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Structural Estimation of Variety Gains from Trade Integration in a Heterogeneous Firms Framework

VICTOR RIVAS[†]

ABSTRACT

In this article we develop a simple analytically solvable model of heterogeneous firms. The heterogeneous firm framework presented in this paper is particularly suitable for the structural estimation of variety gains from trade integration, as all structural equations for empirical estimations can be directly derived from the theoretical model.

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[†]Department of Economic History, Houghton Street, London WC2A 2AE, United Kingdom. E-mail: v.rivas@lse.ac.uk. The author is grateful to two anonymous referees and to the Editor of the Journal for many useful comments and suggestions on an earlier draft of the paper. The article represents only the author's view, the usual disclaimer applies.

1 INTRODUCTION

We consider an economy with two factors of production: labour, L, and capital, K. These two production factors are supplied inelastically and are mobile between sectors implying that in the long run wages are equalised across sectors. Factors are used for production of goods and services by a continuum of industries I. Within industries goods are produced by a large number of heterogeneous firms, N_I.

2 CONSUMPTION

As usual, the representative consumer's utility is an increasing function of consumed goods produced by one of I industries, each of which supplies a large number of horizontally differentiated varieties, i, of good I. For simplicity, we assume that the upper tier of utility, U, determining consumption of each industry's output takes the Cobb-Douglas form with the fraction of income spent on industry I 's good equal to α_I:[1]

$$U = \int_0^1 \alpha_I \ln Q_I d_I \tag{1}$$

where the quantity consumed, Q_I, is a consumption index defined over the continuum of horizontally differentiated varieties, i, of good I.[2] The lower tier of utility function determining the consumption of good I takes the CES form:

$$Q_I = \left(\int_0^{N_I} q_i^{\frac{\sigma_I - 1}{\sigma_I}} di \right)^{\frac{\sigma_I}{\sigma_I - 1}} \tag{2}$$

where q_i is consumer demand for variety i, N_I is the number of available varieties of good I and σ_I is the constant elasticity of

[1] We use capital Latin letters for variables referring to industry (sector), small Latin letters for variables referring to individual varieties (firms) and Greek letters for denoting parameters of the model.

[2] We use the terms "good", "sector", and "industry", as synonyms while term "variety" is reserved for horizontally differentiated products within an industry.

substitution between varieties with $\sigma_I > 1$. The utility maximisation under income constraint, Y, yields the demand for individual varieties, q_i:

$$q_i = \frac{\alpha_I Y}{P_I}\left(\frac{p_i}{P_I}\right)^{-\sigma_I} \tag{3}$$

where P_I is the dual price index defined over prices of individual varieties, p_i:

$$P_I = \left(\int_0^{N_I} p_i^{1-\sigma_I}\, di\right)^{\frac{1}{1-\sigma_I}} \tag{4}$$

3 PRODUCTION

A continuum of industries I with $I \in (0,1)$ use both factors (labour and capital) for producing goods and services. Factor intensity varies across industries. The index I ranks industries by their relative factor intensities (L_I / K_I), industries with higher I are more labour intensive. In equation (5) the labour intensity in sector I is captured by parameter β_I. It is higher in sectors which use more intensively labour, that is for sectors which are ranked with a higher I.

Production process involves two types of costs: production costs and, for those firms which enter market I, also market entry costs. The production costs are firm-specific - they vary with firm productivity, ϕ_i, with $\phi_i \in (0,\infty)$. In contrast, market entry costs are industry specific and have two components: variable market transaction cost, V_I and fixed market transaction cost, F_I.[3]

[3]Given that all firms share the same industry-specific market entry costs, we use capital letters in order to distinguish them from the firm-specific production costs indexed i.

In order to avoid undue complexity, analogously to the functional form of the utility function we assume that the cost function, tc_i, takes the Cobb-Douglas form:[4]

$$tc_i(\phi_i) = \frac{q_i}{\phi_i} W^{\beta_I} R^{1-\beta_I} (1+V_I) + F_I \tag{5}$$

where W is wage rate for labour, R is capital rental rate and β_I is elasticity of substitution between labour and capital. Constant returns in the technology for forming the composite input indicates that the sum of the share parameters, β_I, equals to one. The presence of fixed market transaction cost, F_I, implies that in equilibrium each firm chooses to produce a unique variety, all of which are consumed due to the love of varieties of consumers.

Profit maximisation implies that the equilibrium output price is equal to a constant mark-up over marginal cost:

$$p_i(\phi_i) = \frac{W^{\beta_I} R^{1-\beta_I} (1+V_I)}{\rho_I \phi_i} \tag{6}$$

where $\sigma_I = 1/(1-\rho_I) > 1$. With pricing rule (6), firm's equilibrium revenue, $r_i(\phi_i)$, is proportional to firm's productivity, ϕ_i:

$$r_i(\phi_i) = \alpha_I Y \left(\frac{\rho_I P_I \phi_i}{W^{\beta_I} R^{1-\beta_I} (1+V_I)} \right)^{\sigma_I - 1} \tag{7}$$

According to equation (7), given productivity ϕ_i, firm revenue, r_i, is increasing in the expenditure share, α_I, allocated to industry, I, increasing in aggregate consumer expenditure (which equals aggregate income, Y), increasing in the industry price index, P_I, which measures the degree of competition in market I, and increasing in ρ_I, which is an inverse measure of the size of the mark-up over marginal cost. Firm

[4]Despite this simplifying assumption, our analysis generalises to any homothetic cost function for which the ratio of marginal cost to average cost is a function of output alone.

revenue is decreasing in own price and hence own (marginal) production costs, $mc_i = W^{\beta_I} R^{1-\beta_I} / \phi_i$.

Under the equilibrium pricing rule (6), firm profits, π_i, equal firm revenue, r_i, scaled by the elasticity of substitution, σ_I, minus fixed market transaction costs, F_I:

$$\pi_i\left(\phi_i\right) = \frac{r_i\left(\phi_i\right)}{\sigma_I\left(1+V_I\right)} - F_I \tag{8}$$

According to equation (8), firm profits, π_i, are increasing in firm revenue, r_i, and decreasing in fixed market transaction cost, F_I.

4 FIRM PRODUCTIVITY

A firm drawing productivity ϕ_i operates in market I if its revenue, $r_i\left(\phi_i\right)$, covers at least the fixed market transaction cost, F_I, that is, $\pi_i\left(\phi_i\right) \geq 0$. This defines a zero-profit productivity cut-off, $\bar{\phi}_I$, for industry I:

$$r_i\left(\bar{\phi}_I\right) = \sigma_I\left(1+V_I\right)F_I \tag{9}$$

Only those firms drawing productivity equal to or above $\bar{\phi}_I$ operate in market i. Substituting firm revenue (7) into zero profit condition (9) yields the zero-profit productivity cut-off, $\bar{\phi}_I$:

$$\bar{\phi}_I = \left(\frac{F_I \sigma_I\left(1+V_I\right)}{\alpha_I Y}\right)^{\frac{1}{\sigma_I - 1}} \frac{W^{\beta_I} R^{1-\beta_I}\left(1+V_I\right)}{\rho_I P_I} \tag{10}$$

where $Y_I = \alpha_I Y$ is the expenditure share on sector I 's goods, MC_I is marginal cost of the firm marginal firm which makes zero profit from entering market I, and V_I are variable market transaction costs.

In order to make the model operational we need to assume a specific functional form of the productivity distribution function. In order to avoid undue complexity, we assume that each of the M_I potential entrants into market I receive their firm-specific productivity draw ϕ_i from a Pareto distribution with probability density, $g\left(\phi_i\right)$:

$$g\left(\phi_i\right) = \frac{\gamma_I}{\phi_i}\left(\frac{b}{\phi_i}\right)^{\gamma_I} \tag{11}$$

and cumulative distribution, $G\left(\phi_i\right)$:

$$G\left(\phi_i\right) = 1 - \left(\frac{b}{\phi_i}\right)^{\gamma_I} \tag{12}$$

where b is the minimum productivity ($\phi_i \in [b,\infty)$) and γ_I is shape parameter ($\gamma_I > \sigma_I - 1$). Parameter γ_I can be interpreted as an inverse measure of firm heterogeneity in sector i. Sectors with lower γ_I are more heterogeneous in sense that more output is concentrated among the most productive firms.

Choosing the units of measurement such that the minimum productivity, b, equals to unity, the probability density function, $g\left(\phi_i\right)$, and the cumulative distribution function, $G\left(\phi_i\right)$, can be rewritten as follows:

$$g\left(\phi_i\right) = \gamma_I \phi_i^{-\gamma_I - 1} \tag{13}$$

$$1 - G\left(\phi_i\right) = \phi_i^{-\gamma_I} \tag{14}$$

where $1 - G\left(\phi_i\right)$ is the probability that firm, i, will enter market, I, which equals to the fraction of operating firms, N_I, over all firms, M_I, in sector I. Equations (13) and (14) allow us to derive explicit expressions for marginal productivity, average productivity of firms entering market I and for average productivity of firms not entering market I.

First, inverting equation (14) we can express the zero-profit productivity cut-off, $\overline{\phi}_I$, as a function of firms entering market I:

$$\bar{\phi}_I = \left[\frac{N_I}{M_I}\right]^{-\frac{1}{\gamma_I}} \tag{15}$$

according to which zero-profit productivity cut-off is decreasing in the fraction of firms entering market i and firm heterogeneity, γ_I. More firms can enter market I only if the marginal productivity, $\bar{\phi}_I$, decreases with respect to productivity of potential entrants.

Second, from the cumulative distribution function (14) we can derive the CES weighted average productivity, $\tilde{\phi}_I$, for all firms operating in market I with productivity $\phi_i > \bar{\phi}_I$.

$$\tilde{\phi}_I = \left[\frac{1}{1-G\left(\bar{\phi}_I\right)}\int_{\bar{\phi}_I}^{\infty}\phi_i^{\sigma_I-1}g\left(\phi_i\right)d\left(\phi_i\right)\right]^{\frac{1}{\sigma_I-1}} \tag{16}$$

which simplifies to

$$\tilde{\phi}_I = \bar{\phi}_I\left[\frac{\gamma_I}{\gamma_I+1-\sigma_I}\right]^{\frac{1}{\sigma_I-1}} \tag{17}$$

The second term on the right hand side, $\left[\frac{\gamma_I}{\gamma_I+1-\sigma_I}\right]^{\frac{1}{\sigma_I-1}}$, is a constant capturing the higher productivity of entrants.[5] Given that the minimum productivity, b, equals to one and $\bar{\phi}_I > b \Rightarrow \tilde{\phi}_I > \bar{\phi}_I > 1, \forall I$.

Finally, from the cumulative distribution function (14) we can also find the CES weighted average productivity, $\tilde{\phi}_I$, for all firms not entering market I with productivity $\phi_i < \bar{\phi}_I$. The derivation of average productivity for firms not entering market I is analogous to (17).

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[5]Under the estimated parameter values it takes a value between 1.25 and 1.75 implying that the average productivity of firms, N_I, entering market I is 25% to 75% higher than the cut-off productivity of a marginal firm, which earns zero profits from entering market I.

5 GENERAL EQUILIBRIUM

From the demand function (3), the optimal pricing rule (6), zero-profit productivity cut-off (10) and average firm productivity (17) we can derive the aggregate market supply function, X_I, of N_I firms operating in sector I:

$$X_I = \underbrace{\lambda_p P_I^{\gamma} Y_I^{\frac{\gamma}{\sigma_I - 1}} M_I}_{\text{size of sector}} \times \underbrace{\left(W^{\beta_I} R^{1-\beta_I}\right)^{-\gamma} F_I^{1-\frac{\gamma}{\sigma_I - 1}} \left(1+V\right)^{1-\gamma-\frac{\gamma}{\sigma_I - 1}}}_{\text{production and transaction costs}} \tag{18}$$

where λ_p is a constant.[6] According to equation (18), sector I 's output is increasing in the size of sector and declining in costs associated with production and market transactions. The aggregate output is increasing in the output price, P_I, for sector I 's goods with an increasing rate $\left(\gamma > 1\right)$, which is the largest elasticity among the right hand side explanatory variables with respect to output. A higher price index indicates a less fierce competition in sector I, which attracts new firms and expands production of the incumbent firms. Sectoral output is also increasing in the sector I 's expenditure share, $Y_I = \alpha_I Y$ with an increasing rate $\left(\frac{\gamma_I}{\sigma_I - 1} > 1\right)$.[7] Finally, the aggregate sector I 's output is increasing in the number of potential entrants (firms) with a constant rate over the entire interval.

As expected, the costs associated with production and market transactions, MC_I, V_I and F_I, have a negative impact on firm production and hence on the aggregate output. Ceteris paribus, the impact of marginal cost on sectoral output is larger than the impact of market transaction costs, V_I and F_I, because value of the elasticity of marginal cost, γ_I, is larger than the elasticity of fixed and variable market transaction costs.

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[6] $\lambda_p = \left(\frac{\sigma_I}{\sigma_I - 1}\right)^{-\gamma} \frac{\gamma}{\gamma - \sigma + 1} \sigma_I^{1-\frac{\gamma}{\sigma_I - 1}}$.

[7] This implies that a higher demand for sector I 's goods (measured in expenditure share) increases the aggregate production at an increasing rate.

6 CONCLUSIONS

In this article we develop a simple analytically solvable model of heterogeneous firms. The heterogeneous firm framework presented in this paper is particularly suitable for the structural estimation of variety gains from trade integration, as all structural equations for empirical estimations can be directly derived from the theoretical model.

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