p, W+s, V+R, T) + q\, v(p, W - s\,\frac{1-q}{q}, V - M^*, T - P^*)$$

$$\text{(V.34)} \qquad \frac{\partial M^*}{\partial s} = \frac{1-q}{q}$$

$$\left[\frac{v_w(p, W+s, V+R, T) - v_w(p, W - s\frac{1-q}{q}, V - M^*, T - P^*)}{v^F{}_F[p, W+s, F-(W+s)P^* - M^*]} - \right.$$

$$\left. - \frac{v_w(p, W+s, V, T) - v_w(p, W - s\frac{1-q}{q}, V, T)}{v^F{}_F[(p, W+s, F-(W+s)P^* - M^*]}\right]$$

The numerator will be negative iff

$$v_w(p, W+s, V+R, T) - v_w(p, W - s\,\frac{1-q}{q}, V - M^*, T - P^*) <$$

$$< v_w(p, W+s, V, T) - v_w(p, W - s\,\frac{1-q}{q}, V, T)$$

Regardless of whether s is larger or smaller than 0, around $P^* = 0$:

- If $v_{Vw}(p, W, V, T) < 0$, $\quad v_w(p, W+s, V+R, T) < v_w(p, W+s, V, T)$ and $v_w(p, W - s\,\frac{1-q}{q}, V - M^*, T - P^*) > v_w(p, W - s\,\frac{1-q}{q}, V, T)$. Then, $\frac{\partial M^*}{\partial s} < 0$
- If $v_{Vw}(p, W, V, T) > 0$, $\quad v_w(p, W+s, V+R, T) > v_w(p, W+s, V, T)$ and $v_w(p, W - s\,\frac{1-q}{q}, V - M^*, T - P^*) < v_w(p, W - s\,\frac{1-q}{q}, V, T)$. Then, $\frac{\partial M^*}{\partial s} > 0$

(See see Martins (2001) for a formal interpretation of the effect of correlation i n risks on a particular risk premium.)

For $P^* > 0$, the second part of each statement may not hold; we can decompose

$$v_w(p, W - s\,\frac{1-q}{q}, V - M^*, T - P^*) =$$

$$= (T - P^*)\, v^F{}_F[p, W - s\,\frac{1-q}{q}, (W - s\,\frac{1-q}{q})(T - P^*) + V - M^*] +$$

$$+ v^F{}_w[p, W - s\,\frac{1-q}{q}, (W - s\,\frac{1-q}{q})(T - P^*) + V - M^*]$$

Apparently, $P^* > 0$ would favor $\frac{\partial M^*}{\partial s} > 0$ more often than just requiring $v_{Vw}(p, W, V, T) > 0$.

Proposition 7:
1. Criminality increases with spurious uncertainty in wages if, in individuals´ preferences or indirect utility functions, the rate at which substitutability between W and V changes with W decreases with V.
2. Criminality increases with out-of-prison uncertainty in wage rate if individuals´ are risk averse with respect to the wage rate (if probability of detection is high) and/or marginal indirect utility functions with respect to full income exhibit preference towards risk (added to) in W (if probability of detection is low).
3. Provided that P* is small: If non-labor earnings and wages are substitutes, criminality decreases with uncertainty for positively correlated "good states"; it increases if evasion is consonant with low wages and conviction with high.
4. When non-labor earnings and wages are complements (or P* is sizeable), criminality increases with wage dispersion for positively correlated "good states"; it decreases, if evasion is consonant with low wages and detection with high.

In most indirect utility functions, $v^F_{Fw}(p, W, F) < 0$; as $v_{Vw}(p, W, V, T) = v^F_{Fw}(p, W, F) + T\, v^F_{FF}(p, W, F)$ if individuals are also risk averse with respect to full income, $v_{Vw}(p, W, V, T) < 0$ – they fall in case 2. of Proposition 7. Consider individuals that say, face higher unemployment spells, or have seasonal jobs, even if with the same average wage; they face higher uncertainty in the sense described above than stable job-holders. If conviction or detection is consonant with lower wages, criminality will decrease with uncertainty – that is, those types of workers will face an extra deterrent-effect due to higher dispersion in the wage distribution.

Notice, however, that if individuals are risk-lovers, even with $v^F_{Fw}(p, W, F) < 0$, it may occur that $v_{Vw}(p, W, V, T) > 0$ and case 3. may arise.

5.1. Finally, consider price uncertainty:

$$\text{(V.35)} \qquad r\, v^F(p + s, W, F) + (1 - r)\, v^F(p - s\,\tfrac{r}{1-r}, W, F) =$$
$$= (1 - q^*)\,[r\, v^F(p + s, W, F + R) + (1 - r)\, v^F(p - s\,\tfrac{r}{1-r}, W, F + R)] +$$

$$+ \{q^* [r\, v^F(p + s, W, F - M'^*) + (1 - r)\, v^F(p - s \frac{r}{1-r}, W, F - M'^*)]\}$$

Then:

$$\text{(V.36)} \quad \frac{\partial M'^*}{\partial s} = r\, \{(1 - q^*)\, [v^F_p(p + s, W, F + R) - v^F_p(p - s \frac{r}{1-r}, W, F + R)] +$$

$$+ q^* [v^F_p(p + s, W, F - M'^*) - v^F_p(p, W, F - M'^* - s \frac{r}{1-r})] -$$

$$- [v^F_p(p + s, W, F) - v^F_p(p - s \frac{r}{1-r}, W, F)]\, \} \,/$$

$$/ \{q^* [r\, v^F_F(p + s, W, F - M'^*) + (1 - r)\, v^F_F[p - s \frac{r}{1-r}, W, F - M'^*]\, \}$$

The denominator is positive; $\frac{\partial M'^*}{\partial s}$ will be negative iff

$$(1 - q^*)\, v^F_p(p + s, W, F + R) + q^*\, v^F_p(p + s, W, F - M'^*) - v^F_p(p + s, W, F) <$$

$$< (1 - q^*)\, v^F_p(p - s \frac{r}{1-r}, W, F + R) + q^*\, v^F_p(p - s \frac{r}{1-r}, W, F - M'^*) -$$

$$- v^F_p(p - s \frac{r}{1-r}, W, F)$$

For any s larger than 0, the left hand -side measures the difference between the expected value of marginal utility with respect to the price level if income is increased by $s + s \frac{r}{1-r}$. The right hand -side, the same difference but at a smaller implied expected level of p. Define, by analogy:

$$\text{(V.37)} \qquad D^F(p, W, F) = - \frac{v^F_{Fpp}(p, W, F)}{v^F_F(p, W, F)}$$

It may be, thus, interpreted as a measure of how substitutability between F and p in the indirect utility function responds to p. If it increases with F:

$$\text{(V.38)} \qquad v^F_{FFpp}(p, W, F) < v^F_{Fpp}(p, W, F) \frac{v^F_{FF}(p, W, F)}{v^F_F(p, W, F)}$$

Then the risk premium to a risk added to F (or V) increases with uncertainty in p. In this case, $\frac{\partial M'^*}{\partial s} < 0$.

5.2. Suppose conviction does not entail background noise. Then:

$$\text{(V.39)} \qquad r\, v^F(p + s, W, F) + (1 - r)\, v^F(p - s \frac{r}{1-r}, W, F) =$$
$$= (1 - q^*) [r\, v^F(p + s, W, F + R) + (1 - r)\, v^F(p - s \frac{r}{1-r}, W, F + R)] +$$
$$+ q^*\, v^F(p, W, F - M'^*)$$

$$\text{(V.40)} \qquad \frac{\partial M'^*}{\partial s} = r\, \{(1 - q^*) [v^F_p(p + s, W, F + R) - v^F_p(p - s \frac{r}{1-r}, W, F + R)] -$$
$$- [v^F_p(p + s, W, F) - v^F_p(p - s \frac{r}{1-r}, W, F)]\, \} \,/\, [q^*\, v^F_F(p, W, F - M'^*)]$$

If q^* is close to 0, $\frac{\partial M'^*}{\partial s} > 0$ if

$$v^F_p(p + s, W, F + R) - v^F_p(p - s \frac{r}{1-r}, W, F + R) >$$
$$> v^F_p(p + s, W, F) - v^F_p(p - s \frac{r}{1-r}, W, F)$$

That occurs ($s > 0$) for v^F_{pp} increasing with F ($-v^F_{pp}$ decreasing with F), that is, $v^F_{Fpp} > 0$. (For $s < 0$, $\frac{\partial M'^*}{\partial s} < 0$ under the same circumstances – a rise in uncertainty increases criminality.)

If q^* is close to 1 ($s > 0$) $\frac{\partial M'^*}{\partial s} < 0$, when $v^F_{pp} > 0$. Risk-lovers with respect to price will be detained more easily if free-life uncertainty rises – the criminal risk does not involve that uncertainty and becomes relatively less attractive.

One can show – Martins (2001) - that a risk-premium defined in the dimension of V (F) increases with uncertainty in p under the previous circumstances if

$$\text{(V.41)} \quad q^*\, v^F_{FFpp}(p, W, F) + (1 - q^*)\, 2\, v^F_{Fpp}(p, W, F)\, \frac{v^F_{FF}(p, W, F)}{v^F_F(p, W, F)} < 0$$

If (V.41) prevails, $\frac{\partial M'^*}{\partial s} < 0$ and criminality decreases with out -of prison price volatility. (However, this result is not so robust as (V.38).)

5.3. A final aspect is the correlation between "good" and "bad" states:

$$\text{(V.42)} \qquad (1-q)\, v^F(p+s, W, F) + q\, v^F(p- s\,\frac{1-q}{q}, W, F) = (1-q)\, v^F(p+s, W, F+R) + q\, v^F(p- s\,\frac{1-q}{q}, W, F-M'^*)$$

If s > 0, there is positive correlation between the "good" states. If s < 0, the opposite occurs.

$$\text{(V.43)} \qquad \frac{\partial M'^*}{\partial s} = \frac{1-q}{q} \left[\frac{v^F{}_p(p+s, W, F+R) - v^F{}_p(p-s\frac{1-q}{q}, W, F-M'^*)}{v^F{}_F(p-s\frac{1-q}{q}, W, F-M'^*)} - \frac{v^F{}_p(p+s, W, F) - v^F{}_p(p-s\frac{1-q}{q}, W, F)}{v^F{}_F(p-s\frac{1-q}{q}, W, F-M'^*)}\right]$$

The numerator will be negative iff

$$v^F{}_p(p+s, W, F+R) - v^F{}_p(p- s\,\frac{1-q}{q}, W, F-M'^*) < $$
$$< v^F{}_p(p+s, W, F) - v^F{}_p(p- s\,\frac{1-q}{q}, W, F)$$

Regardless of whether s is larger or smaller than 0:

- If $v^F{}_{pF}(p, W, F) < 0$, $v^F{}_p(p+s, W, F+R) < v^F{}_p(p+s, W, F)$ and $v^F{}_p(p- s\,\frac{1-q}{q}, W, F-M'^*) > v^F{}_p(p- s\,\frac{1-q}{q}, W, F)$. Then, $\frac{\partial M'^*}{\partial s} < 0$
- If $v^F{}_{Fp}(p, W, F) > 0$, $v^F{}_p(p+s, W, F+R) > v^F{}_p(p+s, W, F)$ and

$v^F_p(p - s\frac{1-q}{q}, W, F - M'^*) < v^F_p(p - s\frac{1-q}{q}, W)$. Then, $\frac{\partial M'^*}{\partial s} > 0$

But an increase in s has opposite meanings: if $s > 0$, it means an increase in price dispersion; if $s < 0$, we have that an increase in s implies a decrease in price dispersion. (See Martins (2001) for a formal interpretation of the effect of correlation in risks on a particular risk premium.)

Proposition 8:

1. Criminality increases with spurious uncertainty in the price level if, in individuals´ preferences or indirect utility functions, the rate at which substitutability between p and F changes with p decreases with F.
2. Criminality decreases with out-of-prison uncertainty in the price level if individuals´ are risk lovers with respect to the prices (if probability of detection is high) and/or marginal indirect utility functions with respect to full income exhibit aversion towards risk (added to) in W (if probability of detection is low).
3. If individuals´ preferences exhibit income-price substitutability: criminality decreases with uncertainty in the price level if high prices are consonant with evasion and low prices with conviction; it increases if there is positively correlated favorability of price and criminal output movements.
4. If income and prices are complements, criminality increases with volatility in prices for negatively correlated favorability of price and criminal income movements; it decreases if low prices occur with evasion and high prices with apprehension.

In most indirect utility functions, $v_{Fp}(p, W, F) < 0$ and individuals are in case 2. of Proposition 8. If conviction or detection is consonant with lower prices, criminality will decrease with uncertainty in prices.

6. Uncertainty about the punishment can also be modeled in this framework. Let the penalty be $M'^* + s$ with probability r and $M'^* - s\frac{r}{1-r}$ with probability $(1 - r)$. The expected value of the sanction in case of conviction is M'^*, but there is uncertainty around its real value. Unclearer perceptions or on, or knowledge of, the penalty – errors in justice -would be represented by an increase in s when positive, a decrease in s if negative.

(V.44) $\quad v^F(p, W, F) = (1 - q^*)\, v^F(p, W, F + R) +$

$$+ q^*[r\ v^F(p, W, F - M'^* - s) + (1 - r)\, v^F(p, W, F - M'^* + s\,\frac{r}{1-r})]\}$$

(V.45) $$\frac{\partial M'^*}{\partial s} = r\,\frac{v^F{}_F(p, W, F - M'^* + s\frac{r}{1-r}) - v^F{}_F(p, W, F - M'^* - s)}{r\, v^F{}_F(p, W, F - M'^* - s) + (1-r)\, v^F{}_F(p, W, F - M'^* + s\frac{r}{1-r})}$$

The denominator is always positive. If $s > 0$, $\frac{\partial M'^*}{\partial s} < 0$ when $v^F_{FF}(p, W, F) < 0$: risk averse individuals will be detained more easily with a more random punishment system; risk lovers criminality will be enhanced: $\frac{\partial M'^*}{\partial s} > 0$ for $s > 0$ iff $v^F_{FF}(p, W, F) > 0$.

When $s < 0$, uncertainty decreases with s. $-\frac{\partial M'^*}{\partial s} < 0$ when $v^F_{FF}(p, W, F) < 0$ and again criminality is reduced with an increase in the dispersion in M'^* for risk-averters.

Proposition 9: Arbitrariness in law enforcement reduces criminality among risk-averse individuals with respect to “full income”. It increases crime incidence of risk-lovers.

In cases 2.2, 4.2 and 5.2 we discussed situations where randomness or noise were introduced in all but the conviction state. In here, we consider the complementary case: additional randomness is only associated with apprehension.

Indeterminate sentences were discussed by Block and Lind (1975a), but they model it as the trade-off between q and M´ keeping the expected value of the penalty, q M´, constant[38]. They claim that a fixed sentence, say three-year, is more deterrent than a one to five-year sentence with an expected value of three-year length. That is only true in our model if individuals are risk-lovers with respect to full income – that means $v^F_{FF}(p, W, F) > 0$. Block and Lind assume a general form of utility decreasing and convex in punishment, M´, that is, translated in our notation $v^F_{M'M'}(p, W, F, M') > 0$; for our framework $v^F_{M'}(p, W, F, M') = -v^F_F(p, W, F - M')$ < 0, but $v^F_{M'M'}(p, W, F, M') = v^F_{FF}(p, W, F - M')$ can be positive or negative; hence their assumption on the convexity of indirect utility on the sentence, as they acknowledge themselves, is more similar to risk preference towards the sentence than to risk-aversion of the potential criminal.

[38] That approach is more adequate to infer about the relative effectiveness of sanctions and arrest rates, as argued in §4 of section III, than to answer what is the effect of uncertainty in sentences on criminal behavior. Notice that multivariate risk theory was incipient in the 70´s.

The authors justify the hypotheses on the grounds of discounting of future disutility from imprisonment [39]: the marginal disutility from sentence length, $-v_P^F$, would diminish with the length of sentence, P, rendering $-v^F_{PP} < 0$ due to individual discounting of the future. Instead, we highlight the implied pure income effect of loss of time endowment and leave the two risk-attitude alternatives free. In our framework, for a continuous and continuously differentiable utility function, $-v_P^F = v_T^F$ and $-v^F_{PP} = -v^F_{TT}$; taking away individuals time may be increasingly distasteful, rendering $-v^F_{PP} > 0$ – as giving them more time, decreasingly valuable, implying consistently $v^F_{TT} = W^2\, v^F_{FF} < 0$. This type of symmetry is partly removed in Kahneman and Tversky´s (1979) prospect theory [40] – see also Tversky and Kahneman (1991) – or Loomes and Sudgen´s (1982) regret theory [41]. (It may eventually be solved in an intertemporal optimization context.)

Some authors have observed that criminals seem to behave as risk-preferrers [42]. That is indeed the profile that we expect if we consider that a delinquent faces a lottery – under identical circumstances and homogeneous sanctions, a risk-lover would take the lottery more easily than a risk-averter. Yet, the penalty system may also be deterring risk-averters trespassing. (Distribution of) Attitudes towards risk of potential as of actual infractors should be considered in empirical assessment of law enforcement schemes.

Randomness or uncertainty about the output of the theft, R, would work in the same direction – it would discourage risk-averse individuals and would increase risk-lover criminal activities.

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[39] See also Polinsky and Shavell (1999).

[40] Nevertheless, as described, it is a problem of utility function or preferences definition and representation – present for equal right and left (second) derivatives of utility functions -, and not only an uncertainty issue. With symmetric or symmetrically signed right (incremental) and left (decremental) derivatives, v^F_{FF}, and a negative value of the "right" or incremental v^F_{FF}, the individual can be said to be "change resilient" or "change reluctant"; if positive, "change exultant".

[41] See an application to criminal behavior in Garoupa (2001).

[42] See Neilson and Winter (1997) and references therein.

VI. Some Specific Functional Forms.

This sections aims at inspecting regularities in risk attitude indicators in some commonly used functional forms.

The reader can find a thorough account of the linear supply function, the linear expenditure system and the quadratic indirect utility – even in a slightly different format - in Stern (1986). CES direct utility, inverse of translog cost function and of the generalized Leontieff cost functions were derived by solving the implied cost functions of the corresponding technologies – see Lau (1986), Nadiri (1982) and Hamermesh (1993) – in production (that here is translated to utility). Given the importance of F and T, we will use specifications that involve these arguments – i.e., primary references are $v^F(p, W, F)$ and $H^F(p, W, F, T)$.

For simplicity, p is parameterised as 1 or a free constant in most cases. (In most cases, we could recover the full formulations by replacing W for $\frac{W}{p}$ and V (or F) by $\frac{V}{p}$ ($\frac{F}{p}$) – and M by $\frac{M}{p}$.)

We will state for each of the four examples the primordial specification, the labor supply function and the indirect utility function; also the measures of absolute risk aversion; of the rate of decline of $v^F_W(p, W, F)$ with F – that is of substitutability between W and F in the F metric; and of aversion in the presence of independent risks.

Attention is given to theoretical requirements required by duality theory [43]:

1. The direct utility function, U(O,Y), must be increasing and quasi-concave in (O,Y).

2. $v^F(p, W, F)$, the "full income" indirect utility function must be homogeneous of degree 0 in (p, W, F), quasi-convex in (p, W), non-increasing in (p, W), non-decreasing in F. p equals 1; hence, we are left with the requirement that $v^F(W, F)$, is non-increasing in W and non-decreasing in F.

3. v(p, W, T, V), the standard indirect utility function must be homogeneous of degree 0 in (p, W, V), quasi-convex in (p, W) [44], non-increasing in p, non-decreasing in (W, T, V). When p is parameterised as 1, we must guarantee that v(W, T, V), is non-decreasing in (W, T, V).

4. "Full income" labor supply satisfies Roy´s identity with respect to W, $H^F(p, W, F, T) = T + \frac{v^F{}_W(p, W, F)}{v^F{}_F(p, W, F)}$; non-increasing in F (leisure is a normal good) and p, non-decreasing in W

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43 The reader is referred to standard textbooks: Varian (1992), Deaton and Muellbauer (1980), Killingswoth (1983).

44 See Pereira (1989).

(income and leisure are normal goods). $H^F(p, W, F, T)$ is homogeneous of degree 0 in (p, W, F) – as p = 1, it will not be required. Finally, $\frac{\partial H^F(p, W, F, T)}{\partial T} = 1$.

5. Positive substitution effect, using Slutsky decomposition, is satisfied if

$$\frac{\partial H(p, W, V, T)}{\partial W} - H\,\frac{\partial H(p, W, V, T)}{\partial V} > 0 \text{ that is}$$

$$\frac{\partial H^F(p, W, F, T)}{\partial W} + (T - H)\,\frac{\partial H^F(p, W, F, T)}{\partial F} > 0$$

6. $0 < \frac{\partial H(p, W, V, T)}{\partial T} < 1$, i.e., $-1 < W\,\frac{\partial H^F(p, W, F, T)}{\partial F} < 0$

A final requirement of the properties of the indirect utility function is that $v^F(p, W, F)$ is quasi-convex in (p, W). Then if $v^F(p, W, F)$ is convex in (p, W), that property is satisfied. However, being $v^F(p, W, F)$ convex in (p, W) does not guarantee convexity of v(p, W, V, T); notice that:

$$\text{(VI.1)} \qquad v_w(p, W, V, T) = v^F_w(p, W, W\,T + V) + T\,v^F_F(p, W, W\,T + V) > 0$$

and

$$\text{(VI.2)} \qquad v_{ww}(p, W, V, T) = v^F_{ww}(p, W, W\,T + V) + 2\,T\,v^F_{Fw}(p, W, W\,T + V) + T^2\,v^F_{FF}(p, W, W\,T + V)$$

Risk-aversion to full income - that is, $v^F_{FF}(p, W, W\,T + V) < 0$ - and/or substitutability between F and W - $v^F_{Fw}(p, W, W\,T + V) < 0$ - renders concavity of v(p, W, V, T) in W more likely.

Our interest lies on two subjects:

1. the features of the risk measures advanced in sections III and IV, which condition the sign of the impact of changes in the level and uncertainty in the endowments on the deterrent sanction.

2. how expected labor supply of the marginal delinquent compares with that of a non-offender.

3. how labor supply of a convict compares to the labor supply of a non-criminal and an escapee. This determines how labor supply reacts to a rise in the probability of detection.

A. Linear Supply Function

The general linear supply function is given by:

(VI.3) $H^F = T + \alpha W + \beta F + \gamma = T + (\alpha + \beta T) W + \beta V + \gamma$

$\beta < 0$; $\alpha > 0$ (leisure is not an inferior good);

$1 + \beta W > 0$ (H increases with T); $H < T + \dfrac{\alpha}{\beta}$ (Slutsky decomposition)

The corresponding indirect utility will be:

(VI.4) $$v^F(W, F) = \exp(\beta W) \left(F + \frac{\alpha}{\beta} W - \frac{\alpha}{\beta^2} + \frac{\gamma}{\beta}\right) =$$

$$= \exp(\beta W) \left[V + \left(\frac{\alpha}{\beta} + T\right) W - \frac{\alpha}{\beta^2} + \frac{\gamma}{\beta}\right] = v(W, V, T)$$

The Arrow-Pratt measure of risk aversion exhibited by $v^F(W, F)$ with respect to F:

$$\rho^F(p, W, F) = -\frac{v^F{}_{FF}(W, F)}{v^F{}_F(W, F)} = 0$$

Always, $v^F(p, W, F)$ is linear in F: v exhibits constant risk aversion with respect to F. (III.16) holds.

The measure of the rate of decline of $v^F{}_W(p, W, F)$ with F is positive and independent of F:

$$\kappa^F(p, W, F) = -\frac{v^F{}_{WF}(W, F)}{v^F{}_F(W, F)} = -\beta > 0$$

$$\kappa(p, W, V, T) = -\frac{v_{WV}(W, V, T)}{v_V(W, V, T)} = -\beta > 0$$

$\kappa(p, W, V, T) > 0$ and there will be substitutability between V and W in indirect utility. The measures of risk aversion to other noises:

$$N^F(p, W, F) = - \frac{v^F{}_{FFF}(W, F)}{v^F{}_F(W, F)} = 0$$

$$Z(p, W, V, T) = - \frac{v_{VWW}(W, V, T)}{v_V(W, V, T)} = - \beta^2 < 0$$

$N^F(p, W, F)$ is zero and $Z(p, W, V, T)$ independent of V: the consumer does not respond to independent noises in either V or W in what concerns his criminal behavior or decisions.

At P*, expected labor supply will be:

$$E(H) = T + \alpha W + \beta F + \gamma - q P^*$$

At $(1-q) F^1 + q F^{2*} = F$, expected labor supply of the trespasser will be necessarily smaller than the certain labor response by the amount q P*.

$$H^F(p, W, F^1, T) < H^F(p, W, F^{2*}, T - P^*) \quad \text{iff} \; - \beta(F^1 - F^{2*}) > P^*$$

That is, iff, $- \beta(R + M^* + W P^*) > P^* = \frac{(1 - q^*) R - q^* M^*}{q^* W}$ which is equivalent to:

$$- \beta R > q^* P^*$$

That is only if either q* or P* are very small, or R or the income effect on labor supply are large.

B. Linear Expenditure System (and Cobb-Douglas Direct Utility)

We depart from the direct utility function:

$$\text{(VI.5)} \qquad U(O, Y) = A (Y - \bar{Y})^{a_1} O^{a_2}$$
$$a_1, a_2 > 0; \quad a_1, a_2 < 1$$

The implicit labor supply is:

(VI.6) $$H = T - a_2 \frac{F - \bar{Y}}{W(a_1 + a_2)} = \frac{a_1}{a_1 + a_2} T - a_2 \frac{V - \bar{Y}}{W(a_1 + a_2)}$$

H is linear in F; increasing and concave in W.

(VI.7) $$v^F(W, F) = A \; a_1^{a_1} a_2^{a_2} (a_1 + a_2)^{-(a_1+a_2)} (F - \bar{Y})^{(a_1+a_2)} W^{-a_2}$$

$v^F_{ww}(W, F) = a_2 (a_2 + 1) v^F(W, F) / W^2 > 0$ and so, $v^F(W, F)$ is always convex in W. One can show that

$$v_{ww}(W, V, T) = \frac{v(W, V, T)}{(F - \bar{Y})^2 W^2}$$

$$[(a_1 - 1) a_1 (W T)^2 + (a_2 + 1) a_2 (V - \bar{Y})^2 + 2 (1 - a_1) a_2 W T (V - \bar{Y})]$$

For $v_w(W, V, T) > 0$, $a_2 (V - \bar{Y}) < a_1 W T$; then, if $V = \bar{Y}$, $v_{ww}(W, V, T) < 0$, and $v(W, V, T)$ is concave in W; for $V > \bar{Y}$, convexity may occur.

The Arrow-Pratt measure of risk aversion exhibited by $v^F(W, F)$ with respect to F:

$$\rho^F(p, W, F) = - \frac{v^F{}_{FF}(W,F)}{v^F{}_F(W,F)} = \frac{1 - a_1 - a_2}{F - \bar{Y}}$$

It can be positive or negative and has the sign of $(1 - a_1 - a_2)$. It decreases with F if individuals are risk averse; it increases with F if individuals are risk lovers. (Note that if $\bar{Y} = 0$, v^F exhibits constant relative degree of risk aversion, a measure given by F $\rho(p, W, F)$; as is well-known, constant relative risk aversion implies (linearly) decreasing absolute risk aversion if the measure is positive and individuals are risk averse; increasing absolute risk aversion if the measure is negative.)

$\rho^F(p, W, F)$ increases with $\bar{Y}$ - usually interpreted as the subsistence level of income - if the direct utility function is concave, i.e., $1 - a_1 - a_2 > 0$, i.e., if consumers are risk averse with respect to full income. Then M*, hence, criminality decreases when subsistence standards rise in the economy. The reverse happens if they are risk -lovers.

The measure of the rate of decline of $v^F_w(p, W, F)$ with F does not respond to F, but decreases with W:

$$\kappa^F(p, W, F) = -\frac{v^F{}_{WF}(W,F)}{v^F{}_F(W,F)} = \frac{a_2}{W}$$

$$\kappa(p, W, V, T) = -\frac{v_{WV}(W, V, T)}{v_V(W, V, T)} = \frac{a_2}{W} + T\,\frac{1 - a_1 - a_2}{W\,T + V - \bar{Y}} =$$

$$= \frac{(1 - a_1)\,W\,T + a_2\,(V - \bar{Y})}{W\,(W\,T + V - \bar{Y})}$$

Then, $\frac{\partial \kappa(p, W, V, T)}{\partial V} < 0$ for risk -averse individuals – M* increases with the wage level. For risk -lovers, the opposite occurs. In any case, $\kappa(p, W, V, T) > 0$ and there will be substitutability between V and W in indirec t utility provided $a_2\,(V - \bar{Y})$ is small.

$$N^F(p, W, F) = -\frac{v^F{}_{FFF}(W, F)}{v^F{}_F(W, F)} = \frac{(2 - a_1 - a_2)(a_1 + a_2 - 1)}{(F - \bar{Y})^2}$$

$$Z(p, W, V, T) = -\frac{v_{VWW}(W, V, T)}{v_V(W, V, T)} =$$

$$= (a_1 + a_2 - 1)\left[\frac{a_2}{(W\,T + V - \bar{Y})\,W} + \frac{(2 - a_1 - a_2)\,T}{(W\,T + V - \bar{Y})^2}\right]T - \frac{a_2\,(a_2 + 1)}{W^2}$$

$N^F(p, W, F)$ increases with F if there is risk aversion with respect to full income, i.e., if $a_1 + a_2 < 1$; in that case, Z(p, W, V, T) increases with V. If $a_1 + a_2 > 1$ (however, $a_1 + a_2 < 2$ always), the opposite occurs.

M´* = W P* + M* solv es

$$(1 - q^*)\,(F^1 - \bar{Y})^{(a_1+a_2)} + q^*\,(F^{2*} - \bar{Y})^{(a_1+a_2)} = (F - \bar{Y})^{(a_1+a_2)}$$

In this case, if $a_1 + a_2 < 1$, U(Y,O) is concave and v is concave in F; $a_1 + a_2 > 1$, we observe convexity. Hence previous considerations apply: if $a_1 + a_2 < 1$, $(1 - q^*)\,R > q^*\,(W\,P^* + M^*)$, otherwise the inequality is reversed.

$$\frac{\partial M^*}{\partial V} = [(1\text{-}q^*)\,(F^1 - \bar{Y})^{(a_1+a_2-1)}\,q^*\,(F^{2*} - \bar{Y})^{(a_1+a_2-1)} - (F - \bar{Y})^{(a_1+a_2-1)}]\,/$$

$$/\,[q^*\,(F^{2*} - \bar{Y})^{(a_1+a_2-1)}]$$

If $\bar{Y} = 0$, v^F exhibits constant degree of relative risk aversion which implies (linearly) decreasing absolute risk aversion if the measure is positive and individuals are risk averse; increasing absolute risk aversion if the measure is negative. In the first case, M* increases with V. For risk lovers M* decreases with V.

$$\frac{\partial M^*}{\partial W} = T\,\frac{\partial M^*}{\partial V} - P^*$$

Expected labor supply of an individual that commits the crime is given by:

$$E(H) = T - a_2\,\frac{(1-q^*)\,F^1 + q^*F^{2*} - \bar{Y}}{W\,(a_1 + a_2)} - q^*\,P^*$$

If the individual is risk averse, i.e., $(a_1 + a_2) < 1$, $(1\text{-}q^*)\,F^1 + q^*\,F^{2*} > F$ and E(H) is necessarily smaller than if he did not commit the crime. If individuals are risk lovers $(a_1 + a_2) > 1$, $(1\text{-}q^*)\,F^1 + q^*\,F^{2*} < F$; expected labor supply of a criminal can be larger than the labor supply of a non criminal if P* is sufficiently low.

$$H^F(p, W, F^1, T) < H^F(p, W, F^{2*}, T - P^*) \quad \text{iff} \quad \frac{a_2}{W\,(a_1 + a_2)}\,(F^1 - F^{2*}) > P^*$$

That is, iff, $\dfrac{a_2}{W\,(a_1 + a_2)}\,(R + M^* + W\,P^*) > P^*$ which is equivalent to:

$$a_2 (R + M^*) > a_1 \; W \; P^*$$

That is only if either (W x) P* is small, or R or the income effect on labor supply, related to $\frac{a_2}{W\,(a_1 + a_2)}$ are large, labor supply of a convict is larger than of an escapee.

2. If we define U in the logar ithms:

$$\text{(VI.8)} \qquad U(O, Y) = A + a_1 \log (Y - \bar{Y}) + a_2 \log (O)$$
$$a_1, a_2 > 0$$

labor supply (VI.4) stands; the indirect utility function will be:

$$\text{(VI.9)} \qquad v^F(p, W, F) = A + \log [a_1^{a_1} a_2^{a_2} (a_1 + a_2)^{-(a_1+a_2)}] +$$
$$+ (a_1 + a_2) \log(F - \bar{Y}) - a_2 \log(W)$$

$v^F_{ww}(W, F) = \frac{a_2}{W^2} > 0$ and so, $v^F(W, F)$ is always convex in W . One can show that

$$v_{ww}(W, V, T) = \frac{1}{(F - \bar{Y})^2 W^2} [a_2 (W T + V - \bar{Y})^2 - (a_1 + a_2) (W T)^2]$$

For $v_w(W, V, T) > 0$, $a_2 (V - \bar{Y}) < a_1 W T$; then, if $V = \bar{Y}$, $v_{ww}(W, V, T) < 0$, and v(W, V, T) is concave in W; for $V > \bar{Y}$, convexity may occur, provided a_1 is large.

Risk aversion is always positive, decreasing with F and (directly) independent of W:

$$\rho^F(p, W, F) = - \frac{v^F_{FF}(W, F)}{v^F_F(W, F)} = \frac{1}{F - \bar{Y}}$$

$$\kappa^F(p, W, F) = -\frac{v^F{}_{WF}(W, F)}{v^F{}_F(W, F)} = 0$$

$$\kappa(p, W, V, T) = -\frac{v_{WV}(W, V, T)}{v_V(W, V, T)} = \frac{T}{W\,T + V - \bar{Y}}$$

$\kappa(p, W, V, T) > 0$ and there will be substitutability between V and W in indirect utility. $\frac{\partial \kappa(p, W, V, T)}{\partial V} < 0$ always.

$$N^F(p, W, F) = -\frac{v^F{}_{FFF}(W, F)}{v^F{}_F(W, F)} = -\frac{2}{(F - \bar{Y})^2}$$

$$Z(p, W, V, T) = -\frac{v_{VWW}(W, V, T)}{v_V(W, V, T)} = -\frac{2\,T^2}{(W\,T + V - \bar{Y})^2}$$

$N^F(p, W, F)$ increases with F; Z(p, W, V, T) with V. M′* solves

$$(1 - q^*) \log(F^1 - \bar{Y}) + q^* \log(F^{2*} - \bar{Y}) = \log(F - \bar{Y})$$

The left hand-side defines the logarithm of the weighted geometric mean of $F^1 - \bar{Y}$ and $F^{2*} - \bar{Y}$. In this case, as the arithmetic mean is always larger than the geometric mean, for F^1 and F^{2*} that satisfy the equation, $(1 - q^*) (F^1 - \bar{Y}) + q^* (F^{2*} - \bar{Y}) > \exp[(1 - q^*) \log(F^1 - \bar{Y}) + q^* \log(F^{2*} - \bar{Y})] = (F - \bar{Y})$. (Alternatively, $\log(F - \bar{Y})$ is concave in F.) M′* implies that P* is always smaller than (III.20).

$$\frac{\partial M^*}{\partial V} = [(1-q^*)\,(F^1 - \bar{Y})^{(-1)} + q^*\,(F^{2*} - \bar{Y})^{(-1)} - (F - \bar{Y})^{(-1)}] \,/\, q^*\,(F^{2*} - \bar{Y})^{(-1)}$$

The inverse of the harmonic mean i s always larger than the inverse of the geometric mean; hence, the numerator is always positive. (Or the measure of absolute risk aversion, $\frac{1}{F - \overline{Y}}$, is decreasing with income), hence $\frac{\partial M^*}{\partial V} > 0$ always.

$$\frac{\partial M^*}{\partial W} = T \frac{\partial M^*}{\partial V} - P^*$$

If P* is 0, $\frac{\partial M^*}{\partial W} > 0$. It is possible that the sign reverses when M* = 0. We can see whether

$$[(1\text{-}q^*)\,(F^1 - \overline{Y})^{(-1)} + q^*\,(F^{2*} - \overline{Y})^{(-1)} - (F - \overline{Y})^{(-1)}]\,T > P^*\,q^*\,(F^{2*} - \overline{Y})^{(-1)}$$

always – then, $\frac{\partial M^*}{\partial W} > 0$. The expression is equivalent to inquire whether:

$$T\,\{[q^*\,(W\,P^* + M^*) - (1 - q^*)\,R]\,(F - \overline{Y}) + R\,(W\,P^* + M^*)\} >$$
$$> q^*\,P^*\,(F^1 - \overline{Y})\,(F - \overline{Y})$$

We could not rule out that, for high enough P*, the reverse could not occur.

Expected labor supply is the same as before, linear in expected "full income", but we only observe risk averse cases, i.e., $(1\text{-}q^*)\,F^1 + q^*\,F^{2*} > F$: expected labor supply is necessarily smaller than if the individual did not commit the crime.

C. Quadratic Indirect Utility

(VI.10) $v^F(W, F) = \alpha\,(F)^2 + \beta\,W^2 + \gamma\,F\,W + \delta\,W + \varepsilon\,F + \mu$

$2\,\alpha\,F + \gamma\,W + \varepsilon > 0$; with $2\,\beta\,W + \gamma\,F + \delta < 0$.

Convexity of $v^F(W, F)$ in W requires $\beta > 0$. In that case,

$$v_{ww}(W, V, T) = 2\,(\alpha\,T^2 + \beta + \gamma\,T)$$

It can be larger or smaller than 0.

If $\alpha < 0$, $v^F(W, F)$ is concave in F and individuals are risk averse with respect to full income; if $\alpha > 0$, $v^F(W, F)$ is convex in F: they are risk lovers; in any case, risk aversion always increases with F:

$$\rho^F(p, W, F) = -\frac{v^F{}_{FF}(W,F)}{v^F{}_F(W,F)} = -\frac{2\alpha}{2\alpha F + \gamma W + \varepsilon}$$

The rate of decline of $v^F_W(p, W, F)$ with F will have the sign of $-\gamma$. It increases with F if $\alpha\gamma$ is positive, it decreases if $\alpha\gamma$ is negative:

$$\kappa^F(p, W, F) = -\frac{v^F{}_{WF}(W, F)}{v^F{}_F(W, F)} = -\frac{\gamma}{2\alpha F + \gamma W + \varepsilon}$$

$$\kappa(p, W, V, T) = -\frac{v_{WV}(W, V, T)}{v_V(W, V, T)} = -\frac{\gamma}{2\alpha F + \gamma W + \varepsilon} - T\frac{2\alpha}{2\alpha F + \gamma W + \varepsilon}$$

If $2\alpha(2\alpha T + \gamma) > 0$, $\dfrac{\partial\kappa(p, W, V, T)}{\partial V} > 0$.

$$N^F(p, W, F) = -\frac{v^F{}_{FFF}(W, F)}{v^F{}_F(W, F)} = 0$$

$$Z(p, W, V, T) = -\frac{v_{VWW}(W, V, T)}{v_V(W, V, T)} = 0$$

$N^F(p, W, F)$ and $Z(p, W, V, T)$ are zero: the consumer does not respond to independent noises in V or in W in what concerns his criminal behavior or decisions.

(VI.11) $H = T + (2\beta W + \gamma F + \delta) / (2\alpha F + \gamma W + \varepsilon) =$
$= T + [(2\beta + \gamma T) W + \gamma V + \delta] / [2\alpha V + (2\alpha T + \gamma) W + \varepsilon]$

$$\frac{\partial H^F}{\partial W} = [2\beta(2\alpha F + \varepsilon) - \gamma(\gamma F + \delta)] / (2\alpha F + \gamma W + \varepsilon)^2 > 0$$

$$\frac{\partial^2 H^F}{\partial W^2} = - \frac{2\gamma}{2\alpha F + \gamma W + \varepsilon} \frac{\partial H^F}{\partial W}$$

If $\gamma > 0$, $H^F(p, W, F)$ is concave in W. If $\gamma < 0$, it is convex. (However, this does not refer to the convexity of H(p, W, V, T) in V…)

$$\frac{\partial H^F}{\partial F} = [\gamma (\gamma W + \varepsilon) - 2 \alpha (2 \beta W + \delta)] / (2 \alpha F + \gamma W + \varepsilon)^2 < 0$$

$$\frac{\partial^2 H^F}{\partial F^2} = - \frac{4\alpha}{2\alpha F + \gamma W + \varepsilon} \frac{\partial H^F}{\partial F}$$

If $\alpha > 0$, H^F is convex in F (and H(p, W, V, T) in V). If $\alpha < 0$, it is concave.
For the marginal criminal:

$$(1 - q^*) [\alpha (F^1)^2 + (\gamma W + \varepsilon) F^1] + q^* [\alpha (F^{2*})^2 + (\gamma W + \varepsilon) F^{2*}] =$$
$$= [\alpha (F)^2 + (\gamma W + \varepsilon) F]$$

The measure of absol ute risk aversion with respect to F is always increasing in F: $\frac{\partial M^*}{\partial V} < 0$:

$$\frac{\partial M^*}{\partial V} = [(1-q^*) (2 \alpha F^1 + \gamma W + \varepsilon) + q^* (2 \alpha F^{2*} + \gamma W + \varepsilon) - (2 \alpha F + \gamma W + \varepsilon)] /$$
$$/ [q^* (2\alpha F^{2*} + \gamma W + \varepsilon)] = 2\alpha [(1 - q^*) F^1 + q^* F^{2*} - F] / [q^* (2\alpha F^{2*} + \gamma W + \varepsilon)]$$

If $\alpha < 0$, risk averters, $[(1 - q^*) F^1 + q^* F^{2*} - F] > 0$; if $\alpha > 0$, risk lovers, $[(1 - q^*) F^1 + q^* F^{2*} - F] < 0$.

$$\frac{\partial M^*}{\partial W} = T \frac{\partial M^*}{\partial V} - P^* +$$
$$+ [(1 - q^*) (2 \beta W + \gamma F^1 + \delta) + q^* (2 \beta W + \gamma F^{2*} + \delta) - (2 \beta W + \gamma F + \delta)] /$$
$$/ [q^* (2 \alpha F^{2*} + \gamma W + \delta)] =$$
$$= T \frac{\partial M^*}{\partial V} - P^* + \gamma [(1 - q^*) F^1 + q^* F^{2*} - F] / [q^* (2 \alpha F^{2*} + \gamma W + \varepsilon)] =$$
$$= (2 \alpha T + \gamma) [(1 - q^*) F^1 + q^* F^{2*} - F] / [q^* (2 \alpha F^{2*} + \gamma W + \varepsilon)] - P^*$$

It is compatible with the above reasoning on the measure κ(p, W, V, T): iff $2\alpha(2\alpha T + \gamma) > 0$, $\frac{\partial \kappa(p, W, V, T)}{\partial V} > 0$ and, as $[(1 - q^*) F^1 + q^* F^{2*} - F]$ has the opposite sign to α, $\frac{\partial M^*}{\partial W} < 0$. When the reverse occurs, we have an extra negative effect coming from P*.

Expected labour supply is quite messy.

D. CES Utility Function

$$\text{(VI.12)} \qquad U(O, Y) = A\,[a_1 O^{(\sigma-1)/\sigma} + (1 - a_1) Y^{(\sigma-1)/\sigma}]^{\mu\sigma/(\sigma-1)}$$
$$0 < a_1 < 1\,;\ \sigma, \mu > 0$$

$$\text{(VI.13)} \qquad v^F(p, W, F) = A\,F^{\mu}\,[a_1^{\sigma} W^{(1-\sigma)} + (1 - a_1)^{\sigma} p^{(1-\sigma)}]^{\mu/(\sigma-1)}$$
$$0 < a_1 < 1\,;\ \sigma, \mu > 0$$

ρ^F(p, W, F) exhibits the usual pattern of homogeneous utility functions, positive and decreasing with "full income" in the presence of decreasing returns to scale in the direct utility function, that is, when $\mu < 1$; negative and increasing with F with increasing returns to scale.

$$\rho^F(p, W, F) = -\frac{v^F{}_{FF}(W, F)}{v^F{}_F(W, F)} = \frac{1 - \mu}{F}$$

κ^F(p, W, F) is always positive, independent of F and decreasing in the wage rate.

$$\kappa^F(p, W, F) = -\frac{v^F{}_{WF}(W, F)}{v^F{}_F(W, F)} = \frac{\mu\, a_1^{\sigma}}{a_1^{\sigma}\, W + (1 - a_1)^{\sigma}\, p^{1-\sigma}\, W^{\sigma}}$$

κ(p, W, V, T), for risk-averse individuals decreases with V. The opposite occurs for risk lovers.

$$\kappa(p, W, V, T) = -\frac{v_{WV}(W, V, T)}{v_V(W, V, T)} = \frac{\mu\, a_1^{\sigma}}{a_1^{\sigma}\, W + (1 - a_1)^{\sigma}\, p^{1-\sigma}\, W^{\sigma}} + T\,\frac{1 - \mu}{W\,T + V}$$

For risk-averse individuals, κ(p, W, V, T) > 0, but that is not a necessary condition.

$$N^F(p, W, F) = -\frac{v^F{}_{FFF}(W, F)}{v^F{}_F(W, F)} = \frac{(2-\mu)(\mu-1)}{F^2}$$

$$Z(p, W, V, T) = -\frac{v_{VWW}(W, V, T)}{v_V(W, V, T)} = \frac{(\mu-1)}{W\,T+V}$$

$$\left[\frac{(2-\mu)\,T}{W\,T+V} + \frac{\mu\, a_1{}^{\sigma}}{a_1{}^{\sigma}\, W + (1-a_1)^{\sigma}\, p^{1-\sigma}\, W^{\sigma}}\right] T -$$

$$- G(W,p)$$

where G(W, p) denotes a function independent of V or T.

$N^F(p, W, F)$ increases with F if there is risk aversion with respect to full income, i.e., if $\mu < 1$, or if $\mu > 2$; if $1 < \mu < 2$, $N^F(p, W, F)$ decreases with F. If $\mu < 1$, Z(p, W, V, T) increases with V.

Labor supply is positively sloped and linear in F:

$$\text{(VI.14)} \qquad H = T - F\, \frac{a_1{}^{\sigma}}{a_1{}^{\sigma}\, W + (1-a_1)^{\sigma}\, p^{1-\sigma}\, W^{\sigma}}$$

For the marginal criminal, expected labor supply is:

$$E(H) = T - [(1-q^*)\, F^1 + q^*\, F^{2*}]\, \frac{a_1{}^{\sigma}}{a_1{}^{\sigma}\, W + (1-a_1)^{\sigma}\, p^{1-\sigma}\, W^{\sigma}} - q\, P^*$$

Due to homogeneity of the indirect u tility function, risk aversion determines the possibility of a higher expected full income of the marginal criminal than the law -abiding citizen. Risk averse individuals have lower expected labor supply when committing the marginal crime; risk lovers will exhibit the opposite pattern, provided imprisonment is small.

E. Translog Indirect Utility

$$\text{(VI.15)} \quad v^F(W, F) = \alpha\,(\log F)^2 + \beta\,(\log W)^2 + \gamma \log F \log W + \delta \log W + \varepsilon \log F + \mu$$

$$2\,\alpha \log F + \gamma \log W + \varepsilon > 0\;;\; 2\,\beta \log W + \gamma \log F + \delta < 0.$$

$$\rho^F(p, W, F) = -\frac{v^F{}_{FF}(W, F)}{v^F{}_F(W, F)} = (2\,\alpha \log F + \gamma \log W + \varepsilon - 2\,\alpha)/$$

$$/ [F (2 \alpha \log F + \gamma \log W + \varepsilon)]$$

The rate of decline of $v^F_w(p, W, F)$ with F will have the sign of $-\gamma$. It increases with F if $\alpha\gamma$ is positive, it decreases if $\alpha\gamma$ is negative:

$$\kappa^F(p, W, F) = - \frac{v^F{}_{WF}(W,F)}{v^F{}_F(W,F)} = - \gamma / [W (2 \alpha \log F + \gamma \log W + \varepsilon)]$$

$$\kappa(p, W, V, T) = - \frac{v_{WV}(W, V, T)}{v_V(W, V, T)} =$$
$$= \{W\, T\, [2 \alpha \log(W\, T + V) + \gamma \log W + \varepsilon - 2\alpha] - \gamma (W\, T + V)\} /$$
$$/ \{W (W\, T + V) [2 \alpha \log(W\, T + V) + \gamma \log W + \varepsilon]\}$$

$$N^F(p, W, F) = - \frac{v^F{}_{FFF}(W, F)}{v^F{}_F(W, F)} = - \frac{2}{F^2} [\alpha (2 \log F - 3) + \gamma \log W + \varepsilon] /$$
$$/ (2 \alpha \log F + \gamma \log W + \varepsilon)$$

$$Z(p, W, V, T) = - \frac{v_{VWW}(W, V, T)}{v_V(W, V, T)} =$$
$$= - \frac{1}{F^2} \{[2 \alpha (2 \log F - 3) + 2 \gamma \log W + 2 \varepsilon] T - \gamma F/W\} T /$$
$$/ (2 \alpha \log F + \gamma \log W + \varepsilon) +$$
$$+ \gamma / [W^2 (2 \alpha \log F + \gamma \log W + \varepsilon)]$$

(VI.16) $H = T + [F (2 \beta \log W + \gamma \log F + \delta)] / [W (2 \alpha \log F + \gamma \log W + \varepsilon)] =$
$= T + F [2 \beta \log W + \gamma \log(V + WT) + \delta] / \{W [2 \alpha \log(V + WT) + \gamma \log W + \varepsilon]\}$

F. Inverse of Translog Cost Function

(VI.17) $$\log v^F(p, W, F) = - [- \log F + \alpha_1 \log W + \alpha_2 \log p + \frac{\beta_{11}}{2} (\log W)^2 +$$
$$+ \beta_{12} \log p \log W + \frac{\beta_{22}}{2} (\log p)^2 + \mu]\, \lambda$$

Imposing, for homogeneity in (W, p) of the expenditure function

$$\alpha_1 + \alpha_2 = 1 \ ; \ \beta_{11} + \beta_{12} = 0 \ ; \ \beta_{12} + \beta_{22} = 0$$

(VI.18) $$\log v^F(p, W, F) = \lambda \log F - \lambda\, [\alpha_1 \log W + (1 - \alpha_1) \log p + \frac{\beta_{11}}{2} (\log W)^2 - \beta_{11} \log p \log W + \frac{\beta_{11}}{2} (\log p)^2 + \mu]$$

$$\alpha_1 + \beta_{11} \log W - \beta_{11} \log p > 0 \ ; \ \lambda > 0$$

which leads us to (if p is fixed)

(VI.19) $$v^F(p, W, F) = F^{\lambda} \exp\{- \lambda\, [\delta_1 \log W + \frac{\beta_{11}}{2} (\log W)^2 + \varepsilon]\}$$

$$\delta_1 + \beta_{11} \log W > 0 \ ; \ \lambda > 0$$

λ is associated to the returns to scale in the utility function.

$$\rho^F(p, W, F) = - \frac{v^F_{FF}(W, F)}{v^F_F(W, F)} = \frac{(1 - \lambda)}{F}$$

$$\kappa^F(p, W, F) = - \frac{v^F_{WF}(W, F)}{v^F_F(W, F)} = \lambda\, (\delta_1 \frac{1}{W} + \beta_{11} \frac{\log W}{W})$$

$$\kappa(p,W,V,T) = - \frac{v_{WV}(W, V, T)}{v_V(W, V, T)} = \lambda\, (\delta_1 \frac{1}{W} + \beta_{11} \frac{\log W}{W}) + T\, \frac{(1 - \lambda)}{W\,T + V}$$

For risk-averse individuals, $\kappa(p, W, V, T) > 0$, but that is not a necessary condition.

$$N^F(p, W, F) = - \frac{v^F_{FFF}(W, F)}{v^F_F(W, F)} = \frac{(2 - \lambda)(\lambda - 1)}{F^2}$$

$$Z(p, W, V, T) = - \frac{v_{VWW}(W, V, T)}{v_V(W, V, T)} = \frac{(\lambda - 1)}{W\,T + V} \left[\frac{(2 - \lambda)\, T}{W\,T + V} + \lambda\, (\delta_1 \frac{1}{W} + \beta_{11} \frac{\log W}{W})\right] T - G(W,p)$$

where G(W, p) denotes a function independent of V or T.

(VI.20) $$H = T - F(\delta_1 \frac{1}{W} + \beta_{11} \frac{\log W}{W})$$

G. Inverse of Generalized Leontieff (Diewert) Cost Function

(VI.21) $$v^F(p, W, F) = F^{\lambda} (\alpha W + 2 \gamma W^{1/2} p^{1/2} + \delta p)^{-\lambda}$$
$$\alpha + \gamma W^{-1/2} > 0 ; \ \delta + \gamma p^{-1/2} > 0 ; \gamma > 0 ; \ \lambda > 0$$

(VI.22) $$v^F(W, F) = F^{\lambda} (\alpha W + 2 \mu W^{1/2} + \varepsilon)^{-\lambda}$$
$$\alpha + \mu W^{-1/2} > 0 \ ; \ \mu > 0$$

$$\rho^F(p, W, F) = - \frac{v^F{}_{FF}(W, F)}{v^F{}_F(W, F)} = \frac{(1 - \lambda)}{F}$$

$$\kappa^F(p, W, F) = - \frac{v^F{}_{WF}(W, F)}{v^F{}_F(W, F)} = \frac{\lambda(\alpha + \mu W^{-1/2})}{\alpha + 2 \mu W^{1/2} + \varepsilon}$$

$$\kappa(p, W, V, T) = - \frac{v_{WV}(W, V, T)}{v_V(W, V, T)} = \frac{\lambda(\alpha + \mu W^{-1/2})}{\alpha + 2 \mu W^{1/2} + \varepsilon} + \frac{(1 - \lambda) T}{W T + V}$$

For risk-averse individuals, $\kappa(p, W, V, T) > 0$, but that is not a necessary condition.

$$N^F(p, W, F) = - \frac{v^F{}_{FFF}(W, F)}{v^F{}_F(W, F)} = \frac{(2 - \lambda)(\lambda - 1)}{F^2}$$

$$Z(p, W, V, T) = - \frac{v_{VWW}(W, V, T)}{v_V(W, V, T)} = \frac{(\lambda - 1)}{W T + V}$$
$$[\frac{(2 - \lambda) T}{W T + V} + \frac{\lambda(\alpha + \mu W^{-1/2})}{\alpha + 2 \mu W^{1/2} + \varepsilon}] T - G(W,p)$$

where $G(W, p)$ denotes a function independent of V or T.

(VI.23) $$H = T - F \frac{\alpha + \mu W^{-1/2}}{\alpha + 2 \mu W^{1/2} + \varepsilon}$$

H. Final Note

The functional forms are related to W and F rather than W and V. With individual data on W and V, T may be estimable [45] – or may be indexed to and controllable for a function of individual characteristics. Linearity of derivable estimable forms, however, may not exist.

Homogeneous direct utility functions exhibit risk aversion to F according to the degree of homogeneity: risk averse if smaller than one – in which case there is increasing risk aversion to the presence of background noise in non-labor income, and the risk premium decreases (M* increases) with nonlabor earnings and with the wage rate. Individuals will be risk lovers with respect to full income if the degree of homogeneity is larger than one; provided it is not larger than two, individuals decrease risk aversion as background noise rises; the risk premium increases (M* decreases) with nonlabor earnings and with the wage rate.

For homogeneous direct utility functions, there will be substitutability between V and W if (but not only if) individuals are risk-averse with respect to full income – if the degree of homogeneity is smaller than 1.

Also, homogeneous direct utility functions originate full income labor supplies linear in full income.

VII. Other Applications.

Technically, the problem was applied to the study of criminal behavior. However, it can equally be applied to the study of other situations.

1. Consider that the criminal can evade detection at a certain cost S. Then the maximum S he is willing to pay for "full insurance" is S* when R = R′ in the balance equation:

$$\text{(VII.1)} \qquad v^F(p, W, W\,T + V + R' - S^*) = $$
$$= (1-q)\, v^F(p, W, W\,T + V + R) + q\, v^F[p, W, W\,(T - P) + V - M]$$

If we consider that the criminal was caught and is going to trial, if q is the probability of conviction, S* will be the maximum fees he is willing to pay for his release, e.g., bribery, fees of a successful lawyer. Or the maximum out-of-courtroom settlement the defendant is willing to pay (R

[45] Eventually, lifetime data would be required – and hours referred to lifetime hours. Or a life-cycle model specified. See footnote 24.

$= R' > 0$ if he is guilty and keeps the full loot, R, in case of no trial...) to not to go to trial – or the maximum "plea bargain" he will accept [46]. If an individual is risk neutral to F:

$$\text{(VII.2)} \qquad S^* = [R' - (1 - q) R] + q (W P + M)$$

$$\text{(VII.3)} \qquad \text{If } R = R', \qquad S^* = q (R + W P + M)$$

If he is risk averse, he will accept (up to) a larger certain penalty S* than implied by (VII.2) or (VII.3) to avoid going to trial. If he is risk lover, he must be offered a lower value (than the expected value of going to full trial) – leniency is required for risk-lovers.

As in the problem of the previous sections, S* may be independent of V – it will for risk neutral individuals with respect to full income; in that case, it will be positively related to the individual´s wage rate. S* will increase with V if risk aversion increases with F; if it decreases, S* decreases with V - S* has a standard risk-premium interpretation when $R' = 0$. In the second case however, it is still possible that S* increases with W provided a prison sentence is involved.

The results of the previous sections apply to the general trespasser faced with a mechanically reliable punishment system. (Not surprisingly,) the conclusions are totally reversed if we consider the willingness to bribe the executioners. Judicial and police corruption is thus expected to respond to environmental changes – endowment levels or uncertainty - in the opposite direction of conventional delinquency.

2.1. Another application is to the participation in hazardous professions. Suppose M = 0. An individual has the opportunity to work for wage W with no risk. He can engage in a risky activity that reduces his available time by P, say, due to injury, with probability q; to accept it, he must receive compensation. Suppose he receives a composite of fixed premium D and hourly subsidy s to accept it. The general problem above will apply with:

$$\text{(VII.4)} \qquad F^1 = (W + s)\, T + V + D \quad \text{and} \quad F^2 = (W + s)\,(T - P) + V + D$$

The required compensation scheme for the individual to accept the risk involves s* and D* such that:

---

[46] Block and Lind (1975a) formulate plea bargaining in these terms. As they assume convexity of preferences over punishment, they arrive at a required bargained plea smaller than expected value of the sentence.

$$(VII.5)\qquad (1 - q)\ v^F[p, W + s^*, (W + s^*) T + V + D^*] + \\ + q\ v^F[p, W + s^*, (W + s^*) (T - P) + V + D^*] = \\ = v^F(p, W, W T + V)$$

If $s^* = 0$, and we study a constant compensation, conclusions about $\frac{\partial D^*}{\partial V}$ or $\frac{\partial D^*}{\partial W}$ will be symmetric to those on $\frac{\partial M^*}{\partial V}$ and $\frac{\partial M^*}{\partial W}$: D^* behaves as a standard risk premium. A risk neutral individual with respect to F will require

$$(VII.6)\qquad D^* = q\ W\ P$$

That is, a (full) compensati on increasing in the observed wage rate. Under the same condition of risk neutrality with respect to full income, once $v^F_W(p, W, F) < 0$ (assume $v^F_{FW}(p, W, F) = 0$ or relatively small), the required s^* when $D^* = 0$ will be such that the expected "inducing" $q\ F^1 + (1 - q)\ F^2$ will be larger than F and

$$(VII.7)\qquad s^*\ T > q\ (W + s^*)\ P \quad \text{or} \quad s^* > q\ W\ P / (T - q\ P).$$

This implies

$$(VII.8)\qquad s^*\ (T - q\ P) = s^*\ E(O + H) > q\ W\ P$$

However, $s^*\ E(H)$, the expected income payment, maybe smaller than $q\ W\ P$.

Consider the trade-off between D^* and s^* that leaves the worker indifferent, that is obeys (VII.5), or equivalently,

$$(VII.9)\qquad (1 - q)\ v(p, W + s^*, V + D^*, T) + q\ v(p, W + s^*, V + D^*, T - P) = \\ = v(p, W, V, T)$$

Then

$$(VII.10)\ -\frac{\partial D^*}{\partial s^*} = [(1 - q)\ v_w(p, W{+}s^*, V + D^*, T) + q\ v_w(p, W{+}s^*, V{+}D^*, T - P)]\ / \\ / [(1 - q)\ v_V(p, W{+}s^*, V + D^*, T) + q\ v_V(p, W{+}s^*, V + D^*, T - P)] = \\ = (1 - q\ \gamma)\ H(p, W{+}s^*, V + D^*, T) + q\ \gamma\ H(p, W{+}s^*, V{+}D^*, T - P)$$

where

$$\gamma = v_V(p, W+s^*, V + D^*, T - P) / [(1 - q)\, v_V(p, W+s^*, V + D^*, T) + q\, v_V(p, W+s^*, V + D^*, T - P)]$$

There will always be a trade-off between D* and s*: $-\frac{\partial D^*}{\partial s^*} > 0$.

$-\frac{\partial D^*}{\partial s^*}$ is a weighted average of labor supply in case of injury and in case of safety. As labor supply responds negatively to the time endowment, provided $q\gamma < 1$, $H(p, W + s^*, V + D^*, T - P) < -\frac{\partial D^*}{\partial s^*} < H(p, W + s^*, V + D^*, T)$.

If $v^F(p, W, F)$ is concave in F, $v(p, W, V, T) = v^F(p, W, V + W T)$ will be concave in T, as in V. $v_V(p, W + s^*, V + D^*, T) < v_V(p, W + s^*, V + D^*, T - P)$ and $\gamma > 1$. As $H(p, W + s^*, V + D^*, T) > H(p, W + s^*, V + D^*, T - P)$, $-\frac{\partial D^*}{\partial s^*}$ will be smaller than expected labor supply, $E(H) = (1 - q)\, H(p, W+s^*, V + D^*, T) + q\, H(p, W+s^*, V+D^*, T - P)$: with a unit decrease in s*, there can be an increase in D* smaller than expected labor supply, apparently favoring D* in terms of the required costs. If the individual is risk lover with respect to full income and $v^F(p, W, F)$ is convex in F, the opposite occurs.

Consider the effect on [s* E(H) + D*] of a simultaneous increase in s* and reduction in D* obeying the equivalence (VII.5). Then:

$$\text{(VII.11)} \quad \frac{\partial[s^* E(H) + D^*]}{\partial s^*} = [E(H) + s^* \frac{\partial E(H)}{\partial W}] + [s^* \frac{\partial E(H)}{\partial V} + 1] \frac{\partial D^*}{\partial s^*} =$$

$$= \{E(H) + s^* [(1 - q) \frac{\partial H(p, W + s^*, V + D^*, T)}{\partial W} + q \frac{\partial H(p, W + s^*, V + D^*, T - P)}{\partial W}]\} +$$

$$+ \{s^* [(1-q) \frac{\partial H(p, W + s^*, V + D^*, T)}{\partial V} + q \frac{\partial H(p, W + s^*, V + D^*, T - P)}{\partial V}] + 1\} \frac{\partial D^*}{\partial s} =$$

$$= E(H) + \frac{\partial D^*}{\partial s^*}. + s^* [\frac{\partial E(H)}{\partial W} + \frac{\partial E(H)}{\partial V} \frac{\partial D^*}{\partial s^*}] =$$

$$= E(H) + \frac{\partial D^*}{\partial s^*} + s^* [\frac{\partial E(H)}{\partial W} - E(H) \frac{\partial E(H)}{\partial V}] + s^* \frac{\partial E(H)}{\partial V} [E(H) + \frac{\partial D^*}{\partial s^*}]$$

Around s* = 0, $\frac{\partial[s^* E(H) + D^*]}{\partial s^*} = E(H) + \frac{\partial D^*}{\partial s^*}$. Switching to an hourly premium – i.e., increasing s* – will be worthwhile, i.e., $\frac{\partial[s^* E(H) + D^*]}{\partial s^*} < 0$ iff $-\frac{\partial D^*}{\partial s^*} > E(H)$. That is, iff the consumer worker is risk lover with respect to full income. If the consumer is risk averse, as $-\frac{\partial D^*}{\partial s^*} < E(H)$, $\frac{\partial [s^* E(H) + D^*]}{\partial s^*} > 0$ and to induce the workers to accept the risk, by increasing s, the bill will be higher.

$[\frac{\partial E(H)}{\partial W} - E(H)\frac{\partial E(H)}{\partial V}]$, by Slutsky decomposition, measures the substitution effect of an increase in wages and is always positive: sizeable substitution effects on labor supply will favor the higher efficiency of D* over s* in inducing – risk-lovers or risk averse – workers to accept a given injury risk. This is intuitively expected, once with s, hours of work, necessarily curtailed due to the loss P, are unrewarded under the compensating scheme – a higher s is required under such circumstances.

Also, $\frac{\partial E(H)}{\partial V} < 0$; for risk averse individuals, the last term of the expression is negative. As s* rises, it is possible that the influence of the last term becomes sizeable and there is a reversal of the sign of $\frac{\partial [s^* E(H) + D^*]}{\partial s^*}$. A necessary, but not sufficient condition is that:

$$\text{(VII.12)} \qquad -\frac{\partial E(H)}{\partial V}\frac{V}{E(H)} > \frac{V}{s^* E(H)}$$

Then it may be possible that the bill with D* = 0 is smaller than with s* = 0. (VII.12) will probably hold for s* large – which can only occur for high risks.

At least for small risks, P, it is likely that for risk averse individuals it will be more efficient to pay the lump sum D* and not s*: if workers are risk-averse, hedonic premiums to risky activities would be indexed to P and indirectly to W – through the equivalence equation (VII.5) or (VII.9) -, but they do not take the form of an hourly compensation; rather, they will be a simple per period T compensation – i.e., rather, a quasi-fixed cost, paid to the worker in "sickness as in health". Such scheme would be cheaper than to pay an hedonic wage premium indexed linearly to hours worked.

If workers are risk lovers, $\frac{\partial [s^* E(H) + D^*]}{\partial s^*} < 0$ only if the risk or s* are small, or the inequality sign of (VII.12) is reversed. If there are risk lover workers, they will be attracted by hazardous activities; hedonic wage rate profiles with a risk premium linear in hours could be observed (more likely if the substitution effect is small).

If unit costs of the risk were the important concept to minimize from the point of view of the employer, then the relevant derivative would be:

$$\text{(VII.13)} \qquad \frac{\partial[s^* + \frac{D^*}{E(H)}]}{\partial s^*} = 1 + \frac{\frac{\partial D^*}{\partial s^*}E(H) - [\frac{\partial E(H)}{\partial s^*} + \frac{\partial E(H)}{\partial D^*}\frac{\partial D^*}{\partial s^*}]D^*}{E(H)^2}$$

It will be smaller than 0 iff

$$\text{(VII.14)} \qquad E(H)\,[E(H) + \frac{\partial D^*}{\partial s^*}] - [\frac{\partial E(H)}{\partial s^*} + \frac{\partial E(H)}{\partial D^*}\frac{\partial D^*}{\partial s^*}]\,D^* =$$

$$= E(H)\,[E(H) + \frac{\partial D^*}{\partial s^*}] - [\frac{\partial E(H)}{\partial W} + \frac{\partial E(H)}{\partial V}\frac{\partial D^*}{\partial s^*}]\,D^* =$$

$$= E(H)\,[E(H) + \frac{\partial D^*}{\partial s^*}] - [\frac{\partial E(H)}{\partial W} - \frac{\partial E(H)}{\partial V}E(H)]\,D^* - \frac{\partial E(H)}{\partial V}[E(H) + \frac{\partial D^*}{\partial s^*}]\,D^* < 0$$

At $D^* = 0$, the above conclusions stay unaltered it is preferable to increase s^* if $E(H) + \frac{\partial D^*}{\partial s^*} < 0$, that is, for risk lovers (as D^* is already 0, we stay in that point); to decrease s^* for risk averters. As D^* rises, the negativity of $E(H) + \frac{\partial D^*}{\partial s^*}$ is maintained for risk lovers. For risk averse individuals $E(H) + \frac{\partial D^*}{\partial s^*}$ will be positive provided the substitution effect of an increase in wage on labor supply is small.

2.2. Suppose that the risk is proportional to hours of work and that the time loss takes the form Q H. Then, in case of injury $T = O + (1 + Q) H$; replacing in $p\,Y = (W + s) H + V + D$, we get the full income constraint:

$$\text{(VII.15)} \qquad p\,y + \frac{W+s}{1+Q}\,O = \frac{W+s}{1+Q}\,T + V + D$$

The equivalence relation becomes:

$$\text{(VII.16)} \qquad (1-q)\,v^F[p, W + s^*, (W + s^*)\,T + V + D^*] +$$

$$+ q\,v^F(p, \frac{W+s^*}{1+Q}, \frac{W+s^*}{1+Q}\,T + V + D^*) =$$

$$= v^F(p, W, W\,T + V)$$

It is now uncle ar how the risk neutral individual reacts. But if his expected "full income" is larger than W T + V, then

$$s^* T > \frac{q\, W\, Q\, T - D^*(1+Q)}{1+(1-q)\, Q}$$

which (if $v^F_{FW}(p, W, F)$ is negligible) will probably require a larger expected wage than W: with $(1 - q)(W + s^*) + q \frac{W + s^*}{1+Q} > W$ or $s > q \frac{W\, Q}{1+(1-q)\, Q}$.

We can write (VII.16) as:

$$\text{(VII.17)} \qquad (1-q)\, v(p, W + s^*, V + D^*, T) + q\, v\left(p, \frac{W+s^*}{1+Q}, V + D^*, T\right) = v(p, W, V, T)$$

Now:

$$\text{(VII.18)} \; - \frac{\partial D^*}{\partial s^*} = \left[(1-q)\, v_w(p,W+s^*,V+D^*,T) + q \frac{1}{1+Q} v_w\left(p, \frac{W+s^*}{1+Q}, V+D^*,T\right)\right] /$$

$$/ \left[(1 - q)\, v_V(p, W + s^*, V + D^*, T) + q\, v_V\left(p, \frac{W+s^*}{1+Q}, V + D^*, T\right)\right] =$$

$$= (1 - q\gamma)\, H(p, W + s^*, V + D^*, T) + q\, \gamma\, H\left(p, \frac{W+s^*}{1+Q}, V + D^*, T\right) -$$

$$- \; q\, \gamma \frac{Q}{1+Q} H\left(p, \frac{W+s^*}{1+Q}, V + D^*, T\right)$$

where

$$\gamma = v_V\left(p, \frac{W+s^*}{1+Q}, V + D^*, T\right) /$$

$$/ \left[(1 - q)\, v_V(p, W + s^*, V + D^*, T) + q\, v_V\left(p, \frac{W+s^*}{1+Q}, V + D^*, T\right)\right]$$

$- \frac{\partial D^*}{\partial s^*}$ is a weighted average of labor supply in case of safety and in case of injury minus an additional term. If labor supply responds positively to the wage rate , and $\gamma q < 1$, $- \frac{\partial D^*}{\partial s^*} < H(p, W + s^*, V + D^*, T)$. If it is negatively sloped, $- \frac{\partial D^*}{\partial s^*} < H\left(p, \frac{W+s^*}{1+Q}, V + D^*, T\right)$.

If v(p, W, V) exhibits substitutability between W and V, i.e., $v_{Vw}(p, W, V, T) < 0$, $v_V(p, W+s^*, V + D^*, T) < v_V\left(p, \frac{W+s^*}{1+Q}, V + D^*, T\right)$ and $\gamma > 1$. If labor supply is positively sloped, as $H(p, W + s^*, V + D^*, T) > H(p, \frac{W + s^*}{1+Q}, V + D^*, T)$, $-\frac{\partial D^*}{\partial s^*}$ will be smaller than expected labor supply: with a unit decrease in s*, there can be an increase in D* smaller than expected labor supply. If there is complementarity between V and W and $v_{Vw}(p, W, V, T) > 0$, the opposite will occur if Q is small.

Being labor supply negatively sloped: with complementarity, $\gamma < 1$ and $-\frac{\partial D^*}{\partial s^*}$ will be smaller than expected labor supply; with substitutability, it can be larger than expected labor supply when Q is small .

The risk incurred is proportional to hours of work. We could rephrase the efficiency appraisal in terms of unit hourly costs. Consider the effect on $[s^* + \frac{D^*}{E(H)}]$ of a simultaneous increase in s* and reduction in D* obeying the equivalence (VII.17). Then:

$$\text{(VII.19)} \qquad \frac{\partial[s^* + \frac{D^*}{E(H)}]}{\partial s^*} = 1 + \frac{\frac{\partial D^*}{\partial s^*} E(H) - [\frac{\partial E(H)}{\partial s^*} + \frac{\partial E(H)}{\partial D^*}\frac{\partial D^*}{\partial s^*}] D^*}{E(H)^2}$$

It will be larger than 0 iff

$$\text{(VII.20)} \qquad E(H)\,[E(H) + \frac{\partial D^*}{\partial s^*}] - [\frac{\partial E(H)}{\partial s^*} + \frac{\partial E(H)}{\partial D^*}\ \frac{\partial D^*}{\partial s^*}]\, D^* =$$

$$= E(H)\,[E(H) + \frac{\partial D^*}{\partial s^*}] - [\frac{\partial E(H)}{\partial W} - q\ \frac{Q}{Q+1}\ \frac{\partial H(q, \frac{W+s^*}{1+Q}, V, T)}{\partial W} + \frac{\partial E(H)}{\partial V}\ \frac{\partial D^*}{\partial s^*}]\, D^* =$$

$$= E(H)\,[E(H) + \frac{\partial D^*}{\partial s^*}] - [\frac{\partial E(H)}{\partial W} - \frac{\partial E(H)}{\partial V}\ E(H)]\, D^* - \frac{\partial E(H)}{\partial V}\ [\frac{\partial D^*}{\partial s^*} + E(H)]\ +$$

$$+\ q\ \frac{Q}{Q+1}\ \frac{\partial H(q, \frac{W+s^*}{1+Q}, V, T)}{\partial W}\ D^* > 0$$

At $D^* = 0$, $\frac{\partial[s^* + \frac{D^*}{E(H)}]}{\partial s^*} > 0$ and it would be preferable to decrease s* iff $E(H) > -\frac{\partial D^*}{\partial s^*}$. That will occur, has previously discussed, when labor supply is positively sloped and V and W are substitutes – but those are not necessary conditions; or when labor supply is negatively sloped if (but not only if) V and W are complements.

At intermediate solutions, the term $\frac{Q}{Q+1}$ would render the case $\frac{\partial[s^* + \frac{D^*}{E(H)}]}{\partial s^*} > 0$, and D* more efficient than s*, more likely if supply is positively sloped. This effect is more sizable when Q is large.

A large substitution effect of wages on labor supply (more significant when labor supply is positively sloped) favors $\frac{\partial[s^* + \frac{D^*}{E(H)}]}{\partial s^*} < 0$ and s* relative to D*.

2.3. Finally, we could also inquire whether paying D* would be cheaper than an insurance type scheme. That is, a lump sum transfer M*, paid only in case of injury, such that M* < W P* - that is, some D* is being used.

$$\text{(VII.21)} \quad (1\text{-}q)\, v^F(p, W, W\,T + V + D^*) + q\, v^F[p, W, W\,(T - P) + V + D^* + M^*] = \\ = v^F(p, W, W\,T + V)$$

$$\text{(VII.22)} \quad -\frac{\partial M^*}{\partial D^*} = \frac{(1\text{-}q)\, v^F{}_F(p, W, F + D^*) + q\, v^F{}_F(p, W, F - W\,P + D^* + M^*)}{q\, v^F{}_F(p, W, F + D^* + M^* - W\,P)}$$

If workers are risk averse, $-\frac{\partial M^*}{\partial D^*} < \frac{1}{q}$. It is then cheaper to switch to an insurance type scheme, i.e., a compensation paid only in case of injury. If workers are risk lovers, the opposite occurs.

Notice that the compensation, solving (VII.21) for D* = 0, is such that M* = W P and indexed to the wage rate, either for risk-lovers or risk-averse individuals. The expected value with only M* is, thus, q W P. Looking at (VII.21) for M* = 0, we conclude that for risk averse individuals D* > q W P and insurance is cheaper. The opposite occurs for risk lovers.

We could as well perform the relative efficiency of a wage premium and the insurance – even if transitive logic using the previous arguments could close the subject as well.

$$\text{(VII.23)} \quad (1\text{-}q)\, v^F[p, W + s^*, (W + s^*)\,T + V] + \\ + q\, v^F[p, W + s^*, (W + s^*)\,(T - P) + V + M^*] = \\ = v^F(p, W, W\,T + V)$$

Provided some s* is being used – assuming $v^F_{FW}(p, W, F)$ is negligible – $v^F[p, W + s^*, (W + s^*) T + V] > v^F[p, W + s^*, (W + s^*) (T - P) + V + M^*]$, which requires $M^* < (W + s^*) P$.

If $v^F_{FW}(p, W, F)$ is negligible or negative, for a risk neutral or risk averse individual $q M^* + s^* T > q (W + s^*) P$.

$$\text{(VII.24)} \qquad (1-q)\, v(p, W + s^*, V, T) + q\, v(p, W + s^*, V + M^*, T - P] = \\ = v(p, W, W T + V)$$

$$\text{(VII.25)} \qquad -\frac{\partial M^*}{\partial s^*} = \frac{(1-q)\ v_w(p, W + s^*, V, T) + q\, v_w(p, W + s^*, V + M^*, T - P)}{q\, v_V(p, W + s^*, V + M^*, T - P)}$$

$-\frac{\partial M^*}{\partial s^*} < \frac{1}{q} E(H)$ iff $v_V(p, W + s^*, V + M^*, T - P) = v^F_F[p, W + s^*, (W + s^*) (T - P) + V + M^*] > v^F_F[p, W + s^*, (W + s^*) T + V]$. This will occur for a risk averse individual with respect to full income. One can show that:

$$\text{(VII.26)} \qquad \frac{\partial[s^* E(H) + q M^*]}{\partial s^*} = [E(H) + q \frac{\partial M^*}{\partial s^*}] + s^* \frac{\partial E(H)}{\partial W}$$

At s* = 0, (VII.26) will be positive for risk-averse individuals: insurance is cheaper. For risk lovers, the reverse occurs. These relative comparisons may switch, however, for higher risks – and s* - only if supply is negatively sloped for risk averse individuals, positively sloped for risk lovers.

Finally, in all cases, the required compensation fee will be, in general, positively related to wages. Risky activities will be more easily accepted by individuals of lower wage rates.

3. Consider the following context: an individual knows that he can have an illness that implies a loss of time endowment P with probability q. He can buy an insurance L paying insurance premium S. Then

$$\text{(VII.27)} \qquad F^1 = W T + V - S \quad \text{and} \quad F^2 = W (T - P) + V + L - S$$

The maximum premium S* he is willing to accept for obtaining a given L* in the event of the illness will be such that

$$(VII.28)\qquad (1-q^*)\, v^F(p, W, WT+V - S^*) + q^*\, v^F[p, W, W(T-P^*)+V + L^* - S^*] =$$
$$= (1-q^*)\, v^F(p, W, W T + V) + q^*\, v^F[p, W, W (T - P^*) + V]$$

Risk lovers, facing a lower dispersion in " full income" under insurance, will require a higher expected full income, hence, $S^* < q^* L$: no insurance company will accept it. A risk neutral individual with respect to F, will accept, and be indifferent, to pay at most a premium independent of W or V: $S^* = q L^*$.

If the individual is risk -averse, he will be willing to pay till, with the insurance, equality between the two states holds, i.e., prefers full insurance:

$$(VII.29)\qquad v^F(p, W, W T + V - S^*) = v^F[p, W, W (T - P^*) + V + L^* - S^*]$$

That is:

$$(VII.30)\qquad L^* = W P^*$$

Risk averse individuals ask for health insurance in proportion to the wage rate and the expected illness duration.

Then the maximum S^* he is willing to pay for such insurance is:

$$(VII.31)\qquad v^F(p, W, W T + V - S^*) =$$
$$= (1 - q)\, v^F(p, W, W T + V) + q\, v^F[p, W, W (T - P) + V]$$

As he is risk averse,

$$(VII.32)\qquad W T + V - S^* < (1-q) (W T + V) + q [W (T - P) + V]$$

That is:

$$(VII.33)\qquad S^* > q W P > q L$$

S^* will increase with V symmetrically to the sign $\frac{\partial M^*}{\partial V}$ of the criminal´s probl em. Provided P is large, the insurance premium S^* that the individual will be willing to pay for full insurance will most likely increase with W.

4.1. Finally, with a family utility function and a family "full income" budget constraint, P could be interpr et as a deduction in life expectancy and the previous type of situation we could picture life insurance.

It is not reasonable to depict the death sentence or life risks under the above framework – death reduces any possibility of enjoying full income, whet her in leisure or in income consumption and may terminate full income, that is, both V and T, are not enjoyed. To modify the model to encompass such sentence we can retrace Ehrlich´s (1975) [47] modeling of the penal system and apply it to our framework:

Assume that if an individual is caught – which still occurs with probability q - he suffers fine and imprisonment with probability 1 – r and a death sentence with probability r. Alternatively, we could interpret r as the probability of being shot and killed by the police or by the victims when caught. Death completely destroys full income, which is reduced to 0 [48].

$$\text{(VII.34)} \qquad v^F(p, W, W\,T + V) =$$
$$= (1\text{-}q^*)\, v^F(p, W, W\,T + V + R) + q^*\,(1 - r^*)\, v^F[p, W, W\,(T - P^*) + V - M^*] +$$
$$+ q^*\, r^*\, v^F(p, W, 0)$$

4.2. For the risk neutral individual:

$$\text{(VII.35)} \qquad r^* = \frac{R\,(1 - q^*) - q^*\,(M^* + W\,P^*)}{q^*\,[F - (M^* + W\,P^*)]}$$

(VII.35) suggests an intuition for why more developed societies would have, or were able to, switched from capital punishment to other sanctions. With longer life expectancy, higher wages and non-labor income, capital punishment would have become a relatively heavier penalty, and could be applied less often. Notice that in (VII.35) $\frac{\partial r^*}{\partial R} = \frac{1 - q^*}{q^*(F - M'^*)}$ and $\frac{\partial M'^*}{\partial R} =$

[47] Ehrlich (1975) studies murders, contemplating interdependent preferences between individuals in the (individual) utility function. Instead, we are interested in contrasting the effect of different penalty parameters in the presence of an unspecified general offence. However, his analysis of the relative impact of the crime-deterrent parameters is independent of the type of the felony.

[48] Ehrlich (1975) also assumes death may lead to zero consumption. See Hirshleifer and Riley (1992), p 64-66, for a similar hypothesis for evaluation of life.

$\frac{1-q^*}{q^*(1-r^*)}$; as F = W T + V rises, even for risk neutral individuals , $\frac{\partial r^*}{\partial R}$ decreases, even if not $\frac{\partial M'^*}{\partial R}$.

4.3. We can compare the relative efficiency of the death sentence relative to the arrest rate according to the criteria previously stated in §4 of section III in the following way: Denote $M' = M + W\,P$, $F^1 = W\,T + V + R$ and $F^2 = W\,T + V - M'$; an increase in q* of 1% requires, for the expected "full income" of the criminal to remain unchanged, an increase $\frac{\partial r^*(W\,T + V - M'^*)}{R + r^*(W\,T + V) + (1-r^*)\,M'^*}$ of 1 %. Under (V II.34):

$$\text{(VII.36)} \qquad -\frac{\partial q^*}{\partial r^*}\,\frac{R + r^*(W\,T + V) + (1-r^*)\,M'^*}{q^*(W\,T + V - M'^*)} =$$

$$= \frac{v^F(p, W, F^1) - v^F(p, W, 0)}{v^F(p, W, F^1) - (1-r^*)\,v^F(p, W, F^2{}^*) - r^*\,v^F(p, W, 0)} \times$$

$$\times \frac{R + r^*(W\,T + V) + (1-r^*)\,M'^*}{W\,T + V - M'^*}$$

(VII.36) will be smaller than one if raising the probability of conviction is more effective than increasing the application of the death sentence: a decrease in r* of $\frac{\partial r^*(W\,T + V - M'^*)}{R + r^*(W\,T + V) + (1-r^*)\,M'^*}$ % requires an increase in q* smaller than 1%.

That will occur iff

$$\text{(VII.37)} \qquad \frac{v^F(p, W, F^1) - v^F(p, W, F^2{}^*)}{R + M'^*} > \frac{v^F(p, W, F^2{}^*) - v^F(p, W, 0)}{W\,T + V - M'^*}$$

This will occur only if v(p, W, F) is convex in F: if individu als are risk-lovers with respect to full income. The sign of the inequality will be reversed – and raising the death sentence, or allowing police forces and private citizens to use weapons more freely against criminals, is more efficient than raising the a rrest rate - iff potential criminals are risk -averse.

Also, a change $\frac{\partial r^*}{1-r^*}$ in r* and $-\frac{\partial M'^*}{W\,T + V - M'^*}$ of M′* of the same magnitude leave the expected value of full income unaltered. Then:

$$\text{(VII.38)} \qquad -\frac{\partial M'^*}{\partial r^*} \; \frac{1 - r^*}{W\,T + V - M'^*} = \frac{v^F(p, W, F^{2*}) - v^F(p, W, 0)}{(W\,T + V - M'^*)\, .\, v^F_F(p, W, F^{2*})}$$

It will be smaller than 1, implying that sanctions M´* are more efficient than capital punishment iff:

$$\text{(VII.39)} \qquad \frac{v^F(p, W, F^{2*}) - v^F(p, W, 0)}{(W\,T + V - M'^*)} < v^F_F(p, W, F^{2*})$$

This occurs when punishment is aimed at risk-averse consumers. For risk lovers, the opposite result holds.

Summarizing: If potential criminals are risk-lovers, raising the probability of detection/conviction is more effective than raising the application of the death sentence, and raising the death sentence is more effective than raising (other) sanctions. If they are risk-averse, relative effectiveness follows the reverse order: raising sanctions is more effective than raising the application of death sentence, which is more effective than raising the probability of general conviction.

4.4. An equation describing the adequate fighting potential needed to prevent a war and yet, win if R is the per capita surplus in income that the other side expects from not complying / surrendering could be derived from:

$$\begin{aligned}
\text{(VII.40)} \qquad & v^F(p, W, W\,T + V) = \\
= \; & (1-q^*-t^*)\, v^F(p, W, W\,T + V + R) + q^*\, v^F[p, W, W\,(T - P^*) + V - M^*] + \\
& + t^*\, v^F(p, W, 0)
\end{aligned}$$

Under risk-neutrality:

$$\text{(VII.41)} \qquad t^* = \frac{R\,(1-q^*) - q^*\;(M^* + W\,P^*)}{F + R} = \frac{R\,(1-q^*) - q^*\;(M^* + W\,P^*)}{W\,T + V + R}$$

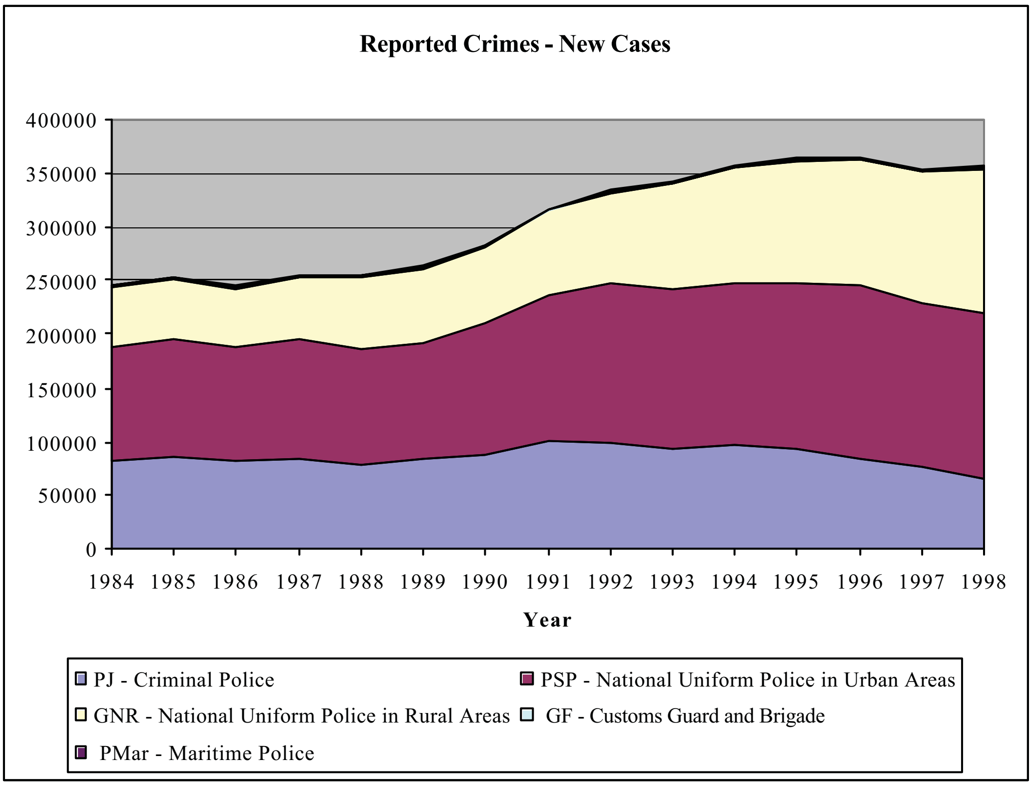
t* represents the killing rate, q* injury rate with average life-time loss of P*, and M* property damages.

t*, as r* above in (VII.35), increases with R; it decreases with W but also with T and V, i.e., with F = W T + V – poorer people have less to live for.

VIII. Portuguese Evidence.

1. In 1998, 341122 crimes were reported by police agencies – 155902 thefts, (39879 corporal offences) 6677 bad checks, 3538 drug traffic. That amounts to 3418 crimes per 100,000 residents – which, if lower than the 5079 for the US in 1997 [49], is sizeable.

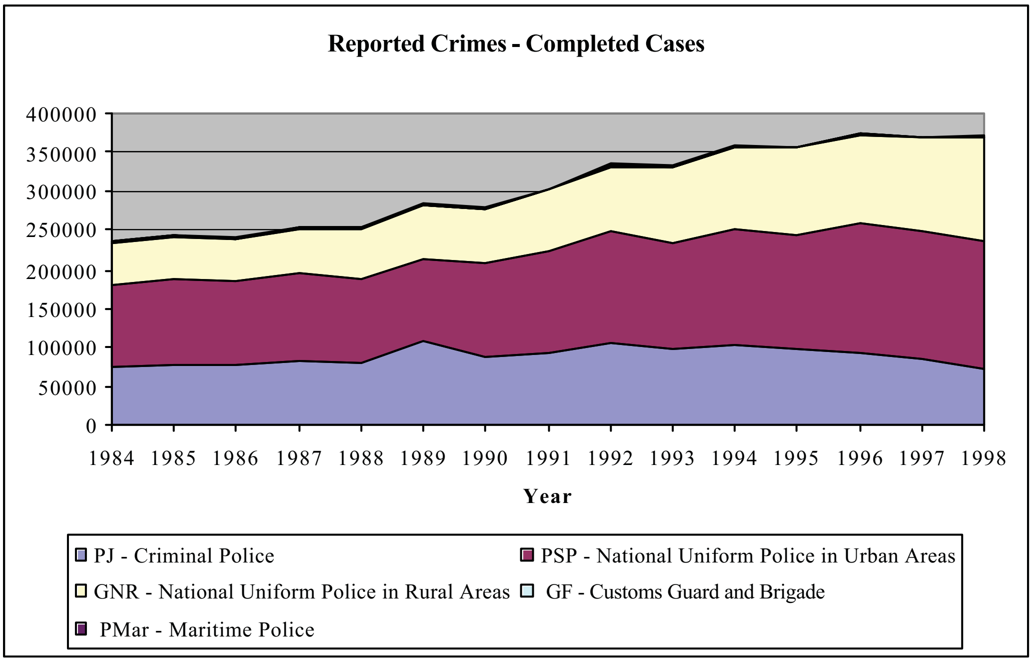
New and Completed Cases registered by criminal police and crimes registered in some other agencies are presented below [50]:



Source: Estatísticas da Justiça, INE.

[49] Freeman (1999).

[50] Portuguese population has been quite stable over the last decade: 1998.31.XII: 9979,5 thousand people (4805,2 men); 1990.31.XII: 9877,5 thousand (4761,7 men); 1988.31.XII: 9955,1 thousand (4800,4 men) 1980.31.XII: 9819,0 thousand (4730,8 men). Therefore, per capita aggregates would show the same pattern.

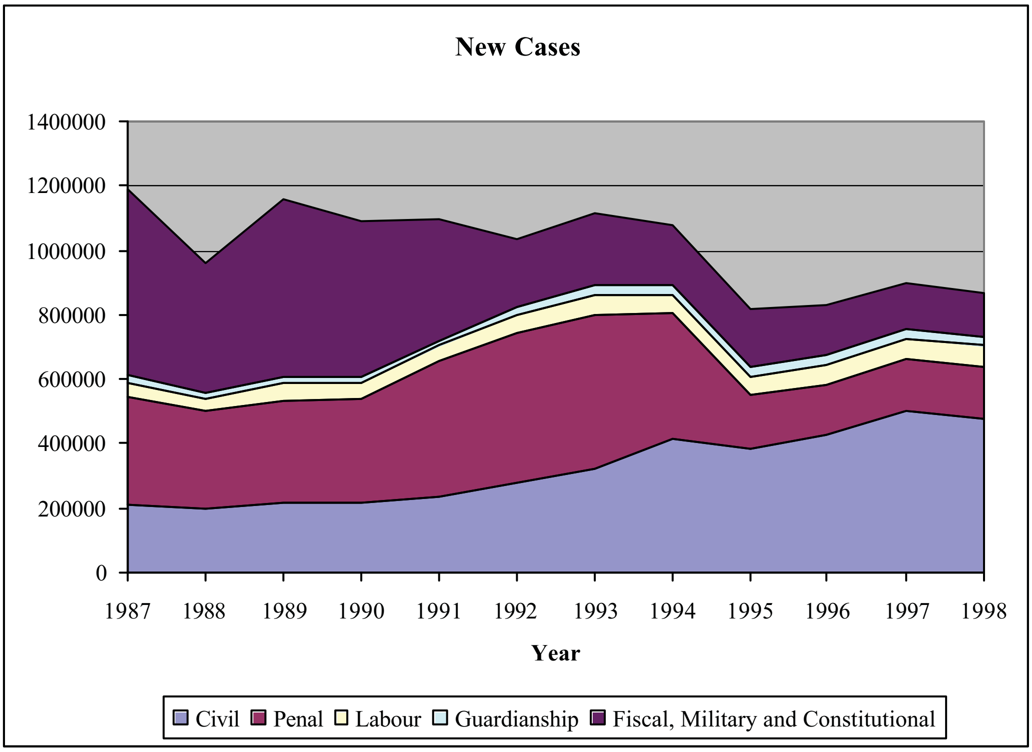


Source: Estatísticas da Justiça, INE.

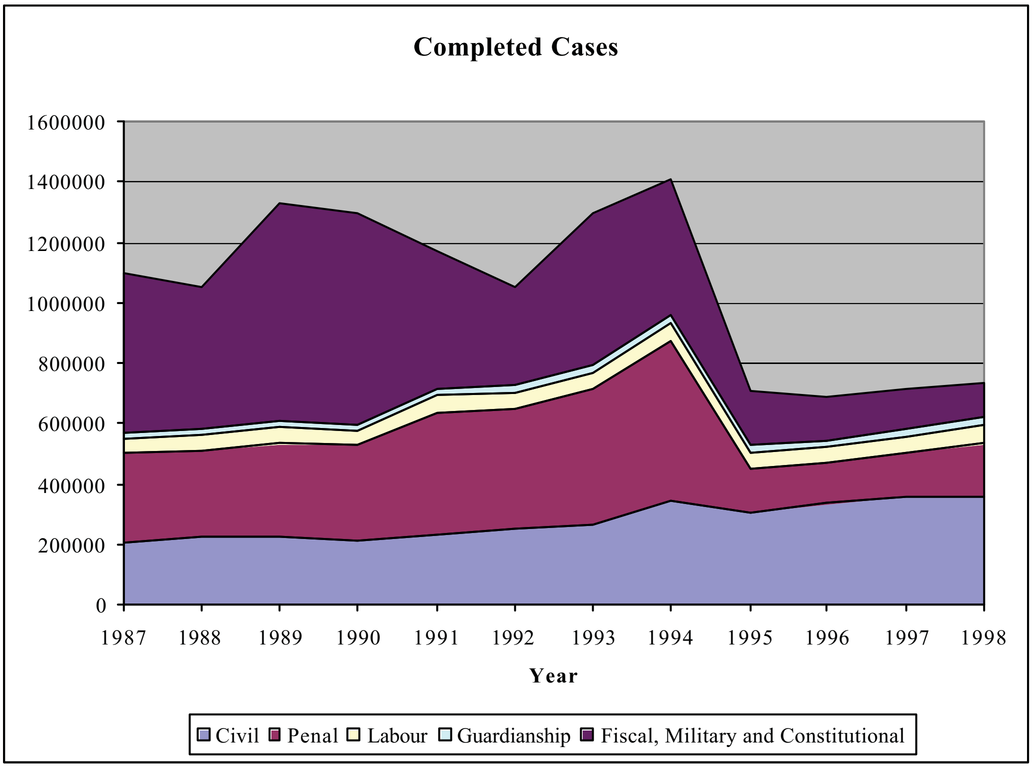
Criminal activities in Portugal seem to have risen sharply in the last decade, both in rural as in urban areas.

2. Cases dealt in Common Courts, after a rise in the early 90´s, and specially penal cases, have decreased – while civil cases rose systematically. Cases in administrative and fiscal, military and Constitutional and Account Courts decreased substantially over the last decade. The graphs below show courts movements since 1987.

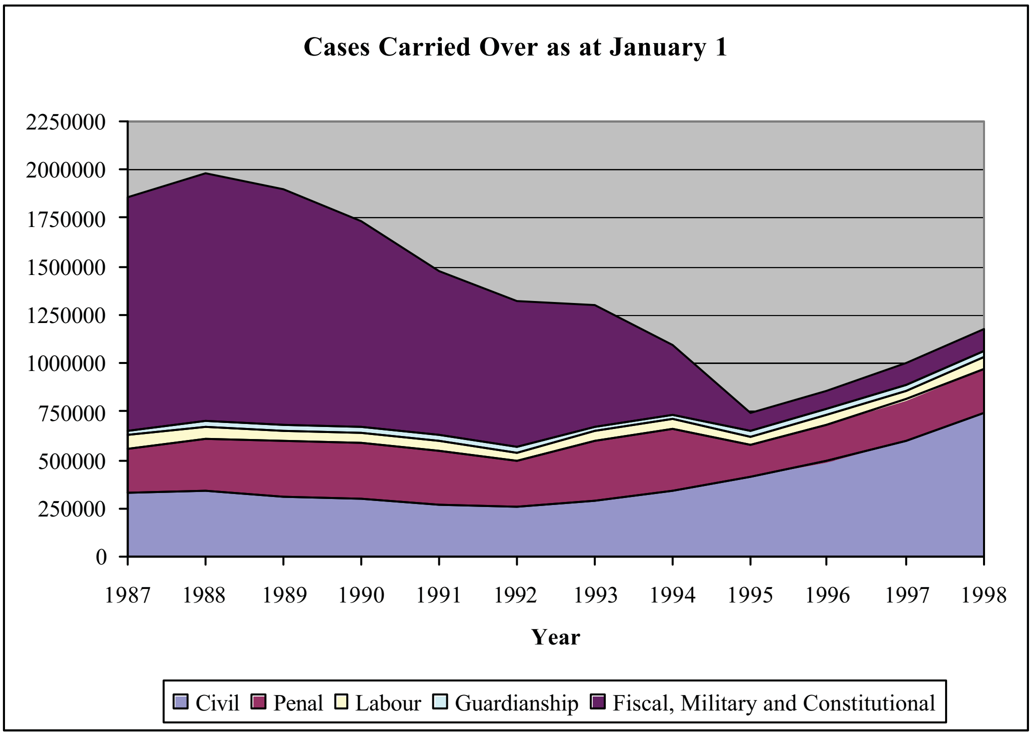
In 1998, there were 1062355 cases carried over as at January 1st; 731057 new cases, 619529 completed cases. (These numbers exclude administrative and fiscal courts; military courts and Constitutional and Account Courts – 131385 new cases in 1998). Out of the 731057 new cases, 471801 (64,5%) were civil; 165908 (22,7 %) penal; 63247 (8,7%) labor; 30101 (4,1%) guardianship.



Source: Anuário Estatístico, INE.



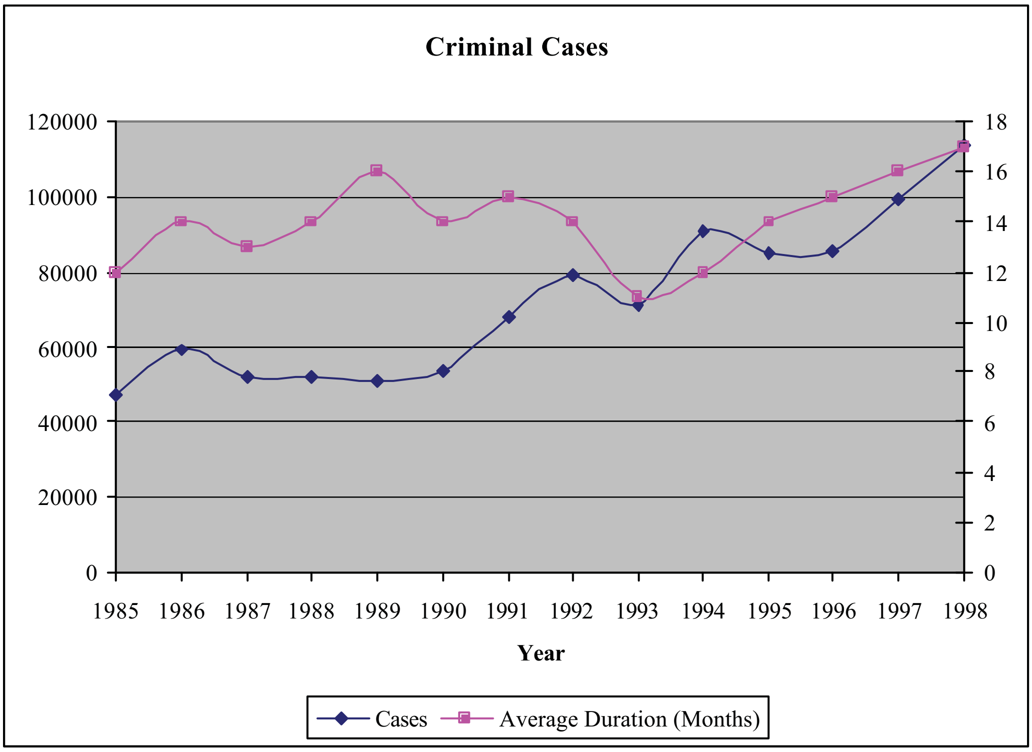
Source: Anuário Estatístico, INE.



Source: Anuário Estatístico, INE.

Given that reported crimes increased sharply, yet penal cases decreased, either (some of) the former are channelled through other courts or we are led to the conclusion that the probability of detection-arrest has declined substantially.

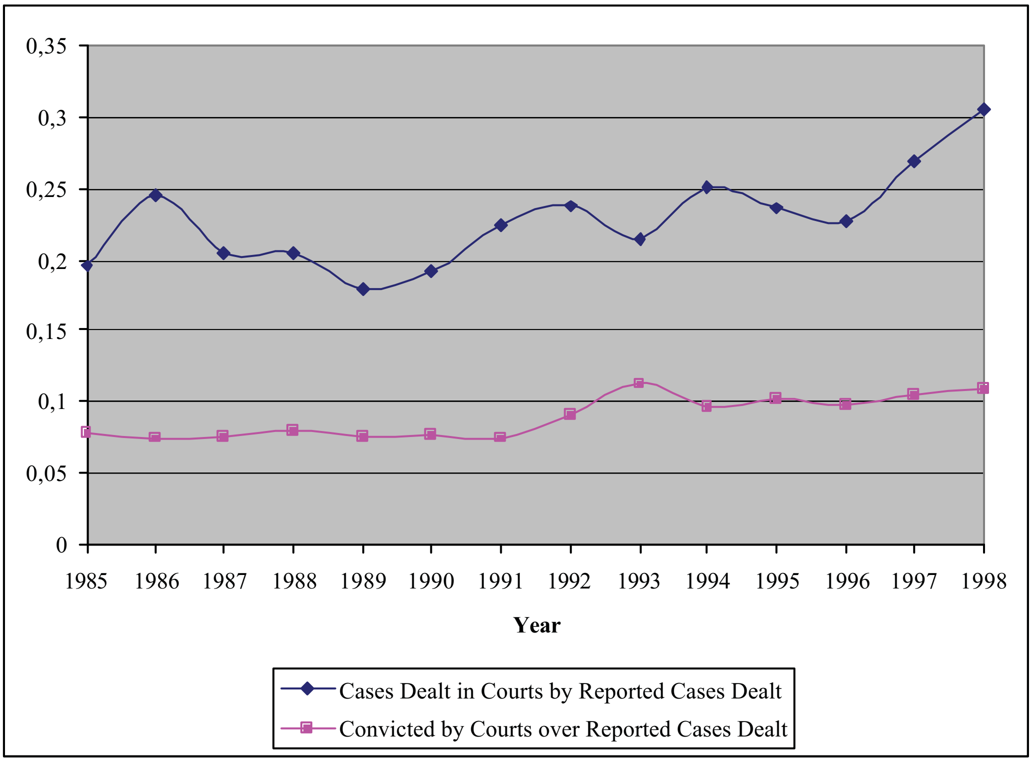
2. Criminal cases co mpleted – which do not have correspondence in the statistics with completed penal cases - have risen and also average duration for completion since 1993:



Source: Anuário Estatístico, INE.

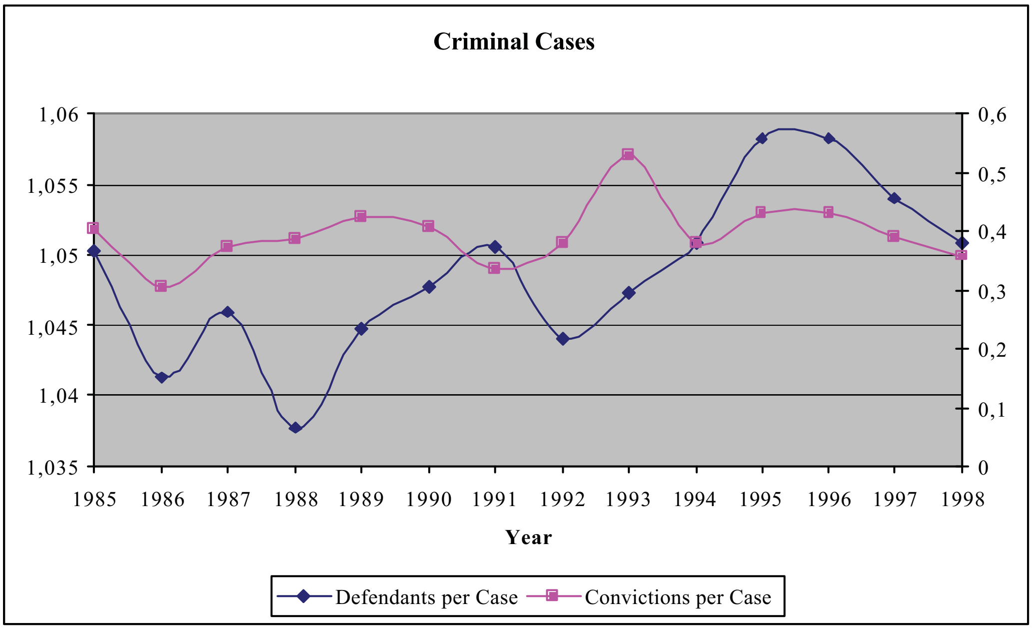
Completed criminal cases in Courts seem to have risen more rapidly than reported crimes in police agencies [51]; yet, that trend is not so clear in number of convicted individuals over reported cases – see below. If lawbreakers per crime (or case) increased, the conviction rate either decreased or, at best, remained stable in the last 5 years of data.

[51] Criminal cases dealt in Courts may not have been reported to the police. Nor does a case in Court correspond necessarily to the conviction of the infractor(s) – nor even to an actual crime. The interpretation of the ratios must therefore be careful.



Source: Anuário Estatístico, INE.

Below, we confirm that defendants per case of criminal nature rose - suggesting that number of trespassers per case rose: that is, organized or associated criminal activities (gangs). Yet, the number of convictions per case presented to courts (scale in the right axis) stayed relatively stable – either less cases were considered crimes and crimes involved a higher number of individuals; or a smaller number of defendants per case considered to represent a crime were found involved.



Source: Anuário Estatístico, INE.

4. We can find below the evolution of the number of convicted defendants disaggregated by type of crime and by type of sanction.

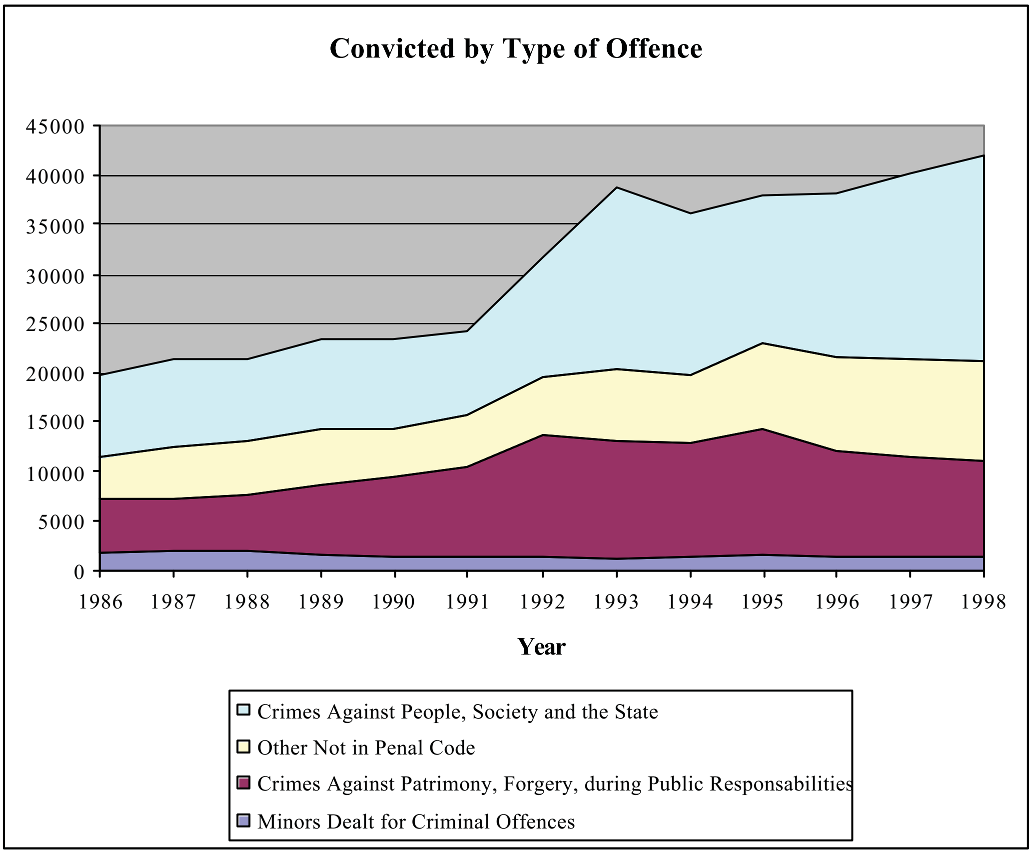
The next figure discriminates sanctions by type of offence. In Portuguese statistics, and following infractions considered in the Penal Code, crimes are classified as Crimes against People, against Society, against P roperty, and against the State. We tried to inquire whether crimes were mainly economically driven – and aggregate them in accordance.

Forgery (forgery of documents, counterfeiting - 1183 in 1998) is a crime against Society; crimes committed during the car rying out of public responsibilities (corruption, embezzlement - 94 in 1998) are crimes against the State; Crimes against Property (include theft, damage, fraud, and crimes against patrimonial rights – 8360 in 1998). We included the three in one category (and not in crimes against People, Society, and the State.)

Crimes not specified in Penal Code (10109 in 1998) are relatively new crimes, but they would have a patrimonial counterpart (they include Economic Crimes – against quality, speculation -, Customs a nd Fiscal Crimes, Drug Related Crimes (4538; of which, consumption: 2442 in 1998), Bad Checks (1226 in 1998), Copyrights, Computer Crimes, Illegal Gambling.

The two previous general items would (very…) roughly correspond to patrimonial offences committed by adults. Their level has increased – even crimes against People, Society and the State increased more rapidly.

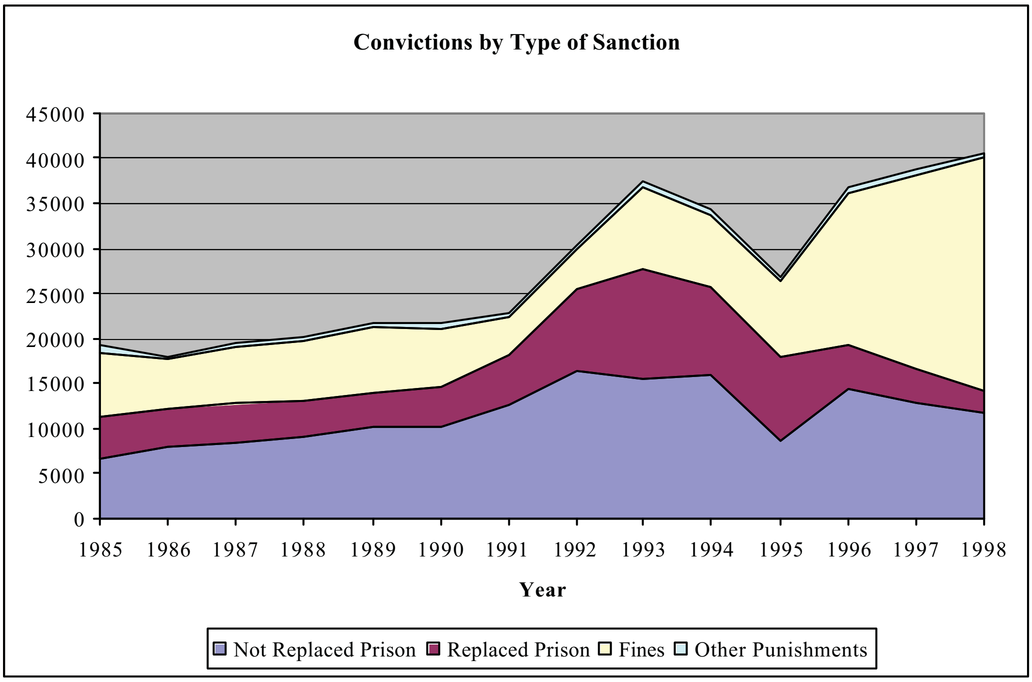
Minors dealt for criminal offences were also added to the numbers (1321 in 1998 – 1044 were involved in crimes against property.) They did not rise over the decades, even if there is a generalized feeling that youngsters are more involved in crime than before – maybe because their conviction rate is low.



Source: Anuário Estatístico, INE.

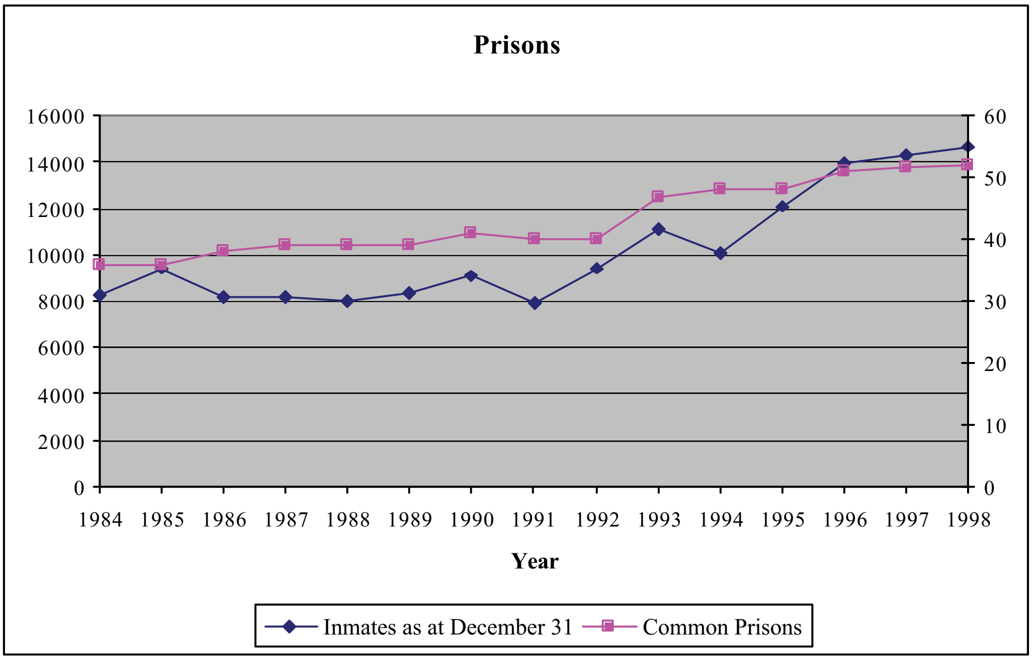
The next graph shows the decomposition of the applied penalties by type of sanction. In 1998, 40622 sanctions were applied, of which 25793 were fines, 14152 prison sentences - 11528 convictions had a not replaced prison sentence -, and 677 suffered other punishments.) (1995 – numerical problem in original data. 1997 is the average of 1996 and 1998.)

The rise in convictions was mainly due to fines in the last five years; prison sentences are much higher than in the 80´s, but stabilizing or declining in the last years of data.



Source: Anuário Estatístico, INE.

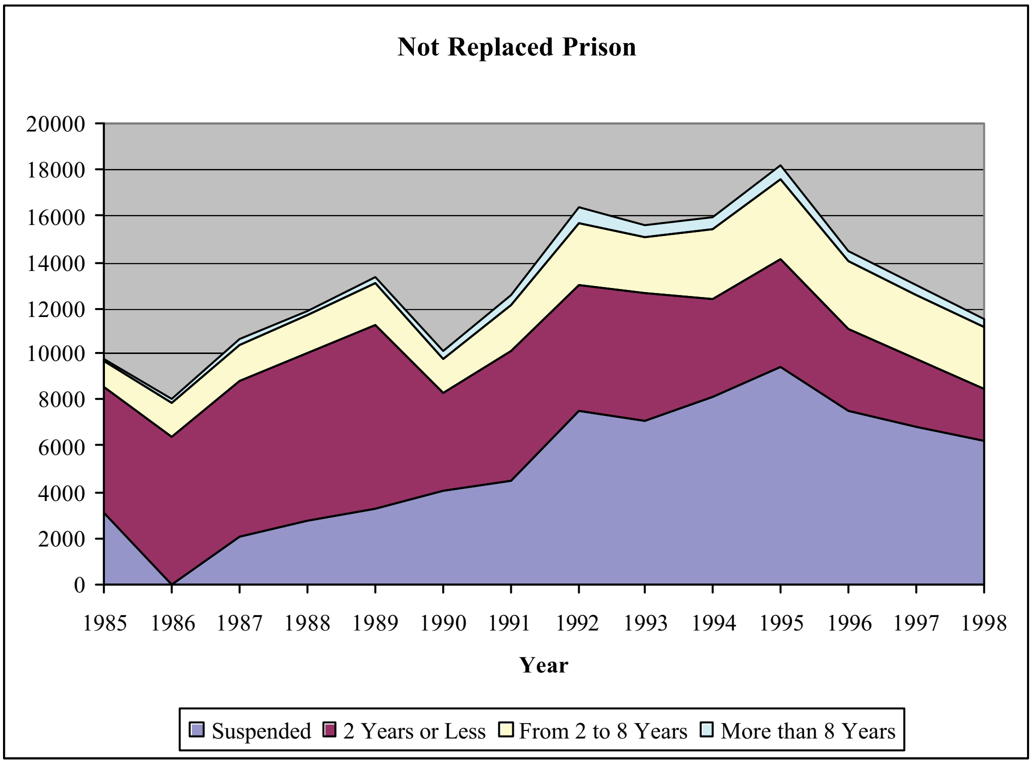
5. Inmates in common prisons almost doubled from 1991 (7877 – 7370 men) to 1998 (14701 – 13291 men). (Numbers do not include minors.).



Source: Anuário Estatístico, INE.

Prison inmates – as capacity and number of prisons, even if with a systematic reported overbooking - increased substantial and continually in the 90's. The rise in inmates can be accounted for a rise in convictions, or a decline in leaves due to an increase in aggregate prison time sentenced some years before. We noticed in §4 that individuals sentenced to prison, after a sharp rise in early 90´s, stabilized or diminished in the last years; (Unless decisions come from other courts,) the continuing rise in prisoners in the last period must be linked to an increase in the average length of prison sentences – to a rise in worse crimes, or sanctions applied for a given infraction were raised.

Disaggregating prison sanctions by length, suspended prison is now the major category; sentences of less than 2 years suffered a sharp decline relative to the eighties. Yet, sentences implying longer than 2 years in prison seem to have risen in the 90´s.



Source: Anuário Estatístico, INE.

Even if data may not be compatible, the system seems to have responded to the rise in crime by raising sanctions rather than the conviction rate – even if more due to the inability to deal with the rise in the denominator: total infractions. The numerator – absolute apprehensions/convictions - also rose, but could not keep up with such exponential increase.

IX. Conclusions.

1. The research followed the implications of including time penances in the standard consumer-worker problem. It was argued that confinement may fruitfully be interpreted as a deduction to the individual - consumer -worker – time endowment in case of detection (and conviction). As such, the model implies a trade-off between the deterrent effects of fines and seclusion which is proportional to and only dependent on the wage rate. Both penalties induce a negative pure income effect, but weighted by the wage rate for the case of imprisonment. Attention is given to how magnitude and uncertainty in endowments condition the minimal deterrent scheme.

The analysis is purely non-ethical and relies only on the technical requirements for an effectively dissuasive penalty system. That is, the approach focus not on how crimes should be punished in fair terms – issues like compensation to victims or for damages are not referenced -, but how to prevent them. No reference is made to differences in propensity for crime: the basic deductions are aimed at optimizing citizens that enjoy different (legal) wage and non-labor earnings opportunities and may commit an unspecified income yielding illegal action (which can be to evade taxes or tariffs, drug dealing, anti-trust law trespassing, shoplifting or pick-pocketing). Individuals only recognize its illicitness by the extent of the associated sanction. The individuals´ income endowment is increased by the monetary value of the offence (net of perpetration costs) in case of success; in case of failure, both the income and time endowment are reduced through law enforcement. Their satisfaction from committing the crime is only dependent upon their general preferences towards (general) income and leisure – and the subjective probabilities they assign to being caught and chastised.

2. There will be a crime-deterrent basket, as in traditional criminal literature[52], being a given sanction (fine-cum-imprisonment) bundle more dissuasive for risk-averse individuals with respect to full income. The analysis above reproduces the usual findings in these type of models: the sign of the attitude towards risk determines whether the required deterrent composite is such that the expected value of the "returns" to criminality must be positive or negative for an individual to find it worthwhile to commit the crime. In general, this statement is not new; however, this research specifically includes the opportunity cost of seclusion in case of detection/conviction, as being considered by individuals, in those calculations.

On the other hand, the minimal efficient dissuasive scheme requires fines or imprisonment increasing in individual´s non-labor income iff (absolute) risk aversion decreases with non-labor income, a much commonly invoked attitude in economics literature. Again, this reproduces the conclusions that were derived from standard models of uncertainty in consumer behavior and criminal studies (but with respect to wealth); additionally, we can infer that the deterrent sanction will decrease with age – implying delinquency diminishes with age – and increase with life expectancy under the same pattern of preferences - decreasing risk-aversion with respect to "full income". However, the relation to labor earnings is quite different: both the minimal deterrent fine and seclusion time may decrease with the wage rate for a much wider range of consumer´s preferences, provided imprisonment is being applied – conclusion that could be inferred from

[52] Sjoquist (1973), Block and Lind (1975).

previous studies [53] but was not modeled in our terms. That is, for the same offence, the deterrent imprisonment or fine may be decreasing in wages even if not in non-wage wealth [54].

Note that the analysis of a crime-deterrent composite does not shed much light on how crimes should be punished in equitable terms. If it would be common sense to accept that the judicial system (as it roughly does) should apply fines and prison sentences that vary in the same direction of the size of the felony, it would be arguably justifiable that penances should vary inversely with the wage rate. On such terms, we could infer instead that society – and not the Law – should strive to increase the individuals´ wages: if criminal opportunities are equally distributed and individuals´ preferences are homogeneous, there will be a negative link between infractions and wages [55].

It does not tells either whether incarceration should be preferred to fines – technically, their deterrent capacity is (in our framework) the same when the length of the former is adequately indexed to the wage rate. Rather, it points to the fact that attaching an imprisonment sentence and affecting a given apprehension rate to a particular infraction determines the wage and non-labor income status of the marginal infractor that society may expect or accepts for that offence.

As a general additional statement, it stands that (within a uniform penalty system) improving the wage opportunities – by education or training; or by direct subsidization of hours of work - of potential criminals is expected to reduce crimes punished with imprisonment much more effectively than providing them with lump-sum transfers. The latter will only decrease the propensity to commit a crime if risk aversion increases with (non-labor) income; otherwise, as sometimes accepted in economics literature, they will increase it.

3. Effect of uncertainty in wage and non-labor earnings was also inspected.

A general rise of uncertainty in non-labor earnings is expected to increase criminality if a measure of risk-aversion to additional noise in income decreases with income, regardless of whether individuals are risk-averse or risk lovers or of how simple risk aversion changes with income.

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[53] Block and Lind (1975a), for example.

[54] Even if also for other reasons, Becker (1968) arrives at similar conclusions, p. 195196.

[55] The empirical fact that women, even if in general, facing a lower wage than men, are not major criminals would seem inconsistent with this assertion, as noticed by Freeman (1999). Yet, he argues that women may face an extra-deterrent effect due to child-rearing time requirements. Also, infractions facing confinement usually require physical strength, in which women have clear disadvantage – this would be eventually captured in the model in a higher probability of failure and/or a higher degree of risk-aversion.

A rise in wage rate instability will increase criminality if (a proposed measure of) how wage-nonlabor income substitutability in indirect utility functions increases with the wage is diminished by a rise in non-labor income – as some, but not all, theoretical functional forms we inspected (some used in empirical labor supply research) imply for risk-lover consumers with respect to "full income". This scenario would be consistent with seasonal job-holders or higher unemployment inflicted work – at given average wage – presenting higher crime rates, (for reasons other than just availability of time to commit the crime, once crime is not considered here a time-consuming activity). And also with increased wage dispersion empirically observed in the past two [56] decades having affected positively the crime rate – for reasons other than interpersonal comparisons arising from eventual externalities in each individual´s utility function (e.g., envy).

"Compounded mishaps" work differently than just spurious uncertainty, environment invoked in the previous paragraph. General risk aversion with respect to full-income is, in this case, necessary and sufficient to guarantee that criminality decreases with non-labor earnings volatility. With non-labor income substitutability with respect to wages, criminality decreases with uncertainty of the wage rate. That is, when endowment rewards are positively correlated to freedom, if workers are risk-averse or their utility function exhibits wage-nonlabor income substitutability, we expect that higher dispersion in the endowments distributions will decrease criminality.

Arbitrariness in the application of penances, for given mean level of law enforcement, decreases delinquency among risk-averse individuals. It increases risk-lovers criminal activities. That is, uncertainty in punishment, contrary to common belief, may have a deterrent effect: it compounds risk, therefore, it discourages (further) those who do not like it.

The inclusion of multiple arguments in the individual´s utility function is useful for the interpretation of some empirical evidence. It also suggests that disincentives to criminal activities may pass through providing or detracting the adequate substitutes-complements, or noises that affect the primary risk involved in the decision of a potential trespasser. These may not be a substitute for sanctions – they may not even be monetarily valuable -, yet render the latter more effective.

4. In general, forfeiters are expected to have a lower labor supply than complying citizens – if both goods are normal, the loot of the crime buys income as well as leisure in case of success; penalties reduce both – imprisonment also reduces labor supply. Even the "marginal" criminal will if, but not only if, only imprisonment is used as penalty.

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[56] Or one and a half. Portugal has also risen foreign residents, specially of African origin, a low-pay category.

The conclusions drawn were systematically independent - in direct terms – on how labor supply responds to the wage. Rather, the direction of the responses discussed above was more related to features of the concavity in full income or pattern of change with the wage rate of the indirect utility function. In what concerns this study, sign and sensitivity of the response of the corresponding measures to "full income", non-labor earnings and wages proved to be more relevant. Such measures seem to be related to the slope of labor supply and, naturally, how they respond to the endowments conditions second derivatives – hence concavity/convexity – of the labor supply function. Further empirical research on the characteristics of labor supply, implicit indirect utility functions and embedded properties of preferences towards risks – and of eventual differences between offenders and non-offenders - may shed light on the relevance of the model in explaining the patterns of criminality.

5. Slight modification of assumptions allowed us to briefly inspect the consequences of introducing the death sentence (or allowing police forces, or victims, to shoot and kill criminals more freely) as an alternative punishment form, and modeling corruption within the judicial system. An explanation of the long-trend decrease in the application of capital punishment was forwarded. Willingness to bribe the executioners seems to respond to exogenous conditions in the opposite direction of standard delinquency.

Applications to health insurance and compensation to hazardous jobs, based on the parallel effect of imprisonment and sickness or injury recovery and incapacitation, were also outlined.

Further theoretical developments of the approach would suggest the use of a life-cycle labor supply-consumption model. Yet, conclusions and dependence of the results on the preference patterns may not be so clearly derivable.

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