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The Overall Seasonal Integration Tests Under Non-stationary Alternatives

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ABSTRACT

Few authors have studied, either asymptotically or in finite samples, the size and power of seasonal unit root tests when the data generating process [DGP] is a non-stationary alternative aside from the seasonal random walk. In this respect, Ghysels, lee and Noh (1994) conducted a simulation study by considering the alternative of a non-seasonal random walk to analyze the size and power properties of some seasonal unit root tests. Analogously, Taylor (2005) completed this analysis by developing the limit theory of statistics of Dickey and Fuller Hasza [DHF] (1984) when the data are generated by a non-seasonal random walk. del Barrio Castro (2007) extended the set of non-stationary alternatives and established, for each one, the asymptotic theory of the statistics subsumed in the HEGY procedure. In this paper, I show that establishing the limit theory of F-type statistics for seasonal unit roots can be debatable in such alternatives. The problem lies in the nature of the regressors that these overall F-type tests specify.

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Keywords: Fisher test, seasonal integration, non-stationary alternatives, Brownian motion, Monte Carlo Simulation.

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1 INTRODUCTION

The stochastic nature of the seasonality seems to gain ground in empirical studies. Several aspects related to seasonal unit root tests were treated in the literature. In this respect, the power of these tests against non-stationary alternatives is an important issue that recently acquired some concern. To the best of our knowledge, Ghysels, Lee, and Noh [GLN] (1994) are the first authors who studied this problem. In fact, in a Monte Carlo study, they showed that against a non-seasonal random walk, the power of the tests of Dickey, Hasza and Fuller (1984) lies well lower than that of the tests of Hylleberg, Engle, Granger and Yoo [HEGY] (1990). Ghysels et al. guessed that "the Dickey et al. test may not separate unit roots at each frequency" (p. 432). The restriction behind the Dickey et al. procedure is that all the unit roots (conventional and seasonal roots) have a modulus of one. Thus, it is clear that the conventional random walk does not fulfil this requirement. However, Rodrigues and Osborn (1999) showed that if this restriction holds, the power of the tests of Dickey et al. would have a proper superiority in finite samples with regard to that of the tests of Hylleberg and al. (1990). In an interesting contribution, Taylor (2003) analysed the large sample behaviour of the seasonal unit root tests of Dickey et al. when the data generating process (DGP) is a non-seasonal random walk, i.e. when the series only admits a zero frequency unit root. In such case and as shown by Taylor (2003), all the Dickey et al. statistics have non-degenerate limiting distributions. These results theoretically explain the empirical findings of GLN. Furthermore, Taylor (2005) showed that asymptotically the statistics of the Dickey et al. augmented test will also do not diverge. In the same context, del Barrio Castro (2006) generalized the results of Taylor (2003) to a set of non-stationary alternatives which include the non seasonal random walk. He found also that the Dickey et al. statistics did not have standard limiting distributions and did not diverge. Based on the same methodology, del Barrio Castro (2007) established the limit theory of the statistic of Fisher and those of Student subsumed by the HEGY procedure. Accordingly, he theoretically derived the effect that can asymptotically have one unit root on the others at different frequencies. Following the terminology of Busetti and Taylor (2003), we have, in these situations, "unattended unit roots". However, del Barrio Castro (2007), in his large sample analysis, did not directly consider the effects of non-stationary alternatives on the overall F-type statistic of seasonal integration[1] which is complementally specified for the HEGY procedure by GLN (1994).

The well-known Fisher tests of the overall seasonal integration null hypothesis are those of Kunst (1997) and Hylleberg et al. (1990). Although these two tests are asymptotically related, they have a main difference in the nature of the explanatory variables used in their basic regressions. Indeed, Kunst's regressors are original variables; however, the HEGY procedure involves regressors that are obtained by non-singular linear transformation of those used by Kunst. Consequently, the HEGY regressors show an ultimate property, to wit, the asymptotic orthogonality.

In a recent paper, Osborn and Rodrigues (2002) developed an appealing and unifying approach for deriving asymptotic results regarding the statistics of the most commonly used seasonal unit root tests. The data generating process (DGP) considered by these authors is the seasonal random walk. This approach is based on the use of circulant matrices which could, in seasonal context, retrieve the limit theory of the involved statistics as well as conveniently traducing the dynamics of time series and its evolution across different seasons. In a similar vein, Haldrup, Montanes and Sanso (2005) used this approach to show the effects of outliers on the limit theory of seasonal unit root tests.

However, most often economic time series are not simultaneously affected by all seasonal unit roots. This empirical finding is all the more consolidated since the practitioners jointly use the deterministic seasonality and seasonal unit root tests as advised by Hylleberg (1995). Therefore, it is interesting to consider non-stationary alternatives, aside from the seasonal random walk assumption, in the finite and large sample studies on the part of seasonal unit root tests.

Here I focus on whether the overall F-type statistic limit theory can be directly and rigorously established when the observed series is generated from the non-stationary alternatives treated by del Barrio Castro (2006). I mean by the word "directly" that in establishing the

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[1]In this paper, we adopt the seasonal integration definition of Ghysels and Osborn (2001, p. 43)

limit theory in question, we do not resort to transformations of the involved regressors. More specifically, I show that when the DGP is one of these non-stationary alternatives and the considered regressors do not satisfy the asymptotic orthogonality, the approach proposed by Osborn and Rodrigues (2002) doesn't work.

The paper proceeds as follows. In the following section, I give some preliminaries about the overall F-tests for seasonal unit roots. I expose the most famous ones, namely those of Hylleberg et al. (1990) and Kunst (1997). Next, I specify a possible set of the data generating processes (DGP) for observed quarterly series. The study can be extended to another data observation frequency, but I retain the quarterly case for illustrative purposes. Put differently, considering quarterly time series affords a clear analysis on account of the reduced number of involved unit roots. The third section discusses how specious asymptotic results, regarding F-type statistics for seasonal unit roots in their entirety, can be reached under non-stationary alternatives. This theoretical study is accompanied by a simulation exercise where I allow for possible augmentation with lagged terms of dependent variable in the regression models corresponding to the studied tests, in order to assess their performance against non-stationary alternatives. In the last section, I conclude.

2 OVERALL SEASONAL INTEGRATION TESTS

2.1 The Kunst test

The Kunst test for quarterly time series is based on the following regression

$$\Delta_4 y_t = \alpha_1 y_{t-1} + ... + \alpha_3 y_{t-3} + \delta y_{t-4} + \varepsilon_t, \quad t = 1,...,T, \tag{1}$$

which is an F-type test of the form

$$F^*_{\hat{\alpha}_1,...,\hat{\alpha}_3,\hat{\delta}} = (T-4)(\hat{\varepsilon}_0^{'}\hat{\varepsilon}_0 - \hat{\varepsilon}^{'}\hat{\varepsilon}) / (\hat{\varepsilon}^{'}\hat{\varepsilon}), \tag{2}$$

where $\hat{\varepsilon}_0$ and $\hat{\varepsilon}_1$ are vectors of residuals estimated under the null $H_0 : \alpha_1 = ... = \alpha_3 = \delta = 0$ and alternative hypotheses of the test. I assume without any loss of generality that al the initial values required by Eq. (1) are null. We can remark that Kunst did not divide the numerator of the statistic (2) by 4, the number of restrictions, as we did it to perform a conventional Fisher test.

2.2 The HEGY test

The basic regression for the HEGY test, without any augmentation and with no deterministic terms, is:

$$\Delta_4 y_t = \pi_1 y_{1t-1} + \pi_2 y_{2t-1} + \pi_3 y_{3t-2} + \pi_4 y_{3t-1} + \varepsilon_t, \quad t = 1,...,T, \tag{3}$$

where

$$\begin{aligned} y_{1t} &= (1 + L + L^2 + L^3) y_t, \\ y_{2t} &= -(1 - L + L^2 - L^3) y_t, \\ y_{3t} &= -(1 - L^2) y_t, \end{aligned} \tag{4}$$

with L is the lag operator.

GLN has extended the HEGY approach with a joint test statistic F_{1234} for the null hypothesis, $H_0 : \pi_1 = \pi_2 = \pi_3 = \pi_4 = 0$, implying all unit roots in data observed at quarterly frequency. H_0 is an overall hypothesis for seasonal integration SI (1) in accordance with the notation of Ghysels and Osborn (2001).

Note that we have

$$\begin{bmatrix} y_{1t-1} \\ y_{2t-1} \\ y_{3t-2} \\ y_{3t-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ y_{t-2} \\ y_{t-3} \\ y_{t-4} \end{bmatrix} \tag{5}$$

We can deduce from (5) that the regressors of the Kunst test are non-singular linear transformations of those of the HEGY test. Consequently, the F-type statistics, F_{1234} and $F^*_{\hat{\alpha}_1,...,\hat{\alpha}_3,\hat{\delta}} / 4$, will have the same limit theory. Given that the two statistics are asymptotically related, the analysis is confined to that of Kunst in the sequel.

Hence, we can observe that there exist a little bit differences between the critical values of both statistics. In general, such critical values are tabulated supposing that the DGP of y_t is:

$$y_t = y_{t-4} + e_t . \tag{A.0}$$

In this paper, I assume that the DGP of y_t drawn from one of the following stochastic processes:

$$y_t = y_{t-1} + e_t, \tag{A.1}$$

$$y_t = -y_{t-1} + e_t, \tag{A.2}$$

$$y_t = y_{t-2} + e_t, \tag{A.3}$$

$$y_t = -y_{t-2} + e_t, \tag{A.4}$$

and

$$y_t = -y_{t-1} - y_{t-2} - y_{t-3} + e_t. \tag{A.5}$$

By using the double subscript notation, we can define the following annual vectors:

$$Y_n = (y_{1n}, y_{2n}, y_{3n}, y_{4n})',$$

and

$$E_n = (e_{1n}, e_{2n}, e_{3n}, e_{4n})',$$

where we suppose that $n = 1,...,N$ and in the T observations there is N years, simply let $T = 4N$. To keep matters tractable, I suppose that $Y_0 = (y_{10}, y_{20}, y_{30}, y_{40})' = (0,0,0,0)'$.

The error processes in the alternatives (A.1)-(A.5) follows a stationary AR(p)

$$\phi(L)e_{sn} = v_{sn},$$

where $\phi(z)e_{sn} = 1 - \sum_{i=1}^{p} \varphi_i z^i$ and s=1,…,4.

The roots of $\phi(z) = 0$ all lie outside the unit circle $|z| = 1$. As for the error sequence $\{v_{sn}\}$, it depicts an innovation process with constant conditional variance σ^2 (see Spanos, 2003, p. 443). Similarly to what has been conjectured by del Barrio Castro (2007) regarding to the error structure in the non-stationary alternatives described above, I suppose that the vector E_n has the following dynamics:

$$E_n = \sum_{j=0}^{\infty} \Gamma_j^* V_n,$$

where $v_n = (v_{1n}, v_{2n}, v_{3n}, v_{4n})'$, and I define the sequence of 4×4 matrices as:

$$\Gamma_0^* = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \gamma_1 & 1 & 0 & 0 \\ \gamma_2 & \gamma_1 & 1 & 0 \\ \gamma_3 & \gamma_2 & \gamma_1 & 1 \end{bmatrix} \quad \Gamma_0^* = \begin{bmatrix} \gamma_{j4} & \gamma_{j4-1} & \gamma_{j4-2} & \gamma_{j4-3} \\ \gamma_{j4+1} & \gamma_{j4} & \gamma_{j4-1} & \gamma_{j4-2} \\ \gamma_{j4+2} & \gamma_{j4+1} & \gamma_{j4} & \gamma_{j4-1} \\ \gamma_{j4+3} & \gamma_{j4+2} & \gamma_{j4+1} & \gamma_{j4} \end{bmatrix}, \text{ for } j = 1,2,\ldots..$$

with

$$\gamma(z) = 1 - \sum_{j=1}^{\infty} \gamma_j z^j$$

being the inverse of $\phi(z)$. Finally, $\Gamma^*(1)$ is defined as

$$\Gamma^*(1) = \sum_{j=0}^{\infty} \Gamma_j^*.$$

del Barrio Castro (2006) used the vector of moving average representation in order to express the alternatives (A.i), i =1,…5, in a vector of quarters representations where the observations of each year are stacked in the above defined vectors Y_n et E_n, let

$$(1-B)Y_n = (\Theta_0^i + \Theta_1^i B)E_n, \quad i = 1,2,\ldots,5, \tag{6}$$

where B is the annual backward operator. The 4×4 matrices Θ_0^i and Θ_1^i (corresponding to the alternatives A.1-A.5) are defined as follows

$$\Theta_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \ \Theta_1^1 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ for (A.1),} \tag{7.1}$$

$$\Theta_0^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 1 \end{bmatrix}, \ \Theta_1^1 = \begin{bmatrix} 0 & -1 & 1 & -1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ for (A.2),} \tag{7.2}$$

$$\Theta_0^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \ \Theta_1^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ for (A.3),} \quad (7.3)$$

$$\Theta_0^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \ \Theta_1^4 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ for (A.4),} \quad (7.4)$$

$$\Theta_0^5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \ \Theta_1^5 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ for (A.5)} \quad (7.5)$$

The following result was established by del Barrio Castro (2007):

$$\frac{1}{\sigma\sqrt{N}} Y_{[rN]} \rightarrow_d B_i(r), \ \ B_i(r) = C_i \Gamma^*(1) B(r), \ \ C_i = \Theta_0^i + \Theta_1^i, \ \ i = 1,2,...,5, \quad (8)$$

where the symbol "$\rightarrow_d$" denotes the convergence of probability measures, $B_i(r)$ is a 4×1 vector Brownian motion process with variance matrix $\Omega_i = \sigma^2 C_i \Gamma^*(1)\Gamma^*(1)' C_i'$ and $B(r)$ is a vector Brownian motion with variance matrix $\sigma^2 I_4$. The subscript i corresponds to the alternative (A.i), i = 1,..., 5.

Note that the rank of C_i, $i = 1,...,5,$ is the number of (seasonal) unit roots implied by the process (A.i), i =1,…,5. In order to determine the number of cointegration relations between the quarters, corresponding to every process (A.i), i =1,…,5, we have to subtract from the periodicity of the quarterly data, i.e. 4, the rank of the matrix C_i, $i = 1,...,5$. We can rewrite Eq. (8) more precisely by identifying the stochastic processes $B_i(r)$, $i = 1,2,..5,$ on the grounds that there is always cointegration among the quarters of the time series; see del Barrio Castro (2007, p.915).

3 LIMIT THEORY OF THE KUNST TEST UNDER NONSTATIONARY ALTERNATIVES

I first introduce the following lemma which can be directly deduced from the preceding result of del Castro Barrio (2007) and lemma A.1 of Osborn and Rodrigues (2002).

Lemma. Suppose that the DGP of y_t in (1) is given by the alternatives (A.1)-(A.5) and suppose that the vector $(e_{1n},...,e_{4n}), \forall n,$ satisfies the assumption 1 of Phillips (1986, p.313), we have under the null of the Kunst test as $T \to \infty$

a) $N^{-2}\sum_{n=1}^{N} Y_n Y_n^{'} \to_d \sigma^2 \int_0^1 M_i B(r)B(r)^{'} M_i^{'} dr,\ i = 1,2,...,5.$

b) $N^{-1}\sum_{n=1}^{N} Y_{n-1}\varepsilon_n^{'} \to_d \sigma^2 M_i \int_0^1 B(r)dB(r)^{'} M_i^{'},\ i = 1,2,...,5.$

c) $T^{-2}\sum_{t=1}^{T} y_{t-k}^2 \to_d \dfrac{\sigma^2}{16}\int_0^1 B(r)^{'} M_i^{'} M_i B(r)dr,\ k = 1,...,4,\ \ i = 1,2,...,5.$

d) $T^{-2}\sum_{t=1}^{T} y_{t-k} y_{t-j} \to_d \dfrac{\sigma^2}{16}\int_0^1 B(r)^{'} M_i^{'} H_k^{'} H_j M_i B(r)dr,\ \ k \neq j,\ i = 1,2,...,5.$

e) $T^{-1}\sum_{t=1}^{T} y_{t-k}\varepsilon_t \to_d \dfrac{\sigma^2}{4}\int_0^1 B(r)^{'} M_i^{'} H_k^{'} M_i dB(r),\ \ k = 1,...,4,\ \ i = 1,2,...,5,$

where $\varepsilon_n = (\varepsilon_{1n}, \varepsilon_{2n}, \varepsilon_{3n}, \varepsilon_{4n})'$ and $M_i = C_i \Gamma^*(1)$.

The matrix $H_k,\ \ k = 1,2,3,4,$ is a particular permutation matrix order 4 which produces the following elementary operations: let a matrix K having 4 lines, the operation $H_1 K$ moves the last row of K to the top row of $H_1 K$ and the other rows moved down one place. More generally, $H_i K$ shifts the final iths rows to the top of the matrix while the remaining rows correspondingly moved down. Note that $H_4 = I_4$ (see Golub and Van Loan, 1996, p. 109-112, for details).

The preceding lemma is mathematically correct however its use, under certain circumstances, may not be valid as will be explained in the sequel.

Let denote by $\hat{\alpha}$ the OLS estimator of the vector $\alpha = (\alpha_1, \alpha_2, \alpha_3, \delta)'$ defined in the Eq. (1). When the DGP of y_t in (1) is given by one of the alternatives (A.1)-(A.5), we can achieve this nugatory asymptotic result by using the preceding lemma:

R.1) $\frac{T}{4}(\hat{\alpha} - \alpha) \to_d F^{-1} f$, where

$$F = \begin{bmatrix} \int_0^1 B(r)' M_i' H_1' H_1 M_i B(r) dr & \int_0^1 B(r)' M_i' H_1' H_2 M_i B(r) dr & \int_0^1 B(r)' M_i' H_1' H_3 M_i B(r) dr & \int_0^1 B(r)' M_i' H_1' M_i B(r) dr \\ \int_0^1 B(r)' M_i' H_2' H_1 M_i B(r) dr & \int_0^1 B(r)' M_i' H_2' H_2 M_i B(r) dr & \int_0^1 B(r)' M_i' H_2' H_3 M_i B(r) dr & \int_0^1 B(r)' M_i' H_2' M_i B(r) dr \\ \dots & \dots & \dots & \dots \\ \int_0^1 B(r)' M_i' H_1 M_i B(r) dr & \int_0^1 B(r)' M_i' H_2 M_i B(r) dr & \int_0^1 B(r)' M_i' H_3 M_i B(r) dr & \int_0^1 B(r)' M_i' M_i B(r) dr \end{bmatrix}$$

$$f = \begin{bmatrix} \int_0^1 B(r)' M_i' H_1' M_i dB(r) \\ \int_0^1 B(r)' M_i' H_2' M_i dB(r) \\ \dots \\ \int_0^1 B(r)' M_i' H_3' M_i dB(r) \\ \int_0^1 B(r)' M_i' M_i dB(r) \end{bmatrix} \forall i = 1, 2, \dots, 5.$$

Before starting the spuriousness of the asymptotic result R.1), I should give some explanation regarding the properties of the matrix F: the elements of the main diagonal of F are all equal. Besides, the elements of F along each diagonal line parallel to the principal diagonal are equal. Thus, F is a Toeplitz.

Toeplitz matrices belong to the larger class of persymmetric matrices. A square matrix B of order n is persymmetric if it symmetric about its northeast-southwest diagonal, i.e., $b_{ij} = b_{n-j+1, n-i+1}$ for all i and j.

Moreover, from the properties of the matrices H_k, $k = 1, 2, 3, 4$, it can be shown that the matrix F is also symmetric as well as its inverse. The equation (1) can be written in matrix form:

$$Y = X\alpha + \varepsilon, \tag{9}$$

where $X = \begin{bmatrix} y_0 & y_{-1} & y_{-2} & y_{-3} \\ y_1 & y_0 & y_{-1} & y_{-2} \\ \dots & \dots & \dots & \dots \\ y_{T-1} & y_{T-2} & y_{T-3} & y_{T-4} \end{bmatrix}$ and $\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \dots \\ \varepsilon_T \end{bmatrix}$.

We have also: $\hat{\alpha}-\alpha=(X^{'}X)^{-1}X^{'}\varepsilon$, where

$$X^{'}X=\begin{bmatrix}\sum_1^T y_{t-1}^2 & \sum_1^T y_{t-1}y_{t-2} & \sum_1^T y_{t-1}y_{t-3} & \sum_1^T y_{t-1}y_{t-4}\\ \sum_1^T y_{t-2}y_{t-1} & \sum_1^T y_{t-2}^2 & \sum_1^T y_{t-2}y_{t-3} & \sum_1^T y_{t-2}y_{t-4}\\ \sum_1^T y_{t-3}y_{t-1} & \sum_1^T y_{t-3}y_{t-2} & \sum_1^T y_{t-3}^2 & \sum_1^T y_{t-3}y_{t-4}\\ \sum_1^T y_{t-4}y_{t-1} & \sum_1^T y_{t-4}y_{t-2} & \sum_1^T y_{t-4}y_{t-3} & \sum_1^T y_{t-4}^2\end{bmatrix}, \text{ and}$$

$$X'\varepsilon=\begin{bmatrix}\sum_1^T y_{t-1}\varepsilon_t\\ \ldots\\ \sum_1^T y_{t-4}\varepsilon_t\end{bmatrix}.$$

We have also: $\frac{T}{4}(\hat{\alpha}-\alpha)=\frac{1}{4}(\frac{X^{'}X}{T^2})^{-1}\frac{X^{'}\varepsilon}{T}$. By way of parts c), d) and e) of the preceding lemma and by using the fact that $H_4=I_4$, we can state spuriously the result R.1). This can be explained by the singularity of the matrix F. More clearly, in view of the result R.1), we deduce that the matrices C_i and M_i are singular. The product $H_j^{'}H_k$ is of a simple form, since it usually yields another H_k matrix. Then, the product term $M_l^{'}H_j^{'}H_kM_l$ inherits the singularity from these factors. As a result, F becomes singular.

Now, if we write the Kunst's F-type statistic as follows: $F^*_{\hat{\alpha}_1,\ldots,\hat{\alpha}_3,\hat{\delta}}=\hat{\alpha}^{'}[(S^2)^{-1}X^{'}X]\ \hat{\alpha}$, where S^2 is the OLS estimator of the residual variance in Eq.(1), we can also state erroneously the following result:

R.2) $F^*_{\hat{\alpha}_1,\ldots,\hat{\alpha}_3,\hat{\delta}}\rightarrow_d f^{'}F^{-1}f$

In cases where some of the frequencies do not admit unit roots, the kunst's statistic $F^*_{\hat{\alpha}_1,\ldots,\hat{\alpha}_3,\hat{\delta}}$, or also the HEGY F-type statistic F_{1234}, diverge to plus infinity at rate T; i.e. the Kunst test is consistent under the alternative it is set up for; see Taylor (2005) and GLN (1994). Consequently, the asymptotic result R.2) stating that F-statistic to be $O_p(1)$ is in error.

The approach of Osborn and Rodrigues (2002) cannot be applied to the Kunst test because its regressors oblige us to work in unstable modes. This is why Lai and Wei (1983) and Chan and Wei (1988) used eigenvector transformations to isolate the stable and unstable modes.

I have generated the empirical quantiles of the Kunst test for the processes (A.1)-(A.5) and associated with nominal levels 90%, 95% and 99%. The sample size considered is 4000 (1000 years) and the number of replications is 20000. I have found that these empirical quantiles tend to be infinite. These results are not exposed here but they are available upon request. Consequently, it is possible to predict that in 100% of cases we reject the null hypothesis for the processes (A.1)-(A.5) and for nominal levels of 5% and 1%, as shown by Table 1 which reports the rejection frequencies for a sample size of 100 (25 years) and a number of replications of 20000. Al simulations were done with the software Matlab.

Table 1: Empirical rejection frequencies of Kunst test under nonstationary alternatives

Kunst Test	Processes					
	(A.0)	(A.1)	(A.2)	(A.3)	(A.4)	(A.5)
$F^{*}_{\hat{\alpha}_1,\ldots,\hat{\alpha}_3,\hat{\delta}}$ nom. size 5%	0.095	1	1	1	1	1
$F^{*}_{\hat{\alpha}_1,\ldots,\hat{\alpha}_3,\hat{\delta}}$ nom. size 1%	0.0158	1	1	1	1	1

Notes: Number of replication: 20000, sample size 4N = 100 observations, nom. size: nominal size.

Also, we have augmented regression (1) corresponding to the Kunst test by lagged values of dependent variable. Thus, this regression becomes:

$$\Delta_4 y_t = \alpha_1 y_{t-1} + \ldots + \alpha_3 y_{t-3} + \delta y_{t-4} + \sum_{i=1}^{p} \Delta_4 y_{t-i} + \varepsilon_t, \quad t = 1, \ldots, T. \qquad (10)$$

In Table 2 below we report the power of the augmented Kunst test against the non-stationary alternatives (A.1)- (A.5).

Table 2: Empirical rejection frequencies of the Kunst augmented test under nonstationary alternatives

		Processes					
		(A.0)	(A.1)	(A.2)	(A.3)	(A.4)	(A.5)
nom.size 5%							
	p=2	0.0592	1	1	1	1	1
	P=4	0.0549	1	1	1	1	0.9920
	P=6	0.0522	1	1	0.9980	0.9976	0.9038
nom.size 1%							
	p=2	0.0140	1	1	1	1	0.9998
	P=4	0.0141	1	1	0.9992	0.9991	0.9271
	P=6	0.0121	0.9990	0.9990	0.97770	0.97460	0.6632

We see from the results in Table 2 that the perfect power is maintained in all the alternatives (A.1)- (A.5) even if we increase the number of lagged terms of dependent variable. At this level, a slight exception to this general finding was detected for the alternative (A.5) and for p = 4 or 6. Particularly, and for this alternative, the exception is much clearer for p = 6 and the nominal level 1%. In fact, the test power decreases and reaches a value around 66%.

4 CONCLUSIONS

A large literature has risen on testing for seasonal unit roots during the last two decades. However, the majority of econometricians, treating this topic, have focused on providing the limit theory of the tests for unit roots at the zero, Nyquist and harmonic seasonal frequencies by considering either an additional determinist component or a modified assumption set concerning the error terms which appear in the regression models associated with such tests. Seldom have studies centered on the power of seasonal unit roots against non-stationary alternatives. Ghysels et al. (1994) early set out this problem and, in a simulation study, they guessed that the DHF test may not separate unit roots at each frequency. Having enriched this analysis by a large sample investigation, Taylor (2003) found that the DHF statistics did not diverge to minus infinity when the DGP of the series is a conventional random walk. del Barrio Castro (2006, 2007) considered an extended set of non-stationary alternatives and studied their asymptotic effects on the DHF and HEGY statistics.

In this paper, I showed that resorting to the approach of Osborn and Rodrigues (2002) can be problematic when the non-stationary alternatives, as defined by del Castro Barrio (2007), are taken into account. Appealing as this approach is, it conceives the DGP only as a seasonal random walk. Consequently, it cannot be appropriately used, under such alternatives, when the regressors are original variables and do not satisfy the asymptotic orthogonality. Indubitably, such a property simplifies the establishment of the asymptotic theory of the statistics in question. Moreover, via a simulation study, I found that the Kunst test maintains high power in cases where some of the frequencies do not admit unit roots. In addition, these high-power properties are preserved when I proceeded to augment the regression model of the test with lagged dependent variables. This clearly shows that Kunst's statistics diverge in these situations.

REFERENCES

Busetti, F. and A. M. R. Taylor. 2003. "Testing against stochastic trend and seasonality in the presence of unattended breaks and unit roots." Journal of Econometrics 117: 21-53.

Chan, N.H. and C.Z. Wei. 1988. "Limiting distributions of least squares estimates of unstable autoregressive processes." Annals of Statistics 16: 367-401.

del Barrio Castro, T. 2006. "On the performance of the DHF test against nonstationary alternatives." Statistics and Probability Letters 76: 291-296.

del Barrio Castro, T. 2007. "Using the HEGY procedure when not all roots are present." Journal of Time Series Analysis 28: 910-922.

Dickey, D. A., D.P. Hasza, and W.A. Fuller. 1984. "Testing for unit roots in seasonal time series." Journal of American Statistical Association 79: 355-367.

Ghysels, E. and D.R. Osborn. 2001. The econometric Analysis of Seasonal Time Series. Cambridge University Press.

Ghysels, E., H.S. Lee, and J. Noh. 1994. "Testing for unit roots in seasonal time series: Some theorical extensions and a Monte Carlo investigation." Journal of Econometrics 62: 415-442.

Golub, G. H. and C. F. van Loan. 1996. Matrix Computations. The Johns Hopkins University Press.

Haldrup, N., A. Montanés, and A. Sanso. 2005. "Measurement errors and outliers in seasonal unit root testing." Journal of Econometrics 127: 103-128.

Hylleberg, S. 1995. "Tests for seasonal unit roots general to specific or specific to general?" Journal of Econometrics 69: 5-25.

Hylleberg, S., R.F. Engle, C.W.J. Granger, and B.S. Yoo. 1990. "Seasonal integration and cointegration." Journal of Econometrics 44: 215-238.

Kunst, R. M. 1997. "Testing for cyclical non-stationarity in autoregressive models." Journal of Time series analysis 18: 123-135.

Lai, T.L. and C.Z. Wei. 1983. "Asymptotic properties of general autoregressive models and strong consistency of least-squares estimates of their parameters." Journal of multivariate analysis 13: 1-23.

Montasser, G. E. 2011. "The overall seasonal integration tests under non-stationary alternatives: A methodological note." EERI Research Paper Series No 11/06, Economics and Econometrics Research Institute, Brussels.

Osborn, D. R. and P.M.M. Rodrigues. 2002. The asymptotic distributions of seasonal unit root tests: a unifying approach. Econometric Reviews 21: 221-241.

Phillips, P.C.B. 1986. "Understanding Spurious Regressions in Econometrics." Journal of Econometrcis 33: 311-340.

Rodrigues, P. M. M. and D. R. Osborn. 1999. "Performance of seasonal unit root tests for monthly data." Journal of Applied Statistics 26: 985-1004.

Spanos, A. 2003. Probability Theory and Statistical Inference: Econometric Modeling with Observational Data. Cambridge University Press.

Taylor, A. M. R. 2003. "On the asymptotic properties of some seasonal unit root tests." Econometric Theory 19: 311-321.

Taylor, A. M. R. 2005. "On the limiting behaviour of augmented seasonal unit root tests." Economics Bulletin 3: 1-10.