Journal of Economics and Econometrics Vol. 53, No. 1, 2010 pp. 90-110

ISSN 2032-9652 E-ISSN 2032-9660

Dynamic Complementarities, Efficiency and Nash Equilibria for Populations of Firms and Workers

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ABSTRACT

We consider an economy with two types of firms (innovative and non-innovative) and two types of workers (skilled and unskilled), where workers' decisions are driven by imitative behavior, and thus the evolution of such an economy depends on the initial distribution of the firms. We show that there exists a continuous of high level steady states and only one low level and asymptotically stable equilibrium. There exists a threshold value on the initial number of firms to be overcome it to located in the basin of attraction of one of the high level equilibrium. We show that in each high level equilibrium there coexists a share of innovative firms with a share of non-innovative firms, and a share of skilled workers (human capital) coexisting with a share of unskilled workers. But if the initial share of innovative firms is lower than the threshold value, then the economy evolves to a low level equilibrium wholly composed by non-innovative firms and unskilled workers. Finally, we characterise the equilibria as the evolutionarily stable strategies against a field.

JEL Classification: C72; C79; D83; O10; O12; O30.
Keywords: Imitative Behavior, Poverty Traps, Strategic Complementarities, Two Population Normal Form Game, Threshold Value.

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1 INTRODUCTION

The notion of strategic complementarities are widely studied and well understood. Thus, the complementarity between investment in R&D and innovative firms on the one hand, and human capital accumulation on the other is commonly accepted as one of the engines of sustained growth. In their seminal papers, Nelson and Phelps (1966) and Schultz (1975) show the major role played by education in helping workers to adapt to new technologies as well as fostering their creation. Redding (1996) formalises such idea within a R&D-based growth model originally developed by Aghion and Howitt (1992), to argue for the presence of strong strategic complementarities between investments of workers in education and of firms' in R&D. He can thus demonstrate the likelihood of a development trap when both investment types are inactive. More recently, various models have shown how skilled labour and high-tech firms complement each other to establish a high level equilibrium (see, in particular, Acemoglu, 1997; 1998).

However, while the issues associated with the strategic complementarities between types of firms and of workers are now fairly well understood, their foundations remain not sufficiently analysed. Hereafter, we propose a dynamic game-theoretical approach to study how such strategic complementarities may lead an economy to settle in a high or a low level equilibrium.

As for the economic intuition, the model considers what is likely to happen in LDCs where often a mismatch arises among economic agents (i.e. firms and workers) with different profiles. Mexico, for instance, is a relatively high-tech country compared to most other Latin-American countries, but it is poor in terms of accumulated human capital. Argentina and Uruguay, on the other hand, are examples of relatively good levels of human capital accumulation coupled with little advanced technology. Such empirical observations can be explained as the outcome of the strategic behaviors adopted by firms and workers on the basis of the given distribution of profiles among economic agents, such profiles being defined as high or low.

Strategic behavior works in this way within our model. Assume that potential workers imitate their neighbors in deciding whether to have a high or low profile. More specifically, they decide as to whether to go to a training school in order to become skilled workers, or be to remain unskilled one without incurring any expenses. Such decisions are rational in the sense that they imitate the best performing strategy given the current state of the economy. On the other hand, firms' decisions depend on the composition of labour profiles available in the economy. That is, a firm decides to be innovative through investing in R&D, if the number (or proportion) of skilled workers is "large enough". Thus, our model studies the existence and properties of multiple equilibria in an economy composed by two structured populations (of firms and workers) acting strategically in the way just defined. It shows that, if the percentage of innovative firms is under a certain threshold value, the economy evolves towards a poverty trap with the number of skilled workers decreasing to zero so that, eventually, it is better for the firms not to invest in R&D. On the contrary, if the initial percentage of innovative firms is higher than such threshold value, the economy will evolve to a high level equilibrium. Our main result is that such equilibrium is a steady state of a dynamical system characterised by the fact that mixed populations may coexist: non-innovative with innovative firms, skilled with unskilled workers. This result matches the experience of many developing countries in which there is a mismatch between R&D department and human capital accumulation (see Ros, 2003).

The low level equilibrium (the "poverty trap"), on the other hand, corresponds to a Pareto-dominated Nash equilibrium of a two-population game in normal form, a property which does not hold true for any of the possible high level equilibria.

Our point is that how a country produce does matter, and not only from the point of view of the international competitions. We understand that, there are profound social and economical differences between a country where a significative percentage of its firms invest in R&D and a country where the most of its firms do not invest in R&D. First, firms that invest in R&D, experience very rapid growth and reductions in cost, spark the development of subsequent industries, and increase the productivity of other sectors of the economy. In essence, spillover effects from the innovative firms are more efficient. Second, jobs in innovative firms require a higher skill level and thus pay more than jobs in no innovative firms.

The paper is organised as follow: Section 2 describes the basic, two-population normal form game characterising strategies and payoffs for firms and workers. Section 3 introduces a dynamic imitation mechanism to analyse the evolution of worker's population. In section 4 we analyse the evolutive behavior of an economy as depending upon its initial conditions. In section 5 the relationships between Nash and dynamic equilibria are analysed and the definition is introduced of an evolutionary stable strategy against a field. In section 6, we introduce a market dynamics for firms, while section 7 draws the conclusions.

2 THE MODEL

We consider an economy composed by two populations: workers, W, and firms, F, each population being further structured in two clubs.[1]

- $W-$ population has the S -club of strategists invest in improving their individual skills (becoming skilled workers), and the $NS-$ club of strategists of low-skill workers.
- The $F-$ population has the I-club of strategists of innovative firms, which are technologically advanced or R&D-prone, and the NI-club of non-innovative firms.

The contractual period between types of firms and workers is characterised by the following assumptions:

- Asymmetric information. At the beginning of the contractual period, workers do not know the type of firm that is going to hire them.[2] However the workers need to certify their skill levels, by means a certificate. So, firms know their profile, a leader-follower information kind of situation (see Fudenberg and Tirole, 1991).
- Training cost. To acquire skill the worker incurs a cost CS, while we will assume (only for simplicity) that no cost has to be born by firms in order to become innovative.
- Income. Let us label $B_i(j)$ the gross-benefit of the i-firm hiring the j-worker, for all $i \in \{I, NI\}$ and $j \in \{S, NS\}$. At any firm, the

[1]A club is a voluntary group deriving mutual benefits from sharing one or more of the following: production costs, members' characteristics, or any good characterised by excludable benefits (Sandler and Tschirhart, 1997). In our case, a club shares a common strategy which gives representative payoffs.

[2]Note that a firm can have been innovating in a previous period and to stop being it in the present one, and reciprocally, a traditionally non innovating can be it in the present period.

$S-$ type worker gets a salary s, while the $NS-$ type gets $\overline{s} < s$.

- Skill premia.[3] Assume that the innovative firms I give premia to their workers, at the end of the contractual period, while NI-firms do not share their benefits.[4] Thus, skilled workers, S, engaged with an innovative firm, I, are assumed to receive a premium $\overline{p}$ while unskilled ones receive a premium p, such that $0 < p < \overline{p}$. Thus, $CS > \overline{s}$, i.e. there are not incentives to be a skilled worker if there are no skill premia.

Moreover, there are strategic complementarities between types of firms as well as between types of workers. So:

- If the firm is innovative, the payoff of the skilled worker is greater than the payoff of the unskilled one, i.e.: $\overline{s} + \overline{p} - CS > s + p$.
- If the firm is non-innovative, the payoff of the unskilled worker is at least as good as the payoff of the skilled worker, i.e.: $s \geq \overline{s} - CS$.
- For a skilled worker,then, the payoffs obtained by the innovative firm are greater than those obtained by the non-innovative firm, i.e., $B_I(S) - \overline{p} > B_{NI}(S)$.
- For a unskilled, the benefits of the non-innovative firm are greater than those of the innovative one, i.e.: $B_I(NS) - p < B_{NI}(NS)$.

In summary,for our two population normal form game, the payoff matrix is represented by,

$W \setminus F$	**I**	**NI**
S	$\overline{s} + \overline{p} - CS,\ B_I(S) - (\overline{s} + \overline{p})$	$\overline{s} - CS,\ B_{NI}(S) - \overline{s}$
NS	$s + p,\ B_I(NS) - (s + p)$	$s,\ B_{NI}(NS) - s$

(1)

[3] A seminal paper about the notion of skill premia is Acemoglu (2003).

[4] Recall that workers do not know the type of contracting firm. So, at the beginning of the productive process, each worker does not know if she is going to receive a premium or not. This piece of information is revealed only at the end of the period, once she learns the type of contracting firm.

The expected payoff of the $S-$ type worker, given the chances of being hired either by the I or NI firm, is:

$$E(S) = \text{prob}(I)\left(\overline{s} + \overline{p}\right) + \text{prob}(NI)\overline{s} - CS \quad (2)$$

where $\text{prob}(I)$ represents the probability of being hired by the innovative firm and $\text{prob}(NI)$ the probability of being hired by the non-innovative firm. Analogously:

$$E(NS) = \text{prob}(I)\left(s + p\right) + \text{prob}(NI)s \quad (3)$$

Hence, workers prefer to be $S-$ type strategists if $E(S) > E(NS)$ and viceversa. This latter happens if and only if $\text{prob}(I)$ is large enough, i.e. when

$$\text{prob}(I) > \frac{CS - (\overline{s} - s)}{(\overline{p} - p)} \quad (4)$$

Workers are indifferent between to be skilled or not, if and only if,[5]

$$\text{prob}(I) = \frac{CS - (\overline{s} - s)}{(\overline{p} - p)} \quad (5)$$

Let us label $\text{prob}(I) = P_u = \frac{CS-(\overline{s}-s)}{(\overline{p}-p)}$, and denote the probability for an innovative firm to employ a skilled worker by $\text{prob}(S)$.

Hence, a firm goes innovative if and only if its expected payoff is greater than the expected payoff of being non-innovative, that is, $E\left(I\right) > E\left(NI\right)$ or,

$$\text{prob}(S) > \frac{B_I(NS) - B_{NI}(NS) - p}{B_I(NS) - B_I(S) + B_{NI}(S) - B_{NI}(NS) + \left(\overline{p} - p\right)} \quad (6)$$

Let's label $\text{prob}(S) = \overline{x}_s$. Hence, the threshold level where economic agents, firms and workers, prefer to be of high-profiles is $\left(\overline{x}_s, P_u\right)$.

We find three Nash equilibria, two of them in pure strategies: $A = \left\{S,I\right\}$ and $B = \left\{NS,NI\right\}$, and a mixed strategy Nash equilibrium given by

$$NE = \left(\overline{X}_s,(1-\overline{X}_s);\ P_u,(1-P_u)\right) \quad (7)$$

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[5]Note that, $0 < \frac{CS-(\overline{s}-s)}{(\overline{p}-p)} < 1$ holds.

It follows that the A equilibrium Pareto-dominates equilibrium B while the latter is the risk dominant equilibrium.

In the next sections, we study the dynamic complementarities between profiles of firms and workers. We consider the dynamics of the workers' population when number of innovative firms, salary levels and education costs are held constant. We characterise dynamic equilibria and derive a threshold value beyond which we exit the low level equilibrium.

3 DYNAMIC IMITATION OF WORKERS

Hereafter, we consider populations of firms and of workers both normalised to 1. Hence, $\text{prob}(I) = PI = QI / Q$ where QI is the number of innovative and Q is the total number of firms. Then, $\text{prob}(NI) = PNI = 1 - PI.$

Let R_i be the probability that the $i-$ strategist, $i \in \{S, NS\}$, raises the question as to whether to change her current behavior. Then, R_i denotes the average time-rate at which a worker, currently using strategy $i \in \{S, NS\}$, reviews her choice.[6]

Let P_{ij} be the probability that such reviewing worker really switches to the strategy $j \neq i$. Then,

$$P(i \to j) = R_i P_{ij} \tag{8}$$

is the probability that a worker of the $i-th$ club changes to the $j-th$ one.[7] In the sequel, $e_S = (1,0)$ and $e_{NS} = (0,1)$ indicate vectors of pure strategies, S or NS.

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[6]This is the behaviorial rule with inertia (see Bjornerstedt and Weibull, 1993; Weibull, 1995 and Schlag, 1998; 1999) that allows an individual to reconsider her action only with probability $R \in (0,1)$ in each round.

[7]In a finite population one may imagine that review times of an S -strategist in population W are modeled as the arrival times of a Poisson process with average (across such individuals) arrival rate R_S, and that at each such arrival time the individual selects a pure strategy according to the conditional probability distribution P_{SNS}. Assuming that all individuals' Poisson processes are statistically independent, the probability that any two individuals happen to review simultaneously is zero, and the aggregate of reviewing time in the W player population among S -strategists is a Poisson process. If strategy choices are

Hence, the expected percentage flow of skilled workers, $\dot{X}_S$, will be equal to the percent probability of unskilled changing to skilled workers minus the percent probability of skilled changing to unskilled workers. For large populations, we may invoke the law of large numbers and model these aggregate stochastic processes as deterministic flows, each flow being set equal to the expected rate of the corresponding Poisson arrival process.

Rearranging terms, we get the system of differential equations characterising the dynamic flow of workers

$$\begin{aligned} \dot{X}_S &= R_{NS}P_{NSS}X_{NS} - R_S P_{SNS} X_S \\ \dot{X}_{NS} &= -\dot{X}_S \end{aligned} \qquad (9)$$

where X_S is the fraction of skilled (X_{NS} of unskilled, respectively) workers.

An imitative dynamics, as the one defined by equation system (9), makes sense if there are at least two distinct behaviors, one of them currently adopted and the other one being a candidate behavior to imitate. Needless to say, in this model, if one of the two populations disappears the incentive to change vanishes with it.

Reviewing workers evaluate their current strategy and decide to imitate only the successful one. An evaluation rule that seems fairly natural in a context of simple imitation, is the average rule, whereby a strategy is evaluated according to the average payoff observed in the reference group (see Apesteguia et al., 2007).[8] Then, assume that potential workers do not observe payoffs of individual neighbors but they can, in some way, compute average payoffs in their neighborhoods and imitate the behavior with the highest average value.

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statistically independent random variables, the aggregate arrival rate of the Poisson process of individuals who switch from one pure strategy S to another NS is $R_S P_{SNS}$.

[8]On imitation theory, Vega-Redondo (1997) and Schalg (1998, 1999) pointed out two approaches based on the idea that individual who face repeated choice problems will imitate others who obtained high payoffs. Anyway, the two models differ along two different dimensions, the informational structure ("whom agents imitate") and the behavioral rule ("how agents imitate"). It can be show that the difference between the two models is mainly due to the different informational assumptions rather than the different adjustment rules. So, it is more important whom one imitates than how imitates (see Apesteguia et al., 2007).

Although a worker does not know all true values of the payoff of all the other workers, she can take a sample of true values in order to estimate the average. Let $\bar{E}(i)$ and $\bar{E}(j)$ be the estimators of the true values $E(i)$ and $E(j)$. Hence, an $i-$ worker changes her current strategy if and only if $\bar{E}(i) < \bar{E}(j)$.

Assume that the probability for an $i-$ worker to become a $j-$ type strategist is such that

$$P[\bar{E}(j) - \bar{E}(i) > 0] \tag{10}$$

then, (7) can be written as

$$\begin{aligned} \dot{X}_S &= R_{NS} P[\bar{E}(NS) - \bar{E}(S) < 0] X_{NS} - R_S P[\bar{E}(NS) - \bar{E}(S) > 0] X_S \\ \dot{X}_{NS} &= -\dot{X}_S \end{aligned} \tag{11}$$

Now, let the value $P[\bar{E}(j) - \bar{E}(i) > 0]$ increase proportionally to the true value $E(j)$ if $E(j) > 0,$ and let such probability be equal to zero if $E(j) < 0$, i.e. $\forall \;\; i,j \in \{S, NS\},$

$$P[\bar{E}(j) > \bar{E}(i)] = \begin{cases} \lambda E(j) & if \;\; E(j) > 0 \\ 0 & if \;\; E(j) < 0 \end{cases} \tag{12}$$

where $\lambda = \frac{1}{|E(NS)+E(S)|}$. Recall that the share PI of innovative firms is constant, and that salaries $(\bar{s}, s)$, premiums $(\bar{p}, p)$, and education costs CS are given. Then, $E(S)$ and $E(NS)$ are constant, too.

Recall also that $E(NS) = (PI)(p) + s \geq 0$ while $E(S) = (PI)(\bar{p}) + \bar{s} - CS$ can be positive or negative depending on the values PI and CS. With salaries, prizes and CS given, $E(S) > 0$ if and only if $PI > \frac{CS - \bar{s}}{\bar{p}}$. Let us write

$$\pi = \frac{CS - \bar{s}}{\bar{p}} \tag{13}$$

as the percentage of innovative firms such that $E(S) = 0.$

Hence, equation system (11) can take one of the following forms:

(I) If $E(S) \leq 0$ and then, $P(\bar{E}(S) - \bar{E}(NS) > 0) = 0,$ the evolution of the skilled share in the workers' population is described by

$$\dot{X}_S = -R_S \lambda E(NS) X_S \tag{14}$$

whose solution is

$$X_S(t) = X_S(0)\exp\left(\frac{-R_S E(NS)}{|\ E(NS) + E(S)\ |}\right)t \tag{15}$$

being $X_S(0)$ the fraction of the high-skill workers at time $t = 0$.

The share in the population of skilled workers decreases until it finally vanishes. But this trend can be modified by changing the parameters of the model: a policy maker can implement policies to reduce training (education) costs and to increase the skill premia of skilled workers.

(II) If $E(S) > 0$, on the other hand, the dynamical system takes the form

$$\begin{aligned} \dot{X}_S &= -\left[R_{NS}E(S) + R_S E(NS)\right]\lambda X_S + R_{NS}\lambda E(S) \\ \dot{X}_{NS} &= -\dot{X}_S \end{aligned} \tag{16}$$

Let label $A = \lambda\left[R_{NS}E(S) + R_S E(NS)\right]$ and $B = R_{NS}\lambda E(S)$.

Then, in this case the solution of the differential equation (16) is

$$X_S(t) = \left(X_S(0) - \frac{B}{A}\right)\exp\left(-At\right) + \frac{B}{A} \tag{17}$$

where $\frac{B}{A} = \frac{R_{NS}E(S)}{R_{NS}E(S) + R_S E(NS)}$. (18)

Note that the share of skilled workers converges to $\frac{B}{A}$. By substitution of expected payoffs, $E(\cdot)$, we get

$$\frac{B}{A} = \frac{R_{NS}\left[\left(PI\right)\left(\overline{p}\right) + \overline{s} - CS\right]}{R_{NS}\left[\left(PI\right)\left(\overline{p}\right) + \overline{s} - CS\right] + R_S\left[\left(PI\right)\left(p\right) + s\right]} \tag{19}$$

1. Considering B / A as a function of the initial percentage on innovative firms PI, its value increases with PI.
2. Notice that, even in the case of all firms being innovative, i.e.: $PI = 1$, it does not follow that at the limit, all workers are going to be high-skill. In this case, at equilibrium their share is

$$B / A = \frac{R_{NS}\left[\overline{p} + \overline{s} - CS\right]}{R_{NS}\left[\overline{p} + \overline{s} - CS\right] + R_S\left[p + s\right]} \tag{20}$$

3. A particularly interesting case is where $PI = P_u = \frac{CS-(\bar{s}-s)}{(\bar{p}-p)}$. Here, the share of innovative firms is such that workers are indifferent between being skilled or unskilled. As $P_u > \pi$, the economy is evolving to a high level equilibrium where

$$\frac{B}{A} = \frac{R_{NS}}{R_{NS} + R_S} \tag{21}$$

is the limit value of the share of skilled workers.

4 INITIAL CONDITIONS MATTER

Does the initial number of innovative firms explain the path of the economy? Consider two countries, 1 and 2. Assume the respective percentage of innovative firms in $t = t_0$ to be: $PI_1 > PI_2$ so that, from the solution of equation (16), the share of skilled in the workers' population in country 1 is, for each $t > t_0$, larger than in country 2, i.e.,

$$X_{1S}(t) > X_{2S}(t),\ \forall t > t_0 \tag{22}$$

then, the equilibrium state is higher in country 1 than in country 2.

Figure 1: Evolution and steady states, initial condition matter

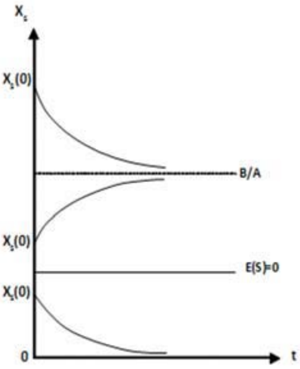


Figure 1 shows the evolution of the dynamical system when the initial percentage of the innovative firms is above or below such threshold value:

1. if $PI > \pi$, then:

- if $X_S(0) > \frac{B}{A}$, skilled workers decrease in the population and their share converges to $\frac{B}{A}$,
- if $X_S(0) < \frac{B}{A}$, skilled workers increase, instead (converging to $\frac{B)}{A}$. In both cases the economy converges to the high level equilibrium.

2. if $PI \leq \pi$:
   - the share of skilled workers is decreasing to zero, $X_S(0) \to 0$. In this case, the economy is in a poverty trap, and the rational behavior on the part of the workers is to opt for being low-skill, and for the firms to be non-innovative. This is the only asymptotically stable Nash equilibrium for the game above.

The foregoing theorem summarises our results.

Theorem 1 *Consider the dynamic flow of workers, given by the system (9). There exists a threshold value,* $\pi = \frac{CS-\bar{s}}{\bar{p}}$, *such that*

1. *If the initial number of innovative firms* PI *is larger than this value, i.e.,* $PI > \pi$ *then, the percentage of skilled workers* $X_S(t)$ *converges to* $\frac{B}{A}$.
2. *If the initial number of innovative firms verifies* $PI \leq \pi$, *then, the percentage of skilled workers* $X_S(t)$ *converges to* 0.

Proof: Is a straightforward conclusion from the solutions of the dynamical systems (16), corresponding the the case $E(S) > 0$ and (14), corresponding to $E(S) \leq 0$.

Definition 1 *Let* Π *the percentage of non-innovative firms in a given economy in time* $t = t_0$ *and let* π *be the threshold value for the economy. Let us now to define the index of potential evolution of the economy:*

$$U = \frac{PI}{\pi} \tag{23}$$

As shown in the following corollary, this number summarises the main characteristics of the potential evolution of the given economy.

Corollary 2 *If the index $U \leq 1$ then the economy is in a poverty trap, i.e., converges to the the low equilibrium where al worker is no skill worker and all firms are no innovative. If the index $U > 1$ then the economy has overcome the poverty trap, and converges to a high level equilibrium, the main characteristics of this equilibrium is given by the quotient B / A given by equation (19).*

Generically, an economy can be located either in a poverty trap or in a high-level equilibrium, depending upon the relation between the share of innovative firms and certain parameters (training costs and premia) of the model. Such relation is summarised by the index of location U.

In our setup, an institutional policy tending to increase the value of U tends also to shrink the basin of attraction of the low equilibrium. Thus, a policy-driven change in the parameters, in the present case by reducing education costs and/or increasing skill premia, may help the economy out of the latter's basin of attraction.

5 DYNAMIC EQUILIBRIA, NASH EQUILIBRIA AND THE EVOLUTIONARY STABLE STRATEGY

There is no possibility to observe the high Nash equilibrium (in pure strategies) $\left(S, I\right) = \left(1,0;1,0\right)$ as it is not a dynamic equilibrium. On the contrary, the low Nash equilibrium in pure strategies $\left(NS, NI\right) = \left(0,1;0,1\right)$ is asymptotically stable, and then the poverty trap arises as a result of the rational conduct of economic agents.

Let us now introduce the concept of an *evolutionary stable strategy against the field* given a profile distribution of the firms' population denoted by y.

Let Δ^w be the set of distributions on the workers' population, and Δ^F be the set of distributions on the firms'. Let $x_w = (x_s, x_{ns}) \in \Delta^w$ be a given distribution on the workers' population and $y_f = (y, 1-y) \in \Delta^F$ a given distribution on the population of firms. Consider a perturbation on the initial distribution y. Let y_ε be the perturbed distribution, let $\varepsilon > 0$ be small enough that the Euclidean distance $| y_f - y_\varepsilon | < \varepsilon$.

Definition 2 Let x_w be a best response against y_f. We say that the distribution on the population of workers x_w, is an evolutionary stable strategy against the field given by y_f, a distribution on the population of the firms, if there exist $\varepsilon > 0$ such that x_w continues being a best response against all distribution y_ε in a neighborhood V_ε of radium ε, centered at y.

Intuitively, this means that, x_w is the unique best response against y_f and that it does better than any other distribution against perturbations (in the distributions of the *field*).

Notice that, when $y \leq \pi$, the distribution $x_w = (0,1)$ (i.e. all workers are unskilled) is an ESS against the field given by y_f.

6 ON THE DYNAMICS OF FIRMS

Until now we have assumed that the percentage of innovative, non-innovative firms is fixed. Workers choose their best responses in a give situation, but is natural to assume that the percentage of innovative firms are changing. We assume now that skilled workers are a fixed input for firms, and when the restriction for this input changes firms maximising again, and now taking account of the new restriction in this input they choose between to be innovative or no-innovative.

The following assertion taken from Ezell and Atkinson (2008) summarise the main results of this section: "Technological and scientific innovation is the engine of U.S. economic growth, and human talent is the main input that generates this growth."

To focus on this strategic complementarities, let us suppose that innovative firms have the production function

$$y = f(z, x_s, x_{ns}) \tag{24}$$

where z is the technology, x_s the number of skilled and x_{ns} of unskilled workers employed by the firm, and y output. Suppose that technology as an input is complementary to skilled labour.[9] Hence, the marginal product of the technology is an increasing function of the number of skilled workers.

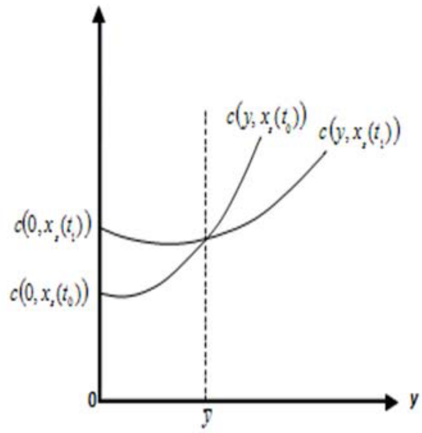
[9]For instance $y = z^\alpha x_s^\beta + x_{ns}$ where $0 < \alpha, \beta < 1$.

Let $x_s(t)$ be the total amount of skilled workers hired by innovative firms at time t. Let $t_0 < t_1$ and assume the amount of skilled workers be increasing over time, i.e. $x_s(t_0) < x_s(t_1)$. Then, from our hypothesis on the technology, follows that

$$\frac{\partial c(y, x_s(t_1))}{\partial y} \leq \frac{\partial c(y, x_s(t_0))}{\partial y} \tag{25}$$

where $c(y, x_s)$ stands for the short run cost function. Hence, there exists $\bar{y}$ such that $c(y, x_s(t_0)) > c(y, x_s(t_1))\ ; \forall y > \bar{y},$ Figure 2 offers a graphic representation.

Figure 2: Short rum costs with increasing disposal of the input skilled workers



Then, if the supply of skilled workers is increasing, short run costs for innovative firms decrease towards the long-run cost. Innovative firms can cash positive profits and there are incentives for non innovative firms to change their decisions.

The following reason reinforces the above argument on the evolution of the firms. *Innovative firms require skilled workers whereas non-innovative firms prefer unskilled ones*, but the number of the latter decreases when the number of innovative firms is increasing. A positive net flow from unskilled to skilled workers would be observed as a consequence of an increasing process of innovation while this same process will be enhanced by an increasing supply of skilled labour.

6.1 Example

To understand the situation just described let us take the following case: Assume the firms to be characterised by the technological function:

$$f(z, x_s, x_{ns}) = kz^{\alpha} x_s^{\beta} + x_{ns}^{\lambda} \tag{26}$$

Where:

$$k = \begin{cases} H & \text{if the firm is innovative} \\ h & \text{if the firm is not innovative} \end{cases}$$

$H > h > 0$ and α, β and λ are positive constants such that $\alpha + \beta = 1$ and $\lambda < 1$

Assume that the technology $z = \overline{z}$ is a given positive constant and that skill premia (the bonus for the skilled worker) are pr. It is easy to see that the short rum cost function is:

$$C(x_{ns}, y, \overline{z}, \overline{x}_s) = (w_s + pr)x_s + w_{ns}\left[y - k\overline{z}^{\alpha} x_s^{\beta}\right]^{\frac{1}{\lambda}} \tag{27}$$

It follows that:

$$C_y'(x_{ns}, y, \overline{z}, \overline{x}_s) = w_{ns}\frac{1}{\lambda}\left[y - k\overline{z}^{\alpha} x_s^{\beta}\right]^{\frac{1}{\lambda}-1}$$

$$C_{y,x_{ns}}''(x_{ns}, y, \overline{z}, \overline{x}_s) = -w_{ns}(\frac{1}{\lambda} - 1)\frac{1}{\lambda}\left[y - k\overline{z}^{\alpha} x_s^{\beta}\right]^{\frac{1}{\lambda}-2} k\overline{z}^{\alpha} x_s^{\beta-1} < 0$$

Then, for innovative firms the cost decreases with the supply x_s of skilled worker faster than for non-innovative ones. So, if at $t = t_0$ the fraction of innovative firms is greater than the threshold value π, the innovative firms can reduce their costs more quickly than non-innovative firms.

Assume that the market price for the final product is p. If firms are competitive, the optimal supply for each firm is given by:

$$\begin{aligned} Y_I^* &= pHz^{\alpha} x_{Is}^* + x_{Ins}^* \\ Y_{NI}^* &= phz^{\alpha} x_{NIs}^* + x_{NIns}^* \end{aligned} \tag{28}$$

Where x_{is}^* and x_{ins}^*, $i \in \{I, NI\}$ stand for the long run demand of inputs for innovative and not innovative firms:

$$x^*_{Ins} = x^*_{NIns} = \left(\frac{w_{ns}}{\lambda p}\right)^{\frac{1}{\beta-1}}, x^*_{Is} = \left(\frac{w_s + pr}{\lambda p H z^\alpha \beta}\right)^{\frac{1}{\beta-1}}, x^*_{NIs} = \left(\frac{w_s + pr'}{\lambda p h z^\alpha \beta}\right)^{\frac{1}{\beta-1}} \quad (29)$$

Let $PI > \pi$ be the number of innovative firms existing at $t = t_0$ and let $X(p)$ be the demand for the final product. The total supply $S(p)$ of the innovative firms will be equal to

$$S(p) = (PI)Y^*_I$$

The number of non innovative firms, at the same time, will be equal to

$$\max\left\{\frac{X(p) - S(p)}{Y^*_{NI}}, 0\right\}$$

Therefore, in the long run, a positive share of innovative firms can coexist with non-innovative ones. To see this, assume that there is a cost to become innovative, $C(h,H)$. Thus, a non-innovative firm has incentive to become innovative if and only if, the benefits are such that:

$$B(NI) < B(I) - C(h,H)$$

This possibility depends, among other things, on the market share the firm can obtain. Were $B(I) - C(h,H) < B(NI)$, the firm would prefer to continue as before.

7 CONCLUSIONS

We have constructed a game theoretical model of the strategic complementarities between types of firms and workers. Workers follow an imitative behavior and firms decide to invest or not in R&D depending on the conditions of labour supply.

As in Accinelli et al. (2007) we shown that, to avoid or to exit a poverty trap, it is necessary to surpass threshold values in human capital and in investment in R&D. In this work, we have shown that rationality on the part of economic agents is not sufficient to avoid poverty traps. Only when initial conditions happen to lie beyond threshold values, rationality leads to an increase in social welfare. Workers will have, then, incentives to improve their skills, while firms would rip greater benefits by investing in R&D: rationality would be associated with a Pareto superior equilibrium. In all other cases, the economy would be going to a poverty trap.

On the other hand, as we have also shown that there is a continuum of high equilibria, each associated with a distinct percentage of innovative firms between the threshold value and 1, we may also think of a continuum of countries which may be in high equilibrium though with different proportions of innovative firms and skilled workers.

In the real world, markets imperfections, costs associated with changes in attitude and myopia on the part of rational agents, render useful the action of a central planer looking at the economy as a whole. In developed economies, a central planner trying to improve the equilibrium level, needs to improve the industry's overall efficiency, for instance by designing mechanisms that promote substitution of non innovative with innovative firms. In less developed economies, a central planner would need to find correct initial conditions such that rationality drives the economy toward the Pareto superior equilibrium. However, were she wish to help that country to exit a poverty trap, she would also another option: to implement a policy that reduces the threshold value π in such way that new feasible trajectories enter the basin of attraction of a high equilibrium. This objective may be attained by reducing educational costs or by introducing incentives for innovative firms to raise their premium for skill. On the basis of our model, the closer a country gets to the threshold, the more growth-enhancing becomes the contribution of investment in education.

In summary, policy makers should find the right mechanism inducing the parties to choose efficient behavior. It is known that policy differences can help us to understand differences in the degrees of development across countries and over time.

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